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All India Council for Technical Education



Mechanics of Materials and their Testing

R. Velmurugan, P. Pragathi

II Year Diploma level book as per AICTE model curriculum
(Based upon Outcome Based Education as per National Education Policy 2020).

The book is reviewed by **Dr. Satish C. Sharma.**

Mechanics of Materials and their Testing

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FOREWORD

Engineers are the backbone of any modern society. They are the ones responsible for the marvels as well as the improved quality of life across the world. Engineers have driven humanity towards greater heights in a more evolved and unprecedented manner.

The All India Council for Technical Education (AICTE), have spared no efforts towards the strengthening of the technical education in the country. AICTE is always committed towards promoting quality Technical Education to make India a modern developed nation emphasizing on the overall welfare of mankind.

An array of initiatives has been taken by AICTE in last decade which have been accelerated now by the National Education Policy (NEP) 2020. The implementation of NEP under the visionary leadership of Hon'ble Prime Minister of India envisages the provision for education in regional languages to all, thereby ensuring that every graduate becomes competent enough and is in a position to contribute towards the national growth and development through innovation & entrepreneurship.

One of the spheres where AICTE had been relentlessly working since past couple of years is providing high quality original technical contents at Under Graduate & Diploma level prepared and translated by eminent educators in various Indian languages to its aspirants. For students pursuing 2nd year of their Engineering education, AICTE has identified 88 books, which shall be translated into 12 Indian languages - Hindi, Tamil, Gujarati, Odia, Bengali, Kannada, Urdu, Punjabi, Telugu, Marathi, Assamese & Malayalam. In addition to the English medium, books in different Indian Languages are going to support the students to understand the concepts in their respective mother tongue.

On behalf of AICTE, I express sincere gratitude to all distinguished authors, reviewers and translators from the renowned institutions of high repute for their admirable contribution in a record span of time.

AICTE is confident that these outcomes based original contents shall help aspirants to master the subject with comprehension and greater ease.


(Prof. T. G. Sitharam)

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This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thought to further develop the engineering education in our country. Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

Dr. R. Velmurugan

Dr. P. Pragathi

PREFACE

Mechanics of materials forms the backbone of engineering disciplines, underpinning the design and analysis of structures, machines, and devices that shape our modern world. Understanding the principles of mechanics of materials is crucial for aspiring engineers but also essential for professionals in fields ranging from civil and mechanical engineering to aerospace and materials science.

This book, "Mechanics of Materials and Their Testing," is crafted with a primary focus on providing a comprehensive understanding of these fundamental principles, catering to the needs of diploma students in engineering disciplines. Whether you are a student embarking on your academic journey, an instructor guiding aspiring minds, or a practitioner in the field, this book aims to be your reliable companion, offering clear explanations, practical examples, and insightful discussions to deepen your understanding of mechanics of materials and their testing.

Key Features of the Book

Comprehensive Coverage: *This book provides an in-depth exploration of essential topics in mechanics of materials, including moment of inertia, stress and strain analysis, shear force and bending moment diagrams, and the calculation of bending and shear stresses in beams and columns.*

Accessible Language: *We have strived to present complex concepts in an easily understandable language, ensuring that students at the diploma level can grasp the fundamental concepts with confidence.*

Practical Insights: *Real-world examples and case studies are integrated throughout the book to bridge the gap between theoretical knowledge and practical application. These insights will help students appreciate the relevance and significance of the subject matter.*

Illustrative Visuals: *The book is enriched with diagrams, graphs, and illustrations that visually demonstrate important concepts, aiding in comprehension and retention of material. These visuals serve as valuable tools for enhancing, understanding and fostering intuitive grasp of abstract principles.*

Exercises and Problem Solving: Each chapter includes exercises and problems that encourage active learning and application of the concepts. Solutions to selected problems are provided to assist students in their self-assessment and progress.

Target Audience: This book is specifically tailored to meet the needs of diploma level students pursuing studies in engineering disciplines. The content is structured to align with the curriculum requirements of diploma-level courses, ensuring that students can easily comprehend and apply fundamental concepts in mechanics of materials and their testing.

Acknowledgement: We extend our sincere appreciation to the educators, students, and professionals whose insights, expertise, and invaluable feedback have greatly contributed to the development of this book on mechanics of materials and their testing. Their dedication to advancing the understanding of material behaviour under various loads and stresses has been pivotal in shaping this comprehensive resource.

We invite all readers to delve into the pages of "Mechanics of Materials and Their Testing for Diploma Students." May this book serve as a beacon, guiding readers towards a deeper comprehension of the principles governing the mechanics of materials and inspiring the engineers and innovators of tomorrow.

Dr. R. Velmurugan

Dr. P. Pragathi

OUTCOME BASED EDUCATION

For the implementation of an outcome-based education the first requirement is to develop an outcome-based curriculum and incorporate an outcome-based assessment in the educational system. By going through the outcome-based assessments, evaluators will be able to evaluate whether the students have achieved the outlined standard, specific and measurable outcomes. With the proper incorporation of outcome-based education there will be a definite commitment to achieve a minimum standard for all learners without giving up at any level. At the end of the programme running with the aid of outcome-based education, a student will be able to arrive at the following outcomes:

Programme Outcomes (POs) are statements that describe what students are expected to know and be able to do upon graduating from the program. These relate to the skills, knowledge, analytical ability, attitude and behaviour that students acquire through the program. The POs essentially indicate what the students can do from subject-wise knowledge acquired by them during the program. As such, POs define the professional profile of an engineering diploma graduate.

National Board of Accreditation (NBA) has defined the following seven POs for an Engineering diploma graduate:

PO1. Basic and Discipline specific knowledge: Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specialization to solve the engineering problems.

PO2. Problem analysis: Identify and analyse well-defined engineering problems using codified standard methods.

PO3. Design/ development of solutions: Design solutions for well-defined technical problems and assist with the design of systems components or processes to meet specified needs.

PO4. Engineering Tools, Experimentation and Testing: Apply modern engineering tools and appropriate technique to conduct standard tests and measurements.

PO5. Engineering practices for society, sustainability and environment: Apply appropriate technology in context of society, sustainability, environment and ethical practices.

PO6. Project Management: Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.

PO7. Life-long learning: Ability to analyse individual needs and engage in updating in the context of technological changes.

COURSE OUTCOMES

By the end of the course the students are expected to learn:

CO-1: Understanding and applying the concept of moment of inertia in engineering and physics to analyze structural stability.

CO-2: Fundamental understanding of the concepts of stress, strain and material properties.

CO-3: Understand the load transferring mechanism in beams and stress distribution due to the shear force and bending moment.

CO-4: Understand the principal stresses and strains, theories of failures of structural members.

CO-5: Basic understanding of the method of superposition, flexibility method, and stiffness method as applied to statically determinate and indeterminate axial and torsional members, bending of beams and buckling of columns.

Mapping of Course Outcomes with Programme Outcomes to be done according to the matrix given below:

Course Outcomes	Expected Mapping with Programme Outcomes (1-Weak Correlation; 2- Medium Correlation; 3- Strong Correlation)						
	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO-1	3	3	3	3	1	2	3
CO-2	3	3	3	3	1	2	3
CO-3	3	3	3	3	1	2	3
CO-4	3	3	3	3	1	2	3
CO-5	3	3	3	3	1	2	3

GUIDELINES FOR TEACHERS

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraint, they should manoeuvre the time to the best advantage of all students.
- They should assess the students only based on the certain defined criterion without considering any other potential ineligibility to discriminate them.
- They should try to improve the learning abilities of the students to a certain level before they leave the institute.
- They should ensure that all the students are equipped with the quality knowledge as well as competence after they finish their education.
- They should always encourage the students to develop their ultimate performance capabilities.
- They should facilitate and encourage group work and team work to consolidate newer approaches.
- They should follow Blooms Taxonomy in every part of the assessment.

Bloom's Taxonomy

Level	Teacher should check	Student should be able to	Possible Mode of Assessment
Create	Students ability to create	Design or Create	Mini project
Evaluate	Students ability to justify	Argue or Defend	Assignment
Analyse	Students ability to distinguish	Differentiate or Distinguish	Project/Lab Methodology
Apply	Students ability to use information	Operate or Demonstrate	Technical Presentation/ Demonstration
Understand	Students ability to explain the ideas	Explain or classify	Presentation/Seminar
Remember	Students ability to recall (or remember)	Define or Recall	Quiz

GUIDELINES FOR STUDENTS

Students should take equal responsibilities in implementing the OBE. Some of the responsibilities (not limited to) for the students in OBE system are as follows:

- Students should be well aware of each UO before the start of a unit in each and every course.
- Students should be well aware of each CO before the start of the course.
- Students should be well aware of each PO before the start of the programme.
- Students should think critically and reasonably with proper reflection and action.
- Learning of the students should be connected and integrated with practical and real-life consequences.
- Students should be well aware of their competency at every level of OBE.

ABBREVIATIONS AND SYMBOLS

List of Abbreviations

General Terms			
Abbreviations	Full form	Abbreviations	Full form
<i>CG</i>	<i>Centre of gravity</i>	<i>cm</i>	<i>Centimetre</i>
<i>MI</i>	<i>Moment of inertia</i>	<i>m</i>	<i>meter</i>
<i>SF</i>	<i>Shear Force</i>	<i>mm</i>	<i>millimeter</i>
<i>BM</i>	<i>Bending moment</i>	<i>GPa</i>	<i>Giga Pascal</i>
<i>UTM</i>	<i>Universal testing machine</i>	<i>MPa</i>	<i>Mega Pascal</i>
<i>SI</i>	<i>System International</i>	<i>kN</i>	<i>Kilo Newton</i>

List of Symbols

Symbols	Description	Symbols	Description
a	Rankine's constant	Z	Section modulus
C	Shear modulus (N/mm ²)	r, θ	Polar co-ordinates
E	Young's modulus (N/mm ²)	α	Coefficient of linear expansion (J°C)
e	Linear strain	θ, β	Angle (rad)
$C.G$	Centre of gravity, Centroid of area	Z	Section modulus
g	Acceleration due to gravity	μ	Poisson's ratio
L_E	Effective length of a column	η	Efficiency
M, m	Mass (kg)	ϵ	Strain
P_{cr}	Critical load of a column	ρ	Density (kg/m ³)
Φ	Shear strain	δl	Change in length
ω	Angular velocity (rad/s)	L/r	Slenderness ratio
σ	Normal stress (N/mm ²)	ΔT	Change in temperature
τ	Shear stress (N/mm ²)	Z	Section modulus

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1

MOMENT OF INERTIA

UNIT SPECIFICS

Upon completion of this chapter, the reader will have acquired the following abilities:

- *Calculate the centroid of the area of the specific section*
- *Identify moments of inertia and second moments of area.*
- *Describe the theorems of perpendicular and parallel axes.*
- *Calculate the second moment of area of the specific section.*
- *Determine the radius of gyration of the specific section.*
- *Determine the second moment of area of symmetrical and unsymmetrical sections.*
- *Describe section modulus and determine its value of a specific section.*

This unit is structured to provide readers with a comprehensive understanding of the foundational aspects of mechanics, particularly focusing on centroid, moment of inertia, and related concepts. It begins by introducing the fundamental principles of centroid and center of gravity, essential for analyzing the distribution of mass within various shapes and sections. Subsequently, readers are guided through the calculation of centroid for specific geometric shapes such as triangles, semicircles, and composite sections, enabling them to accurately determine the center point of mass in different scenarios. Furthermore, readers will explore the

practical applications of moment of inertia in engineering design, as well as related concepts such as radius of gyration, section modulus, and the perpendicular and parallel axis theorems. Through a systematic approach, this unit aims to equip readers with the foundational knowledge and analytical skills necessary for tackling complex mechanics problems and engineering challenges effectively.

Apart from this at the end of the unit, a succinct recapitulation of the overall broad concepts is provided in the form of a unit summary. Besides, a large number of multiple-choice questions as well as descriptive-type questions with Bloom's taxonomy action verbs are included. A list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge. Video resources along with QR codes are mentioned for getting more information on various topics of interest which can be surfed or scanned through mobile phones for viewing.

RATIONALE

The concept of moment of inertia is a fundamental principle in physics and engineering, extensively utilized across various disciplines. In engineering, it plays a pivotal role in structural design, mechanical engineering, and dynamics. Moment of inertia represents an object's resistance to rotational motion about a specific axis and is calculated based on the mass distribution of the object relative to the axis. In structural engineering, the moment of inertia helps determine a beam's ability to resist bending and torsional forces, crucial for designing stable and reliable structures like bridges, buildings, and mechanical components. Beyond

engineering, moment of inertia finds applications in physics, astronomy, and robotics, where it aids in understanding rotational dynamics, planetary motion, and the behaviour of robotic manipulators. Overall, the concept of moment of inertia serves as a cornerstone in various fields, providing valuable insights into the behaviour of rotating systems and facilitating the design and analysis of a wide range of mechanical and structural systems.

PRE-REQUISITES

Before reading this unit, the students are advised to revisit the following:

Engineering Mechanics: Sem I

UNIT OUTCOMES

After completing this unit, you will be able to:

U1-O1: Understand the principles of centroid and center of gravity, and calculate the centroids of different geometric sections, including triangles, semicircles, parabolic spandrels, and composite sections.

U1-O2: Define the unit of moment of inertia and apply it to solve engineering problems.

U1-O3: Compute moment of inertia through integration techniques and understand its application in structural analysis and design.

U1-O4: Determine the radius of gyration and section modulus for different shapes and configurations, aiding in structural optimization.

U1-O5: Calculate the moment of inertia for different sections including rectangular, hollow rectangular, circular, hollow circular, triangular, I-section, T-section, semi-circular, composite, and build-up sections, enabling you to analyze and design structural components effectively.

Unit-1 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES <i>(1-Weak Correction; 2- Medium Correction; 3- Strong Correction)</i>				
	CO-1	CO-2	CO-3	CO-4	CO-5
U1-O1	3	1	1	1	2
U1-O2	3	1	1	1	2
U1-O3	3	1	1	1	2
U1-O4	3	1	1	1	2
U1-O5	3	1	1	1	2

1.1 INTRODUCTION

When any structural member experiences external loads, it undergoes stress and strain. In some situations where structures are exposed to moments, whether due to bending or torsion, the structure shows some resistance to these moments and is explained by the concept of moment of inertia.

The moment of inertia about a given axis is determined by multiplying the area by the square of the perpendicular distance from the axis to the center of gravity of the area. The understanding of the centroid and the moment of inertia is important in analyzing and designing structures.

For the static problems, the term "moment of inertia" is referred to as the second moment of area. However, in dynamic scenarios, it is denoted as the second moment of mass or mass moment of inertia with respect to the reference axis. In calculating the moment of inertia, the centroidal of an area or centre of gravity of an object is important. Hence, the centroidal or centre of gravity of mass is described in this section.

1.2 CENTROID AND CENTRE OF GRAVITY

The point about which the entire weight of the object is concentrated is referred to as its center of gravity, often abbreviated as C.G.

The centroid of a plane figure, such as a square, rectangle, triangle, quadrilateral, circle, etc., is the point where the total area of the object is considered to be concentrated. This point is also denoted as C.G. It is important to note that the

centroid and center of gravity coincide, and this common point holds significance when determining the moment of inertia for any section.

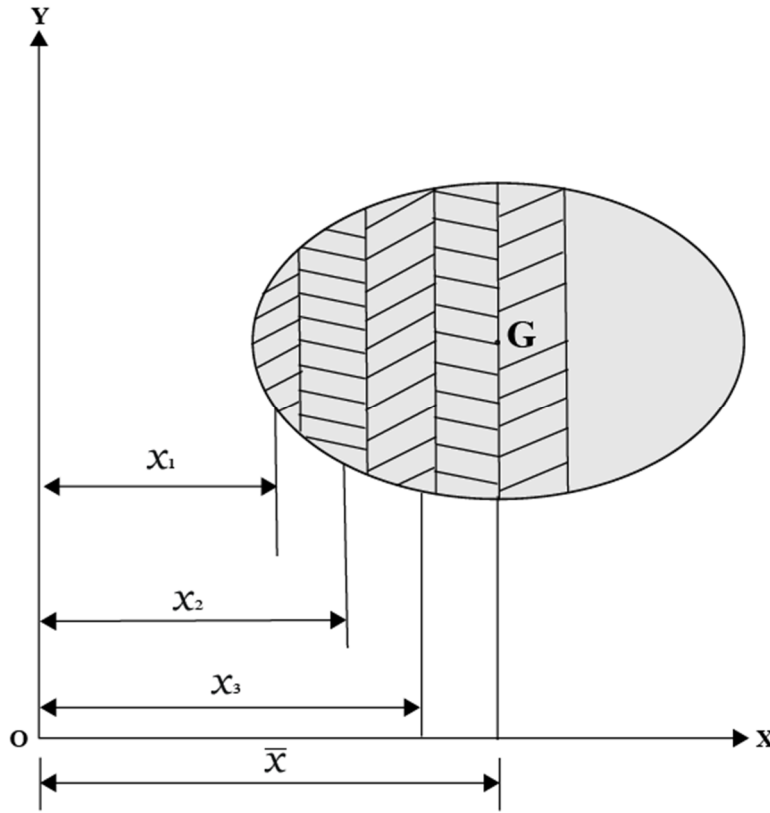


Fig. 1.1: Reference axes of an arbitrary section.

The centroidal distance of different planes with respect to the reference axis is given below.

Suppose the object is divided into 'n' number of segments, and the distance of each segment is denoted by X_i from the reference axis, and A_i is the area of the segment (i varies from 1 to n) then from the y-axis, the centroidal distance $\bar{x}$ is given by

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} \quad \dots \dots (1.1)$$

Similarly, from the x-axis, the centroidal distance $\bar{y}$ is given by

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} \quad \dots \dots (1.2)$$

1.3 CENTROID OF DIFFERENT SECTIONS

The centroid of the various sections commonly employed in solid mechanics comprises of

- Triangle section
- Semicircle section
- Parabolic spandrel
- Composite section

1.3.1 CENTROID OF THE TRIANGLE SECTION

Consider the $\triangle ABC$ with a base b and height h in Fig. 1.2. Let us determine the distance of the centroid from the base. Consider (b_1) as the width of the elemental segment with a thickness of (dy) at a distance (y) from the base. As $\triangle AMN$ and $\triangle ABC$ are similar triangles, we can express:

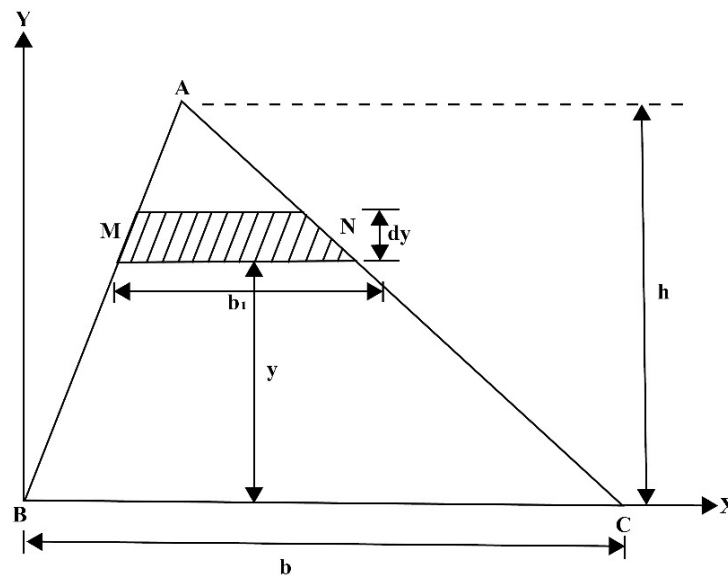


Fig. 1.2: Centroid of the triangle section about y axis

$$\frac{b_1}{b} = \frac{h-y}{h}$$

$$b_1 = \left(\frac{h-y}{h}\right) b$$

$$b_1 = \left(1 - \frac{y}{h}\right) b$$

$$\therefore \text{Area of the element} = dA = b_1 dy \quad \dots \dots (1.3)$$

$$dA = \left(1 - \frac{y}{h}\right) b \cdot dy$$

$$\therefore \text{Area of the triangle } A = \frac{1}{2} bh$$

Using the expression of

$$\text{Centroid, } \bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\int y \, dA}{A} \quad \dots \dots (1.4)$$

Now,

$$\begin{aligned} \int y \, dA &= \int_0^h y \left(1 - \frac{y}{h}\right) b \cdot dy \\ &= \int_0^h \left(y - \frac{y^2}{h}\right) b \cdot dy \\ &= b \left[\frac{y^2}{2} - \frac{y^3}{3h} \right]_0^h \\ &= \frac{bh^2}{6} \quad \dots \dots (1.5) \end{aligned}$$

$$\therefore \bar{y} = \frac{\int y \, dA}{A} = \frac{bh^2}{6} \times \frac{1}{\frac{1}{2}bh}$$

$$\therefore \bar{y} = \frac{h}{3} \quad \dots \dots (1.6)$$

Therefore, the centroid of the triangle is located at a distance of $\frac{h}{3}$ from the base of the triangle, where (h) represents the height of the triangle.

1.3.2 CENTROID OF THE SEMICIRCLE

Consider the semicircle with a radius of (R), as shown in Fig. 1.3. Due to its symmetry, the centroid must lie on the y-axis. To calculate $\bar{y}$, consider an element positioned at a distance r from the centre O of the semicircle. This element is bounded by radii at angles θ and $\theta + d\theta$ with a radial thickness of dr.

$$\text{Area of element} = r \cdot d\theta \cdot dr \quad \dots \dots (1.7)$$

The moment of area about the x-axis is given by:

$$(r \times d\theta \times dr) \times (r \sin\theta) = r^2 \sin\theta \, dr \, d\theta$$

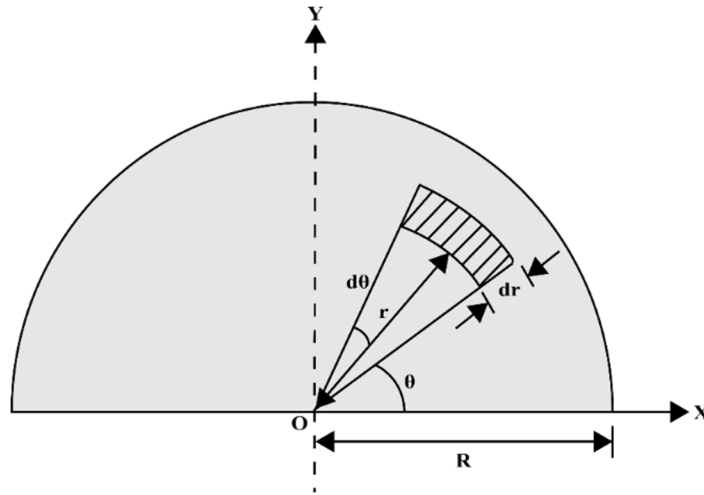


Fig. 1.3: Centroid of the semicircle

$\therefore$ Total moment of area about the central axis,

$$\int_0^\pi \int_0^R r^2 \sin\theta \, dr \, d\theta = \int_0^\pi \left[\frac{r^3}{3} \right]_0^R \sin\theta \, d\theta$$

$$\begin{aligned}
 &= \frac{R^3}{3} [-\cos\theta]_0^\pi \\
 &= \frac{R^3}{3} [1 + 1] = \frac{2R^3}{3} \quad \dots \dots (1.8)
 \end{aligned}$$

$$\therefore \text{Area of the semicircle} \quad A = \frac{1}{2} \pi R^2$$

$$\therefore \bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\left(\frac{2R^3}{3}\right)}{\left(\frac{1}{2}\pi R^2\right)} = \frac{4R}{3\pi} \quad \dots \dots (1.9)$$

Hence, the centroid of the semi circle is located at a distance $\frac{4R}{3\pi}$ from the base.

1.3.3 CENTROID OF THE PARABOLIC SPANDREL

Consider the parabolic spandrel shown in Fig. 1.4, where the height of the element at a distance x from point O is given by $y = kx^2$.

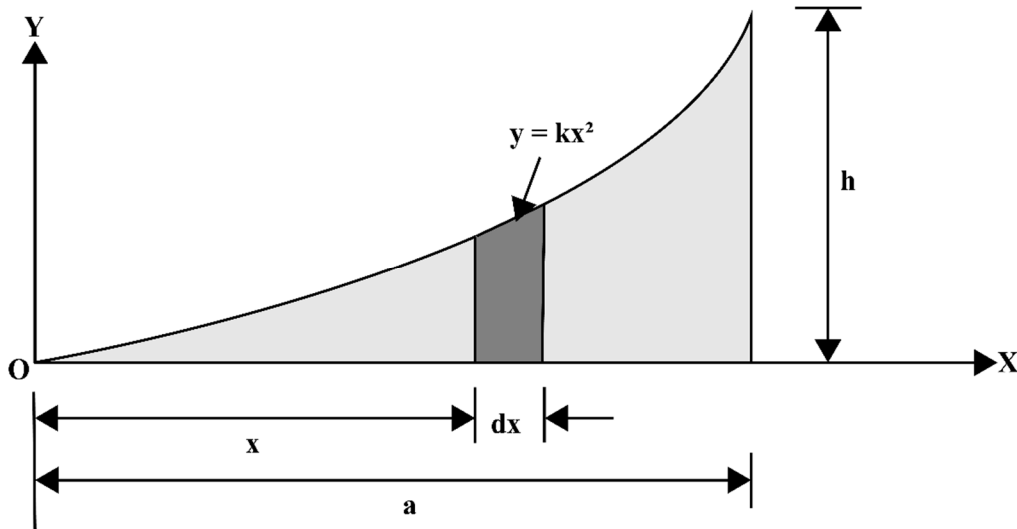


Fig. 1.4: Centroid of the parabolic spandrel

Width of element = dx

$\therefore$ Area of the element $dA = kx^2 dx$

$$\begin{aligned} \therefore \text{Total area of spandrel} &= \int_0^a kx^2 dx \\ &= \left[\frac{kx^3}{3} \right]_0^a \\ &= \frac{ka^3}{3} \end{aligned} \quad \dots \dots (1.10)$$

Moment of the area about y-axis

$$\begin{aligned} &= \int_0^a kx^2 dx \cdot x \\ &= \int_0^a kx^3 dx \\ &= \left[\frac{kx^4}{4} \right]_0^a \\ &= \frac{ka^4}{4} \end{aligned} \quad \dots \dots (1.11)$$

$$\begin{aligned} \text{Moment of area about x-axis} &= \int_0^a dA \cdot \frac{y}{2} \\ &= \int_0^a kx^2 dx \cdot \frac{kx^2}{2} \\ &= \int_0^a \frac{k^2 x^4}{2} dx \\ &= \frac{k^2 a^5}{10} \end{aligned} \quad \dots \dots (1.12)$$

$$\therefore \bar{X} = \frac{\frac{ka^4}{4}}{\frac{ka^3}{3}} = \frac{ka^4}{4} \times \frac{3}{ka^3} = \frac{3a}{4} \quad \dots \dots (1.13)$$

$$\therefore \bar{y} = \frac{\frac{k^2 a^5}{10}}{\frac{ka^3}{3}} = \frac{k^2 a^5}{10} \times \frac{3}{ka^3} = \frac{3ka^2}{10} \quad \dots \dots (1.14)$$

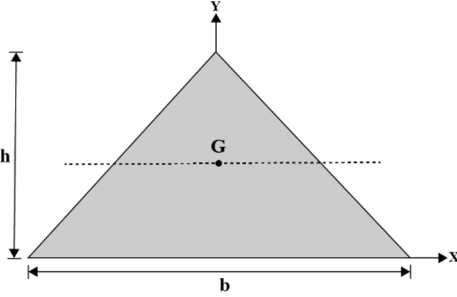
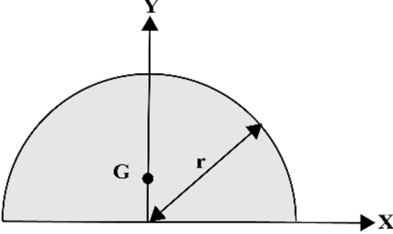
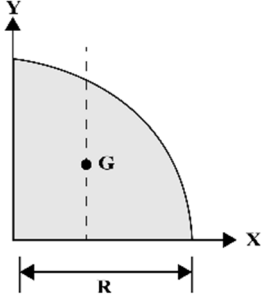
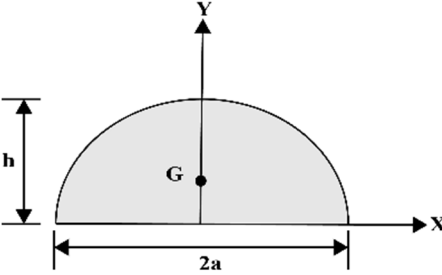
From the Fig. 1.4, at $x = a$, $y = h$;

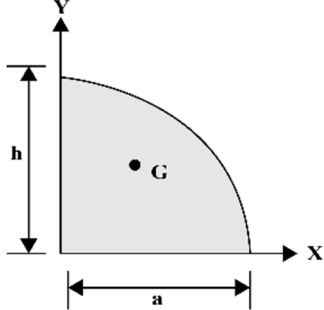
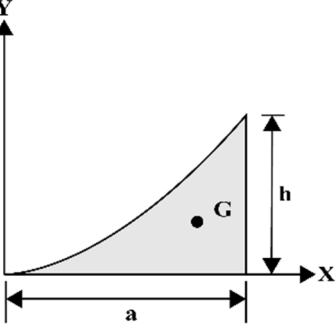
$$\therefore h = ka^2 \text{ or } k = \frac{h}{a^2}$$

$$\therefore \bar{y} = \frac{3}{10} \times \frac{h}{a^2} \times a^2 = \frac{3h}{10}$$

Thus, the centroid of spandrel is $\left(\frac{3a}{4}, \frac{3h}{10}\right)$ (1.15)

Table 1.1: Centroids of some common sections

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{2}$
Quarter Circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{4}$
Parabola		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

Semi parabola		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

1.3.4 CENTROID OF COMPOSITE SECTIONS

We have primarily focused on determining the centroids of basic shapes like rectangles, triangles, circles, and semicircles. However, in engineering practice, it is common to work with composite sections formed by combining multiple simple sections. These types of sections are often referred to as built-up sections or composite sections. When determining the centroid of composite sections, it is not always necessary to use the first principle method (integration). Instead, the composite section can be divided into appropriate simple shapes. Then, the centroid of each simple shape can be determined by applying the standard formulas given in Table 1.1.

If we assume that the area of each simple figure is concentrated at its centroid, we can find the moment of the area about a specific axis by multiplying the area with the distance from its centroid to the reference axis. After calculating the moment of each area about the reference axis, we can determine the distance of the centroid of

the composite section from the same axis by dividing the total moment of the areas by the total area of the composite section.

Solved Problems

Problem 1.1 Find the centroid of the area as shown in Fig. 1.5 with respect to the axis shown.

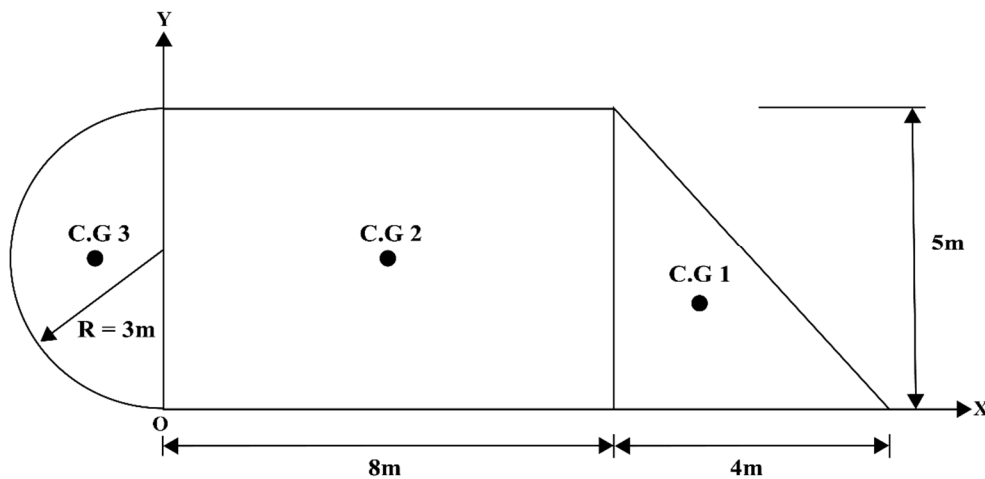


Fig. 1.5: Centroid of the composite section

Solution:

The composite section has been subdivided into three basic shapes: a triangle, a rectangle, and a semicircle.

Now,

$$\text{Area of triangle } A_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4 \times 5 = 10 \text{ m}^2$$

$$\text{Area of rectangle } A_2 = l \times b = 8 \times 5 = 40 \text{ m}^2$$

$$\text{Area of semicircle } A_3 = \frac{\pi r^2}{2} = \frac{1}{2} \times \pi \times 3^2 = 14.137 \text{ m}^2$$

$$\therefore \text{Total area } A = 64.137 \text{ m}^2$$

The coordinates of the centroids of these sections are:

$$x_1 = 8 + \left(\frac{1}{3} \times 4\right) = 9.333 \text{ m}$$

$$y_1 = \frac{5}{3} = 1.666 \text{ m}$$

$$x_2 = 4 \text{ m}$$

$$y_2 = \frac{5}{2} = 2.5 \text{ m}$$

$$x_3 = -\frac{4R}{3\pi} = -\left(\frac{4 \times 3}{3 \times \pi}\right) = -1.2732 \text{ m} \quad \therefore \quad (\text{Note Negative sign of } x_3)$$

$$y_3 = 3 \text{ m}$$

Finding the centroid of each section is given

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A} \quad \dots \dots (\text{Refer 1.1})$$

$$\bar{x} = \frac{(10 \times 9.333) + (40 \times 4) + (14.137 \times (-1.2732))}{64.137}$$

$$\bar{x} = 3.6691 \text{ m}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A} \quad \dots \dots (\text{Refer 1.2})$$

$$\bar{y} = \frac{(10 \times 1.666) + (40 \times 2.5) + (14.137 \times 3)}{64.137}$$

$$\bar{y} = 2.480 \text{ m}$$

Problem 1.2

Find the centroid coordinates of the plane area shown in Fig. 1.6 from the reference axes. Take $X = 50 \text{ mm}$.

Solution:

The composite shape has been subdivided into the following simple figures:

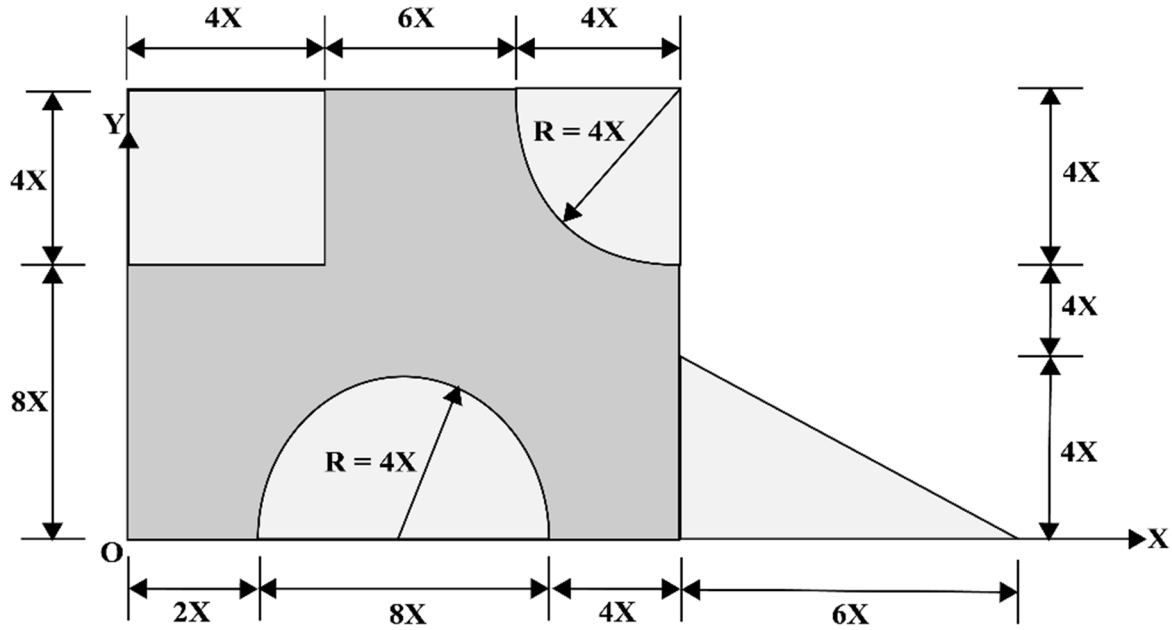


Fig. 1.6: Centroid of the composite section

1) Rectangle

$$A_1 = L \times B = (14x) \times (12x) = 168 x^2$$

$$x_1 = 7x; y_1 = 6x$$

2) Triangle

$$A_2 = \frac{1}{2} \times (6x) \times (4x) = 12 x^2$$

$$x_2 = 14x + 2x = 16x$$

$$y_2 = \frac{4x}{3}$$

3) Square to be subtracted

$$A_3 = (-4x) \times (4x) = -16x^2$$

$$x_3 = 2x; y_3 = 8x + 2x = 10x$$

4) Semicircle to be subtracted

$$A_4 = -\frac{1}{2} \pi (4x)^2 = -8\pi x^2$$

$$x_4 = 6x$$

$$y_4 = \frac{4R}{3\pi} = \frac{4(4x)}{3\pi} = \frac{16x}{3\pi}$$

5) A quarter of the circle to be subtracted

$$A_5 = -\frac{1}{4} \times \pi(4x)^2 = -4\pi x^2$$

$$x_5 = 14x - \frac{4R}{3\pi} = 14x - \frac{4(4x)}{3\pi} = 12.302x$$

$$y_5 = 12x - 4 \times \left(\frac{4R}{3\pi}\right) = 10.302x$$

$$\text{Total area } A = 168x^2 + 12x^2 - 16x^2 - 8\pi x^2 - 4\pi x^2$$

$$A = 126.30x^2$$

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i} \quad \dots \dots (i)$$

$$\begin{aligned} \sum A_i x_i &= (168x^2 \times 7x) + (12x^2 \times 16x) - (16x^2 \times 2x) - (8\pi x^2 \times 6x) \\ &\quad - (4\pi x^2 \times 12.302x) \end{aligned}$$

$$= 1030.608x^3$$

$$\bar{x} = \frac{1030.608x^3}{126.300x^2}$$

$$= 8.1599x = 8.1599 \times 50 \quad (\text{Since } x = 50\text{mm})$$

$$\bar{x} = 407.995 \text{ mm}$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} \quad \dots \dots (ii)$$

$$\begin{aligned} \sum A_i y_i &= (168x^2 \times 6x) + \left(12x^2 \times \frac{4x}{3}\right) - (16x^2 \times 10x) - \left(8\pi x^2 \times \frac{16x}{3\pi}\right) \\ &\quad - (4\pi x^2 \times 10.302x) \end{aligned}$$

$$= 691.870x^3$$

$$\bar{y} = \frac{691.870x^3}{126.300x^2}$$

$$= 5.4780x = 5.4780 \times 50 \quad (\text{Since } x = 50\text{mm})$$

$$\bar{y} = 273.9 \text{ mm}$$

1.4 AREA MOMENT OF INERTIA

Consider the thick segment of area A, as shown in Fig. 1.7.

Let

x = The distance between the centre of gravity of area A and the Y-axis.

y = The distance between the centre of gravity of area A and the X-axis.

The area moment about the Y-axis

$$= \text{Area} \times \text{perpendicular distance between the centroid of the area and the}$$

Y-axis

$$= A \times x \quad \dots\dots (1.16)$$

Eqn (1.3) is referred to as the first moment of the area with respect to the Y-axis.

This first moment of area is used in determining the centroid of the area.

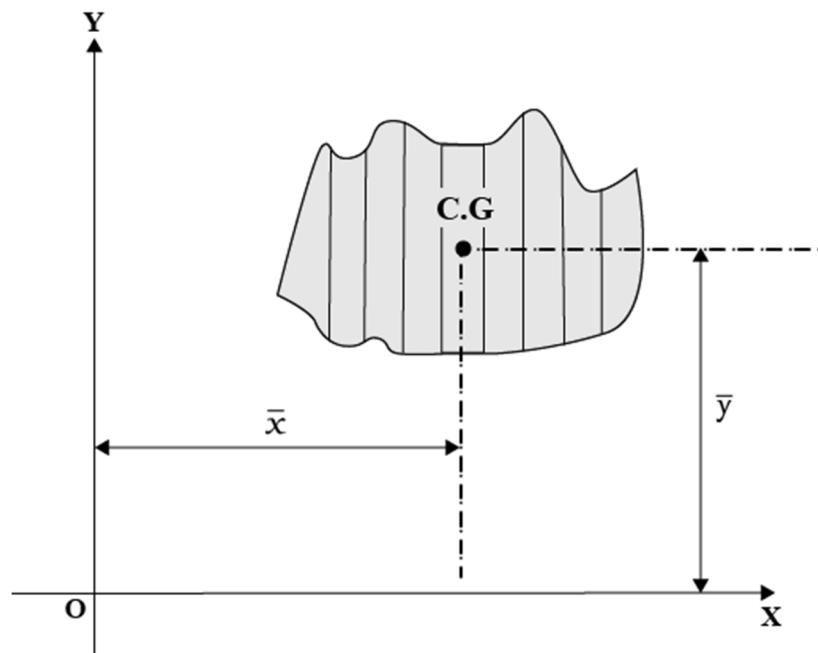


Fig. 1.7: Figure showing the segment area moment about the Y-axis

When the area moment given in Eqn (1.16) is further multiplied by the perpendicular distance between the centroid area and the Y-axis (referred to as distance x), the resulting quantity (Ax). $x = Ax^2$ is called the second moment of area. This second area moment plays a pivotal role in the analysis of solid mechanics.

Similarly, the area moment first moment of area about the X-axis = Ay . Furthermore, the second moment of area about the X- axis = (Ay) . $y = Ay^2$.

When the mass (m) of an object is considered instead of its area, the resulting second moment is referred to as the second moment of mass or the mass moment of inertia. As a result, the moment of inertia when mass is considered about the Y-axis = mx^2 and about X-axis = my^2 .

Therefore, the product of either the mass or area and the square of the distance from the center of gravity of that area or mass to a given axis is referred to as the moment of inertia. This moment of inertia is denoted as "I." Specifically, the moment of inertia about the X-axis is represented as I_{XX} , while about the Y-axis is represented as I_{YY} .

1.5 MOMENT OF INERTIA OF A PLANE AREA

In order to determine the moment of inertia of a plane area, dividing the area into multiple small segments is necessary.

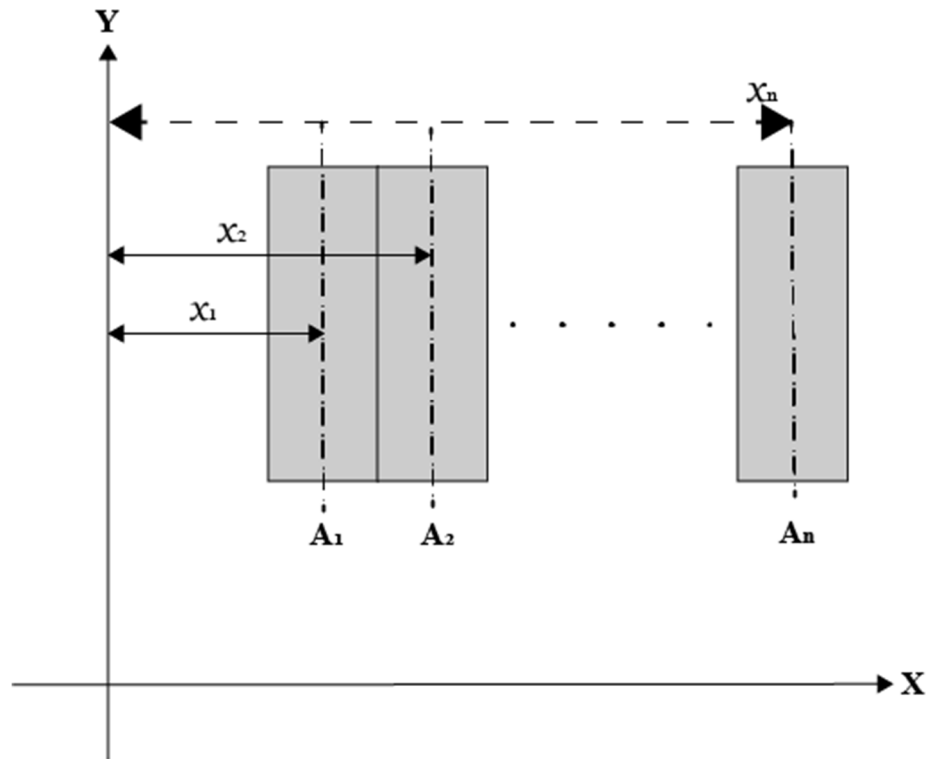


Fig. 1.8: Details of the moment of inertia for the plane area

Consider,

$A_1, A_2, A_3 \dots$ = Area of the segments

$x_1, x_2, x_3 \dots$ = The distances of the individual elements from the line for which the moment of inertia is to be calculated.

Now, the moment of inertia of the area about the reference Y-Y axis is given by

$$I_{yy} = A_1 x_1^2 + A_2 x_2^2 + A_3 x_3^2 + \dots$$

$$I_{yy} = \sum_{i=1}^n A_i x_i^2 \quad \dots \dots (1.17)$$

1.6 UNIT OF MOMENT OF INERTIA

- The unit of moment of inertia of a plane area is obtained from the units of length and area.
- If the area is represented in mm^2 , the length is expressed in mm, and the moment of inertia is expressed in mm^4 .

1.7 MOMENT OF INERTIA BY INTEGRATION

The moment of inertia of any arbitrary section can be determined using the integration or summation approach.

In order to determine the moment of inertia of a plane figure, it is required to find it for both the Y-Y and X-X axes, as shown in Fig. 1.9.

Consider the area which is divided into a number of segments.

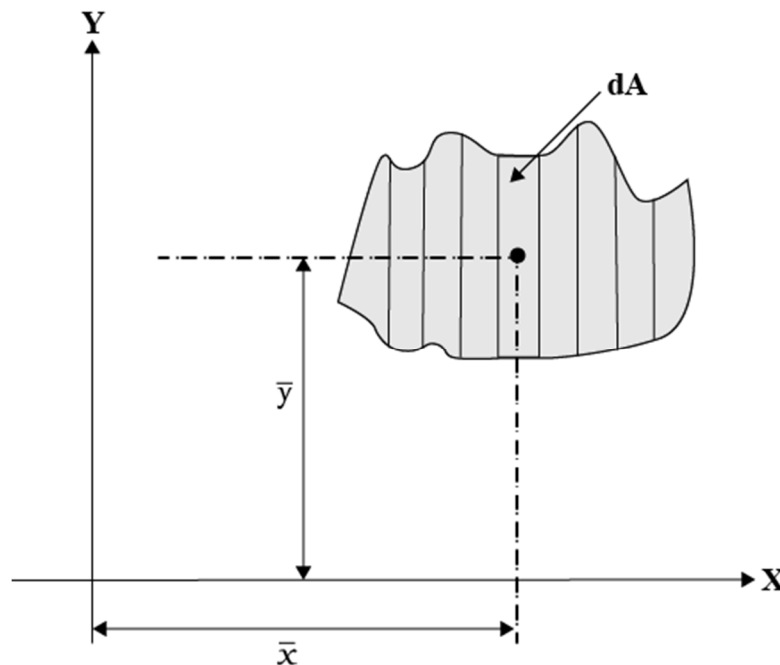


Fig. 1.9: Reference axes of an arbitrary section

Let,

dA = Segmental area of an arbitrary segment

x = Centroid distance of the segment from Y- axis

y = Centroid distance of the segment from X- axis

Hence, the moment of inertia of the segment about the Y-Y axis = $x^2 \cdot dA$

The moment of inertia of the entire area can be calculated by integrating the expression given above.

$$I_{yy} = \Sigma x^2 \cdot dA = \int x^2 \cdot dA \quad \text{..... (1.18)}$$

$$\text{Similarly, } I_{xx} = \Sigma y^2 \cdot dA = \int y^2 \cdot dA \quad \text{..... (1.19)}$$

1.8 RADIUS OF GYRATION

The radius of gyration (k) of the object concerning a specific axis is defined as the distance from that axis, such that when the square of this distance is multiplied by the area of the object (A), it gives the (moment of inertia) of the object about the axis.

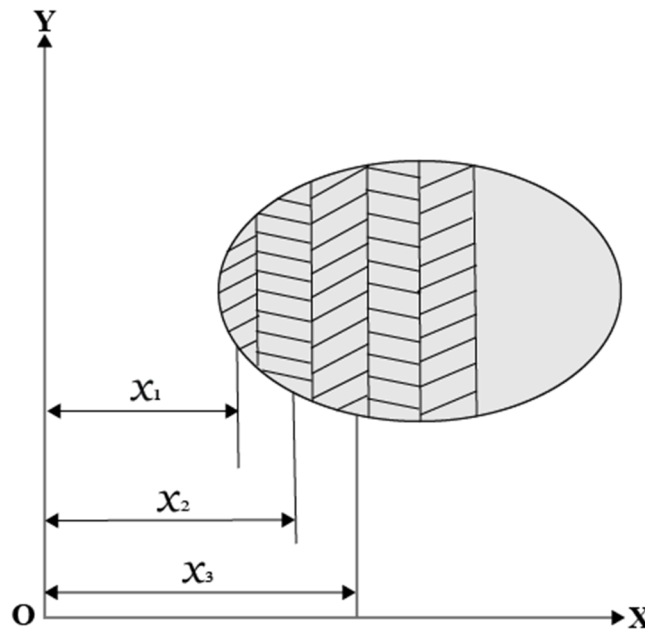


Fig. 1.10: Details for the moment of inertia calculations

For Fig. 1.10, the moment of inertia about the given axis can be calculated using Eqn. 1.8 as follows:

$$I_{yy} = A_1 x_1^2 + A_2 x_2^2 + A_3 x_3^2 + \dots \quad \dots (1.20)$$

Consider the entire surface area concentrated at a distance 'k' from the reference axis. The moment of inertia of the entire area about the given axis will be equal to $A \times k^2$.

Thus, $I = Ak^2$, k is the radius of gyration about the reference axis.

$$k = \sqrt{\frac{I}{A}} \quad \dots (1.21)$$

SI unit of radius of gyration is meter.

1.9 SECTION MODULUS

In Fig. 1.11(a), the beam experiences bending when subjected to transverse loads. Fig. 1.11(b) shows the cross-section of a beam. The axis that passes through the centroid and is parallel to the base of the section is known as the centroid axis. It is essential to note that this centroid axis is perpendicular to the plane of the paper in Fig. 1.11(a).

As the beam bends, each section of the beam undergoes rotation about the centroid axis. This rotational motion results in the effect of the second moment of area, also called the moment of inertia, coming into play in bending beams. The moment of inertia has been discussed in detail in the earlier sections, and it is an important parameter in the bending of beams.

The beam section is symmetric about the Y-axis, but in the case of simple bending, it may not be symmetrical about the horizontal axis. As a result, the centroid will not necessarily be at the center of the depth 'd' but could be located either above or below the horizontal axis. In Fig. 1.11(b), the centroid is positioned below the half depth 'd/2'. Due to this asymmetry, the distances ' y_a ' and ' y_b ' from the centroid axis to the extreme fibers at the bottom are unequal. These distances are known as the distances of the extreme fibers.

The quantities ' I/y_a ' and ' I/y_b ' are called section modulus of the beam section. These values are important parameters in analyzing the bending behavior of the beam and depend on the dispersion of the area about the centroid axis.

Section modulus (Z) is defined as the moment of inertia of the section divided by the distance from the centroid axis to the extreme fiber, whether the top or the bottom. Mathematically, it can be written as:

$$Z = \frac{I}{y} \quad \dots \dots (1.22)$$

Where:

Z = Section modulus

I = moment of inertia of the section about the centroid axis

y = Distance from the centroid axis to the extreme fiber (either top or bottom)

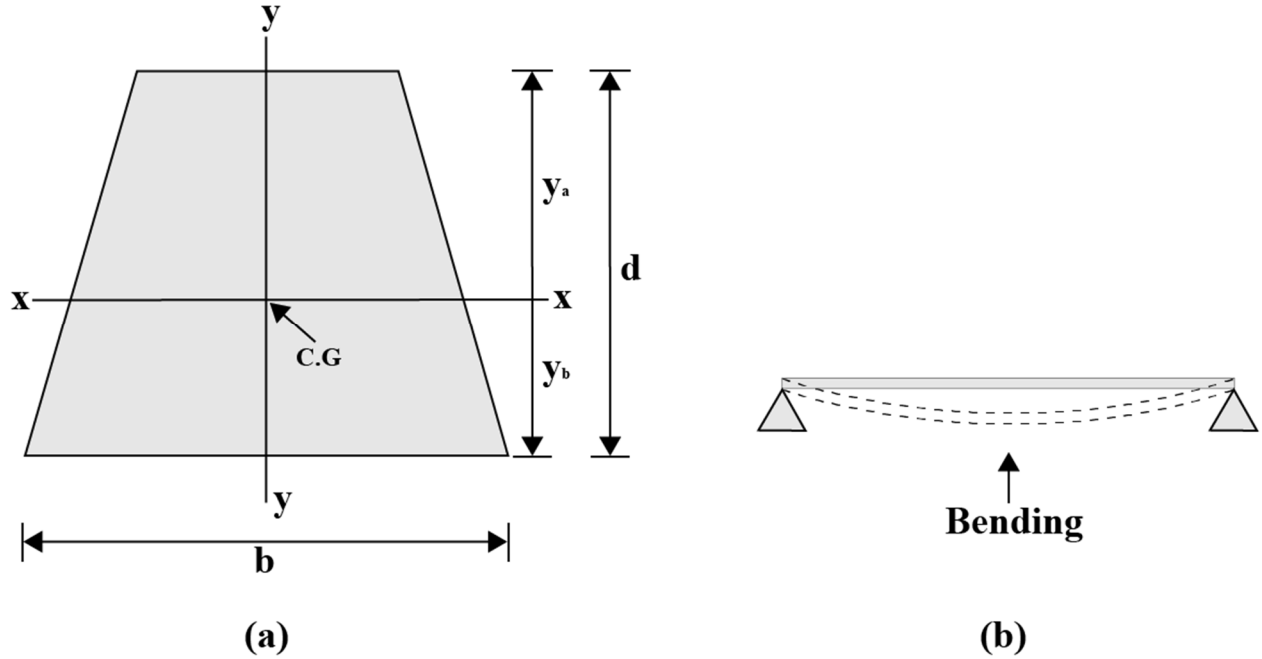


Fig. 1.11(a)-(b): Cross-section of a beam subjected to bending

Section modulus is an important parameter in structural engineering because it governs the design of the beam. The least section modulus of the beam cross section determines its structural strength and resistance to bending. Section modulus is typically denoted by the symbol 'Z', and its unit is given by (L^3) , where 'L' is in length unit (e.g., mm, cm, m, etc.). For example, if the length unit is in meters, the unit of section modulus Z will be m^3 .

1.10 PERPENDICULAR AXIS THEOREM

The moment of inertia of a plane section perpendicular to I_{xx} and I_{yy} axes intersecting at point O is considered as the moment of inertia I_{zz} about the Z-Z axis. The Z-Z axis is perpendicular to the plane and passes through the point where the Y-Y and X-X axes intersect. The relationship for I_{zz} is given by:

$$I_{zz} = I_{yy} + I_{xx} \quad \text{..... (1.23)}$$

In other words, the moment of inertia about the axis Z-Z is equal to the sum of the moment of inertia about the Y-Y and X-X axes. This relationship allows us to determine the moment of inertia about the Z-Z axis using the moment of inertia values of the I_{xx} and I_{yy} axes.

In Fig. 1.1, consider the centroidal distance of area 'A' with coordinates 'y' and 'x' from OX and OY. The plane OZ is perpendicular to OX and OY, and we'll take 'r' as the distance of the segment from the Z-Z axis, such that $OP = r$.

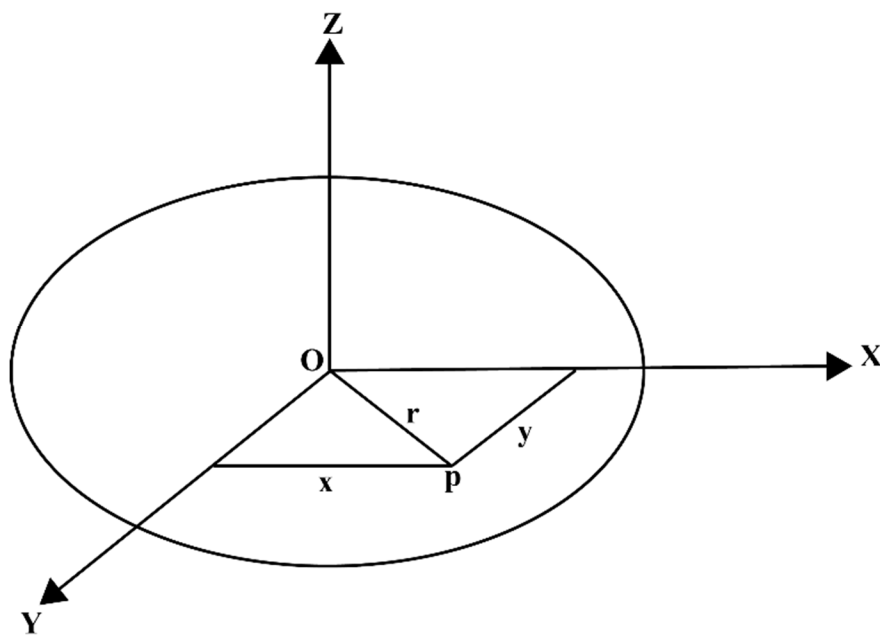


Fig. 1.12: Figure showing the Perpendicular axis theorem

From the geometry of the figure, it can be seen that:

$$r^2 = y^2 + x^2 \quad \text{..... (1.24)}$$

The moment of inertia of the segment about the X-X axis is given by:

$$I_{xx} = A \cdot y^2 \quad \text{..... (1.25)}$$

Similarly, the moment of inertia of the section about the Y-Y axis is:

$$I_{yy} = A \cdot x^2 \quad \text{..... (1.26)}$$

The moment of inertia of the section about the Z-Z axis is:

$$I_{zz} = A \cdot (x^2 + y^2)$$

$$I_{yy} = A \cdot r^2 \quad \text{..... (1.27)}$$

Thus, it can be written as

$$I_{zz} = Ax^2 + Ay^2 = I_{xx} + I_{yy} \quad \text{..... (1.28)}$$

This relationship demonstrates that the moment of inertia about the Z-Z axis (I_{zz}) is equal to the sum of the moment of inertia about the Y-Y axis (I_{yy}) and the X-X axis (I_{xx}). The moment of inertia about the Z axis is known as the polar moment of inertia.

1.11 PARALLEL AXIS THEOREM

Suppose I_G denotes the moment of inertia of a plane area about an axis passing through the centroid. In such case, the moment of inertia of the section about any other axis AB, which is parallel to the centroidal axis and at a distance 'h' from the centroid, can be calculated using the equation:

Where:

I_{AB} = Moment of inertia of the area about the axis AB

I_{xx} = Moment of inertia of the centroid axis

h = Distance from the centroid to the axis AB

A = Total area of the plane section

$$I_{AB} = I_{xx} + (A \cdot h^2) \quad \dots\dots (1.29)$$

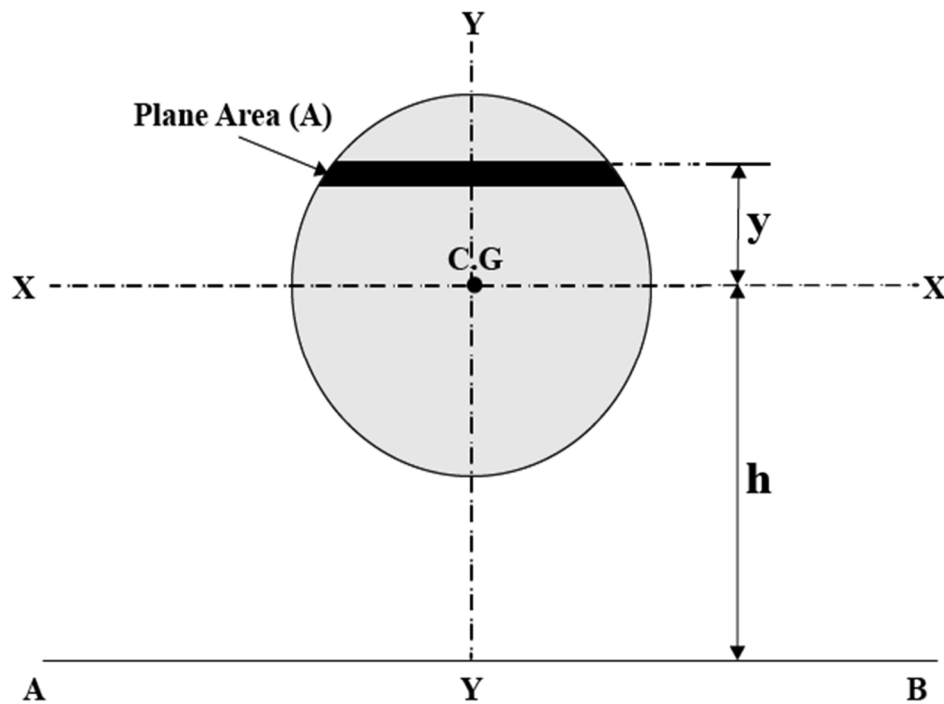


Fig. 1.13: Figure showing the Parallel axis theorem

This equation is useful in determining the moment of inertia of the section which is parallel to the reference axis.

1.12 MOMENT OF INERTIA OF DIFFERENT SECTIONS

The moment of inertia of the different sections that are commonly used in the mechanics of solids include

- Rectangular section
- Hollow rectangular section
- Circular section
- Hollow circular section
- Triangular section

- I-section and C-section
- T-section
- Semi-circular section
- Composite section
- Build-up section
- Parabolic section

The moment of inertia of these sections will be useful to find the moment of inertia of complex sections, which can be segmented into the base sections.

1.12.1 MOMENT OF INERTIA OF A RECTANGULAR SECTION

In Fig. 1.14, consider an elementary segment of thickness δy located at a distance 'y' from the XX axis. Let OY be the axis passing through the centroid of the section.

The moment of inertia of the section about the centroidal axis X-X is given by,

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \cdot b \, dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 \, dy = b \cdot \left(\frac{y^3}{3} \right)_{-\frac{d}{2}}^{+\frac{d}{2}}$$

$$I_{xx} = \frac{bd^3}{12} \quad \dots \dots (1.30)$$

By the parallel axis theorem, the moment of inertia of the section about its base is given by

$$I_{AB} = I_{xx} + Ah^2$$

$$= \frac{bd^3}{12} + bd \cdot \left(\frac{d}{2} \right)^2 \quad \left(\because h = \frac{d}{2} \right)$$

$$I_{uv} = \frac{bd^3}{3} \quad \dots \dots (1.31)$$

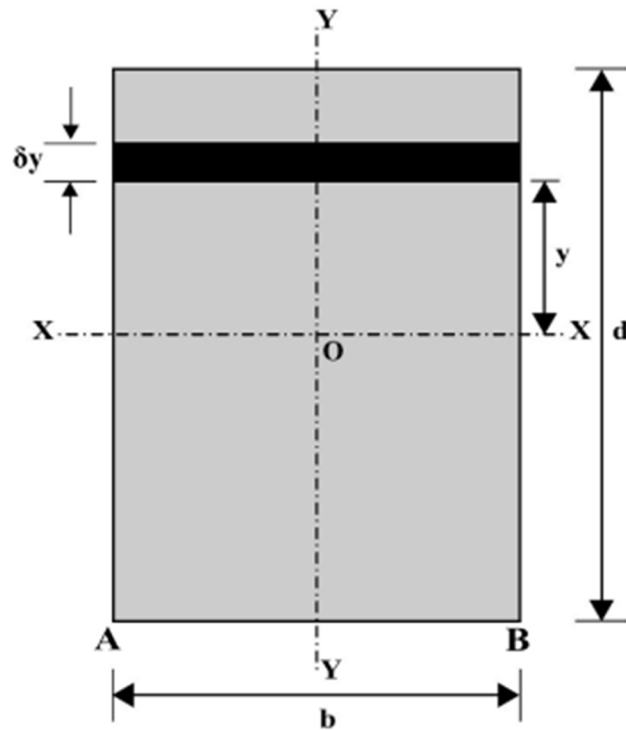


Fig. 1.14: Moment of inertia of the rectangular section

Section modulus,

$$Z = \frac{\text{Moment of inertia about the centroid axis}}{\text{Distance of the farthest point from the centroid axis}}$$

$$Z_x = \frac{I_{xx}}{y_{\max}} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

$$Z_x = \frac{bd^2}{6} \quad \dots \dots (1.32)$$

Similarly, the moment of inertia of the section about its Y -axis will be

$$I_{yy} = \frac{db^3}{12} \quad \dots \dots (1.33)$$

1.12.2 MOMENT OF INERTIA OF A HOLLOW RECTANGULAR SECTION

The moment of inertia of the hollow rectangular section about the centroidal axis X-X, as illustrated in Fig. 1.15, is given by

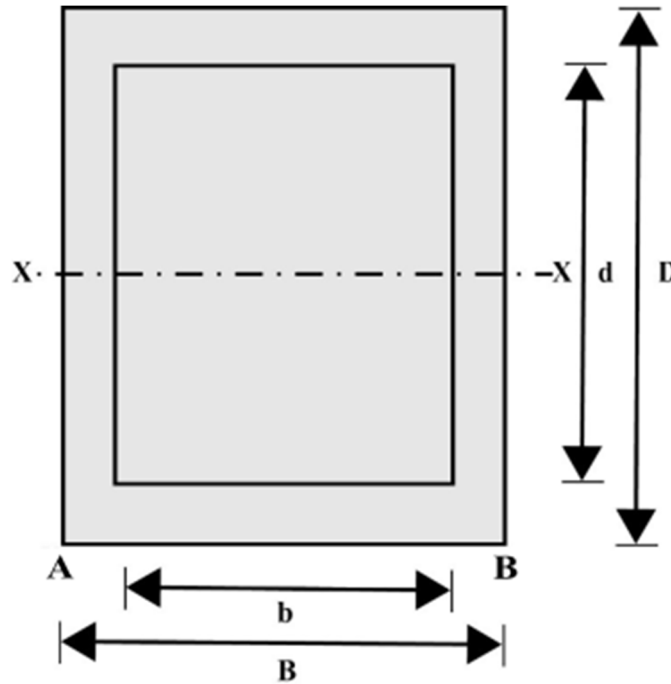


Fig. 1.15: Moment of inertia of the hollow rectangular section

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I_{xx}$$

$$= \frac{BD^3 - bd^3}{12} \quad \dots \dots (1.34)$$

Section modulus is given by,

$$Z_x = \frac{\frac{(BD^3 - bd^3)}{12}}{\frac{D}{2}}$$

$$Z_x = \frac{(BD^3 - bd^3)}{6D} \quad \dots \dots (1.35)$$

1.12.3 MOMENT OF INERTIA OF A CIRCULAR SECTION

The moment of inertia of the circular section about the centroidal axis X-X, as shown in Fig. 1.16.

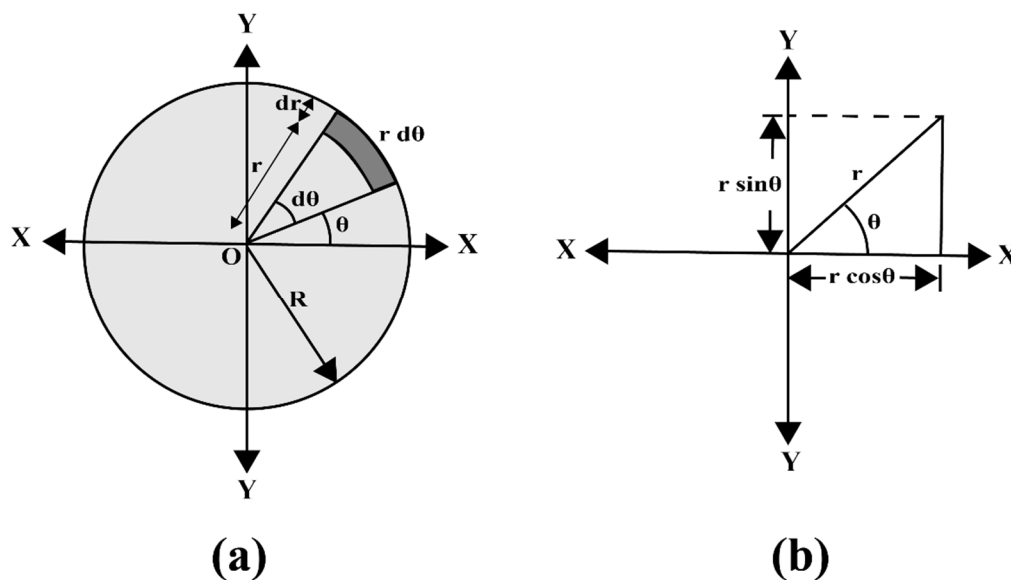


Fig. 1.16: Moment of inertia of the circular section

Consider an elementary circular segment of radius 'r', angle 'dθ', and thickness 'dr'. The area of the circular segment is

$$\text{Area (dA)} = r \cdot d\theta \cdot dr \quad \dots \dots (1.36)$$

Moment of inertia for elementary area:

$$\begin{aligned}
 I_{xx} &= y^2 \cdot dA \\
 &= (r \sin \theta)^2 \cdot r \cdot d\theta \cdot dr \\
 &= r^2 \sin^2 \theta \cdot r \cdot d\theta \cdot dr \\
 I_{xx} &= r^3 \sin^2 \theta \cdot d\theta \cdot dr \quad \dots\dots (1.37)
 \end{aligned}$$

The moment of the whole area is obtained by integrating from θ varying from 0 to 2π

$$\begin{aligned}
 I_{xx} &= \int_0^R \int_0^{2\pi} r^3 \sin^2 \theta \cdot d\theta \cdot dr & \therefore (\cos 2\theta = 1 - 2\sin^2 \theta) \\
 & & \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \\
 &= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1 - \cos 2\theta}{2} \right) \cdot d\theta \cdot dr \\
 &= \frac{r^3}{2} \int_0^R \left(\theta - \frac{\sin 2\theta}{2} \right)_0^{2\pi} \cdot dr \\
 &= \frac{r^3}{2} \int_0^R (2\pi) \cdot dr \\
 &= \int_0^R r^3 \pi \cdot dr \\
 &= \pi \left(\frac{r^4}{4} \right)_0^R \\
 &= \frac{\pi R^4}{4} \\
 &= \frac{\pi}{4} \left(\frac{d}{2} \right)^4 & \left[\therefore R = \frac{d}{2} \right] \\
 &= \frac{\pi}{4} \left(\frac{d^4}{16} \right) \\
 I_{xx} &= \frac{\pi d^4}{64} \quad \dots\dots (1.38)
 \end{aligned}$$

Similarly,

$$I_{yy} = \frac{\pi d^4}{64} \quad \dots \dots (1.39)$$

Section modulus is given by,

$$Z_x = \frac{I_{xx}}{y_{\max}} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}}$$

$$Z_x = \frac{\pi d^3}{32} \quad \dots \dots (1.40)$$

1.12.4 MOMENT OF INERTIA OF A HOLLOW CIRCULAR SECTION

The moment of inertia of the hollow circular section about the centroidal axis X-X, as shown in Fig. 1.17, is given by

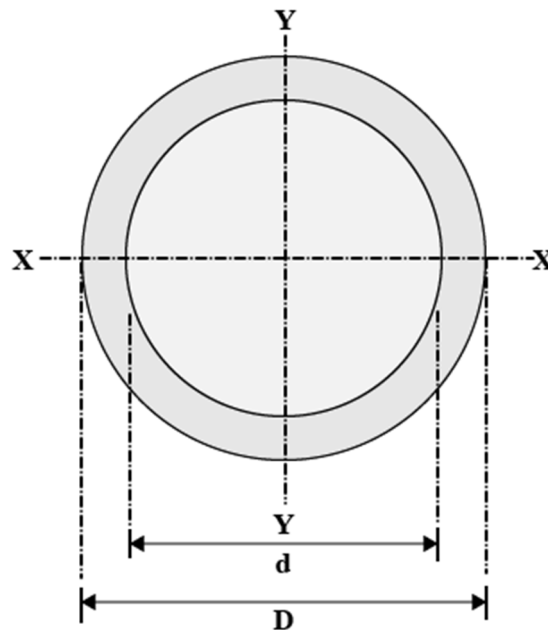


Fig. 1.17: Moment of inertia of the hollow circular section

$$I_{xx} = \frac{\pi(D^4 - d^4)}{64} \quad \dots \dots (1.41)$$

Section modulus,

$$Z_x = \frac{I_{xx}}{y_{\max}} = \frac{\frac{\pi(D^4 - d^4)}{64}}{\frac{D}{2}}$$

$$Z_x = \frac{\pi}{32} \left(\frac{D^4 - d^4}{D} \right) \quad \dots \dots (1.42)$$

Hence, D and d are the outer and inner diameters of the hollow section.

1.12.5 MOMENT OF INERTIA OF A TRIANGULAR SECTION

Consider a triangular section ABC,

Let

b = Base of the triangular section

h = Height of the triangular section.

Now consider a small segment MN thickness dx at a distance of x from the vertex A, as shown in Fig. 1.18. From the geometry of the figure, we find the two triangles, ABC and AMN, are similar. Therefore

$$\frac{MN}{BC} = \frac{x}{h}$$

$$MN = \left(\frac{x}{h}\right) \cdot BC \quad \therefore BC = \text{base} = b$$

$$MN = \frac{bx}{h} \quad \dots \dots (1.43)$$

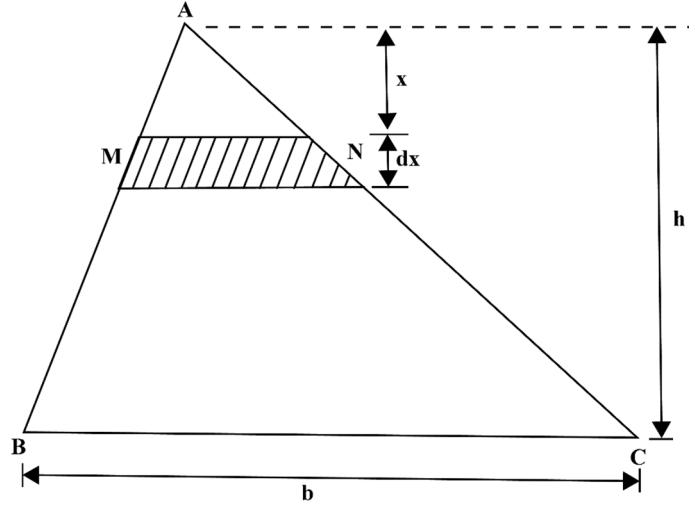


Fig. 1.18: Moment of inertia of the triangular section

We know that the area of the segment $MN = \frac{bx}{h} \cdot dx$ and moment of inertia of the segment about the base $BC = \text{Area} \times (\text{Distance})^2$

$$\begin{aligned} &= \frac{bx}{h} \cdot dx (h - x)^2 \\ &= \frac{bx}{h} (h - x)^2 dx \quad \dots \dots (1.44) \end{aligned}$$

Now, the moment of inertia of the whole triangular section may be found by integrating the above equation for the whole height of the triangle, i.e., from 0 to h.

$$\begin{aligned} I_{BC} &= \int_0^h \frac{bx}{h} (h - x)^2 dx \\ I_{BC} &= \frac{b}{h} \int_0^h x (h^2 + x^2 - 2hx)^2 dx \\ I_{BC} &= \frac{b}{h} \int_0^h (xh^2 + x^3 - 2hx^2) dx \end{aligned}$$

$$I_{BC} = \frac{b}{h} \left[\frac{x^2 h^2}{2} + \frac{x^4}{4} - \frac{2hx^3}{3} \right]_0^h$$

$$I_{BC} = \frac{bh^3}{12} \quad \dots \dots (1.45)$$

We know that the distance between the centre of gravity of the triangular section and base BC,

$$d = \frac{h}{3} \quad \dots \dots (1.46)$$

$\therefore$ Moment of inertia of the triangular section about an axis through its centre of gravity and parallel to the X-X axis.

$$I_{xx} = I_{BC} - Ad^2 \quad \therefore I_{BC} = I_{xx} + Ad^2$$

$$I_{xx} = \frac{bh^3}{12} - \left(\frac{bh}{2} \right) \left(\frac{h}{3} \right)^2 \quad \therefore A = \frac{bh}{2}$$

$$I_{xx} = \frac{bh^3}{36} \quad \dots \dots (1.47)$$

Problem 1.3 Determine the moment of inertia of the triangular section about an axis that passes through the base XX, as shown in Fig 1.19.

Solution:

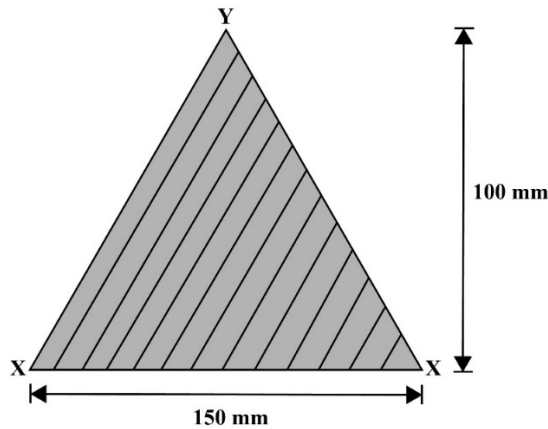


Fig. 1.19: Moment of inertia of the triangular section

Base (b) = 150 mm, and Height (h) = 100 mm.

The moment of inertia of a triangular section about the axis passes through the base is given by

$$I_{xx} = \frac{bd^3}{12}$$

$$I_{xx} = \frac{150 \times 100^3}{12} = 125 \times 10^5 \text{ mm}^4$$

1.12.6 MOMENT OF INERTIA OF AN I-SECTION

The moment of inertia of the I section about the centroidal axis X-X, as shown in Fig. 1.20, is given by

$$I_{xx} = \frac{BD^3 - bd^3}{12} \quad \dots \dots (1.48)$$

Where,

b = B – Thickness of a web

d = D – (2 × Thickness of the flange)

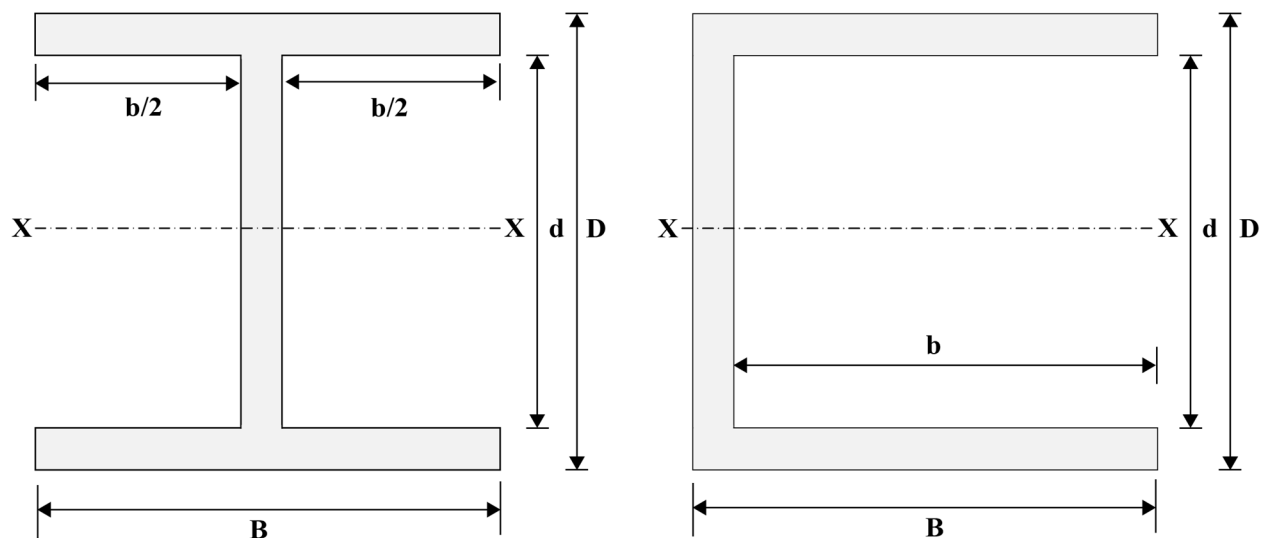


Fig. 1.20: Moment of inertia of the I-section

Section modulus,

$$Z_x = \frac{\frac{(BD^3 - bd^3)}{12}}{\frac{D}{2}}$$

$$Z_x = \frac{(BD^3 - bd^3)}{6D} \quad \dots \dots (1.49)$$

Finding the moment of inertia of each section about the centroidal XX section is given by

$$I_{xx_1} = I_{xx_1} + I_{xx_2} + I_{xx_3} \quad \dots \dots (1.50)$$

Problem 1.4 Determine the moment of inertia of a C-section about the vertical and horizontal axis passing through its centroid. Calculate the polar moment of inertia of the C-section about its centroid axis.

Solution:

To find the centroid of the C-section, it is observed from the figure that the section is symmetric about the Y-Y axis, indicating that the centroid of the section will be on the Y-Y axis.

Divide the section into three rectangle sections, ABCD, BEFG, and EHIJ, for calculating the centroid of the section.

Let $\bar{y}$ = Centroid distance of the section from the base-line HI.

A_1 = Area of the section-1 ABCD = $80 \times 10 = 800 \text{ mm}^2$

$$y_1 = \text{Centroid distance of the area } A_1 \text{ from the base-line HI} = \frac{10}{2} + 230 + 10 = 245 \text{ mm}$$

$$A_2 = \text{Area of the section-2 BEFG} = 230 \times 10 = 2300 \text{ mm}^2$$

$$y_2 = \text{Centroid distance of the area } A_2 \text{ from the base-line HI} = \frac{230}{2} + 10 = 125 \text{ mm}$$

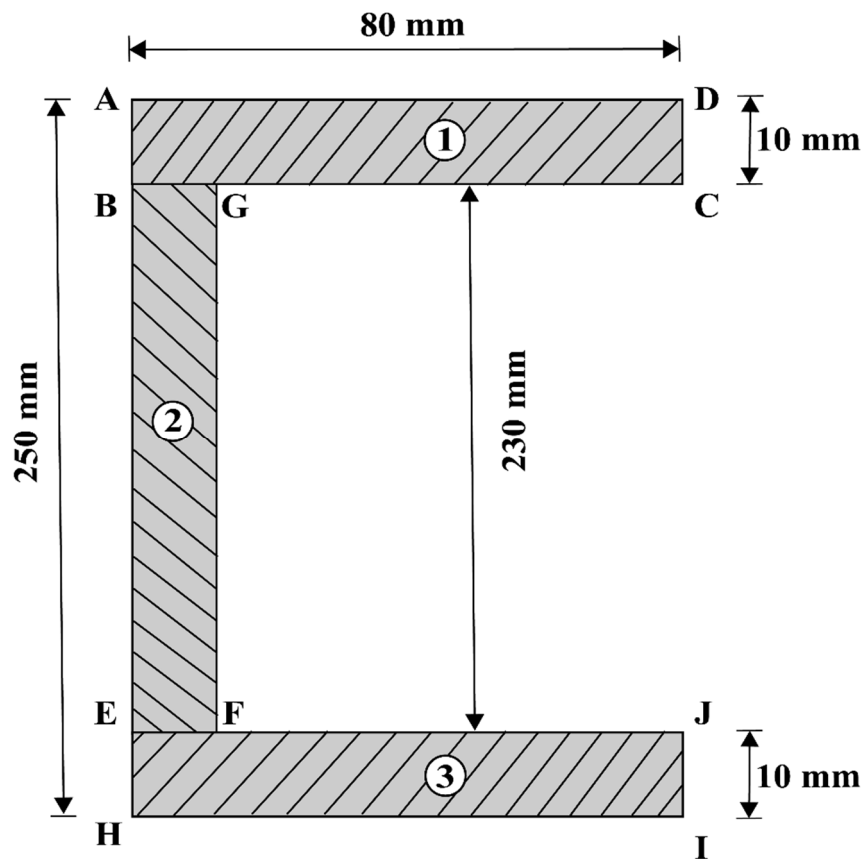


Fig. 1.21: Moment of inertia of a C-section

$$A_3 = \text{Area of the section-3 EHIJ} = 80 \times 10 = 800 \text{ mm}^2$$

$$y_3 = \text{Centroid distance of the area } A_3 \text{ from the base-line HI} = \frac{10}{2} = 5 \text{ mm}$$

The centroid of the section is obtained by using (Eqn 1.2),

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{(800 \times 245) + (2300 \times 125) + (800 \times 5)}{800 + 2300 + 800} = 125 \text{ mm}$$

As a result, the centroid of the C-section is 125 mm away from HI. Determine the moment of inertia of the C-section now.

Let

I_{x1} = Moment of Inertia of (1) about its centroid.

I_{x2} = Moment of Inertia of (2) about its centroid.

I_{x3} = Moment of Inertia of (3) about its centroid.

h_1 = Distance between the centroid of the section (1) and the centroid of the C section

$$h_1 = y_1 - \bar{y} = 245 - 125 = 120 \text{ mm}$$

h_2 = Distance between the centroid of the section (2) and the centroid of the C section

$$h_2 = y_2 - \bar{y} = 125 - 125 = 0 \text{ mm}$$

h_3 = Distance between the centroid of the section (3) and the centroid of the C section

$$h_3 = \bar{y} - y_3 = 125 - 5 = 120 \text{ mm}$$

Now,

$$I_{x1} = \frac{80 \times 10^3}{12} = 6666.66 \text{ mm}^4$$

$$I_{x2} = \frac{10 \times 230^3}{12} = 10139166.67 \text{ mm}^4$$

$$I_{x3} = \frac{80 \times 10^3}{12} = 6666.66 \text{ mm}^4$$

From the parallel axis theorem,

Moment of inertia section (1) about the X-X axis passes through the centroid of a given section.

$$= I_{x1} + A_1 h_1^2 = 6666.66 + (800 \times 120^2) = 11.527 \times 10^6 \text{ mm}^4$$

Moment of inertia section (2) about the X-X axis passes through the centroid of the given section.

$$= I_{x2} + A_2 h_2^2 = 10139166.67 + (2300 \times 0^2) = 10.139 \times 10^6 \text{ mm}^4$$

Moment of inertia section (3) about the X-X axis passes through the centroid of the given section.

$$= I_{x3} + A_3 h_3^2 = 6666.66 + (800 \times 120^2) = 11.527 \times 10^6 \text{ mm}^4$$

∴ Moment of inertia of the C section about the X-X axis passes through the centroid of the given section is,

$$\begin{aligned} I_{xx} &= (11.527 \times 10^6) + (10.139 \times 10^6) + (11.527 \times 10^6) \\ &= 33.19 \times 10^6 \text{ mm}^4 \end{aligned}$$

∴ Moment of inertia of the C section about a Y-Y axis passing through the centroid of the given section is,

$$I_{yy} = \left(\frac{80^3 \times 10}{12} + \frac{10^3 \times 230}{12} + \frac{80^3 \times 10}{12} \right) = 8.7 \times 10^5 \text{ mm}^4$$

The equation of the polar moment of inertia (I_{zz}).

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = (33.19 \times 10^6) + (8.7 \times 10^5) = 3.4 \times 10^7 \text{ mm}^4$$

1.12.7 MOMENT OF INERTIA OF A SEMI-CIRCULAR SECTION

The moment of inertia of the semi-circular section about the centroidal axis X-X, as shown in Fig. 1.22, is to be calculated.

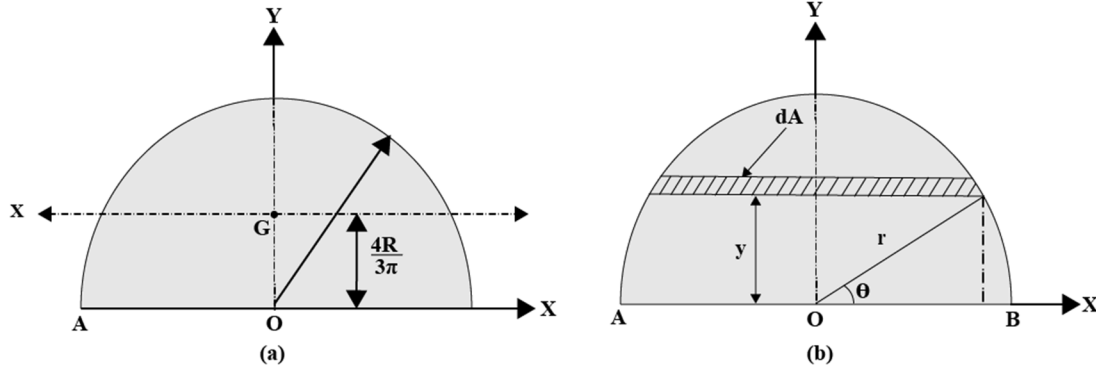


Fig. 1.22: Moment of inertia of the semi-circular section

Consider an elementary circular segment of radius ' r ', angle ' $d\theta$ ', and thickness ' dr '. The area of the circular segment is

$$\text{Area (dA)} = r \cdot d\theta \cdot dr \quad \dots\dots (1.51)$$

Consider an elementary segment at y from the base

$$y = r \sin\theta \quad \dots\dots (1.52)$$

Moment of inertia about AB:

$$\begin{aligned} I_{AB} &= y^2 \cdot dA \\ &= (r \sin\theta)^2 \cdot r \cdot d\theta \cdot dr \\ &= r^2 \sin^2\theta \cdot r \cdot d\theta \cdot dr \\ I_{AB} &= r^3 \sin^2\theta \cdot d\theta \cdot dr \quad \dots\dots (1.53) \end{aligned}$$

The moment of the whole area is obtained by integrating from θ varying from 0 to π

$$\begin{aligned} I_{AB} &= \int_0^R \int_0^\pi r^3 \sin^2\theta \cdot d\theta \cdot dr & \therefore \sin^2\theta &= \frac{1 - \cos 2\theta}{2} \\ &= \int_0^R \int_0^\pi r^3 \left(\frac{1 - \cos 2\theta}{2} \right) \cdot d\theta \cdot dr \\ &= \frac{r^3}{2} \int_0^R \left(\theta - \frac{\sin 2\theta}{2} \right)_0^\pi \cdot dr \end{aligned}$$

$$= \frac{r^3}{2} \int_0^R (\pi) \cdot dr$$

$$= \frac{\pi}{2} \left(\frac{r^4}{4} \right)_0^R = \frac{\pi R^4}{8}$$

$$= \frac{\pi}{8} \left(\frac{d}{2} \right)^4 \quad \left[\because R = \frac{d}{2} \right]$$

$$= \frac{\pi}{8} \left(\frac{d^4}{16} \right)$$

$$I_{AB} = \frac{\pi d^4}{128} \quad \dots \dots (1.54)$$

Using the parallel axis theorem:

$$I_{xx} = I_{AB} - A d^2 \quad [\because I_{AB} = I_{xx} + A d^2]$$

$$I_{xx} = \frac{\pi d^4}{128} - \frac{\pi}{8} \left(\frac{2d}{3\pi} \right)^2 \quad \left[\because \text{distance} = \frac{4R}{3\pi} = \frac{2d}{3\pi} \right]$$

$$I_{xx} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi}$$

$$I_{xx} = 0.11R^4 \quad \dots \dots (1.55)$$

Problem 1.5 Determine the moment of inertia of the shaded region in Fig. 1.23, about the base AB.

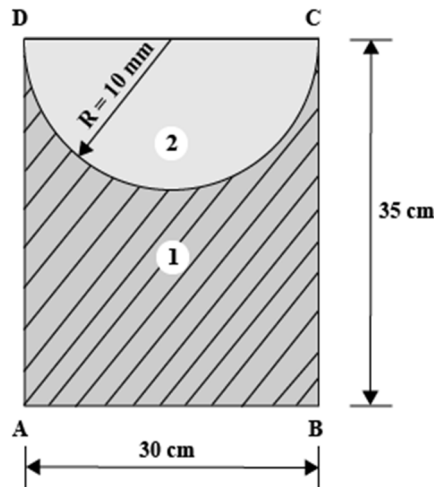


Fig. 1.23: Moment of inertia of the shaded region

Solution:

Semicircle radius (R) = 10 mm

Rectangle width (b) = 30 mm

Rectangle depth (d) = 35 mm

Shaded section moment of inertia about AB = Moment of inertia section-1 about
AB

subtracted by Moment of inertia
section-2 about AB.

$$\text{Moment of inertia section (1) ABCD about AB} = \frac{b d^3}{3} = \frac{30 \times 35^3}{3} = 428750 \text{ mm}^4$$

Moment of inertia section (2) about CD

$$= \frac{1}{2} \times [\text{Moment of inertia of a circle with a 10mm radius about its base}]$$

$$= \frac{1}{2} \times \left[\frac{\pi}{64} \times d^4 \right] = \frac{1}{2} \times \left[\frac{\pi}{64} \times 30^4 \right] = 19870.31 \text{ mm}^4$$

Centroid of a semicircle distance from CD

$$h = \frac{4 r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.246 \text{ mm}$$

$$\text{Semicircle area (A)} = \frac{\pi r^2}{2} = \frac{\pi \times 10^2}{2} = 157 \text{ mm}^2$$

Semicircle moment of inertia about a line through its centroid parallel to CD

$$= (\text{Semicircle moment of inertia about CD}) - [\text{Area} \times (\text{Centroid of a semicircle distance from CD})^2]$$

$$= 19870.31 - [157 \times 4.246^2] = 17039.83 \text{ mm}^2$$

Centroid of the semi-circle distance from AB = $35 - 4.246 = 30.754 \text{ mm}$

$$\begin{aligned} \text{Semi-circle moment of inertia about AB} &= 17039.83 + (157 \times 30.754^2) \\ &= 165530.7 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} \therefore \text{Shaded section moment of inertia about AB} &= 428750 + 165530.7 \\ &= 594280.7 \text{ mm}^4 \end{aligned}$$

1.12.8 MOMENT OF INERTIA OF A T-SECTION

The moment of inertia of the T-section about the centroidal axis X-X, as shown in Fig. 1.24,

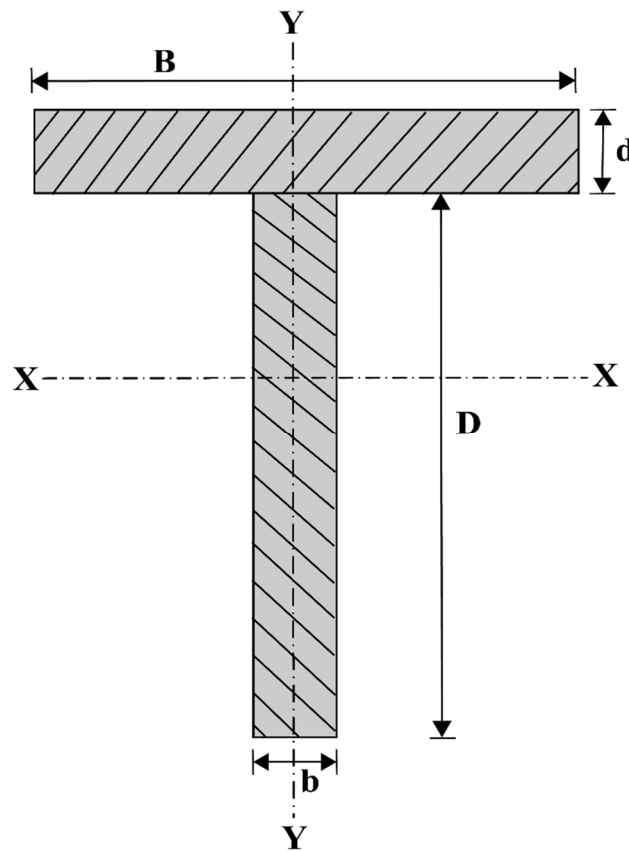


Fig. 1.24: Moment of inertia of the T-section

$$I_{xx} = (I_{x1} + A_1 h_1^2) + (I_{x2} + A_2 h_2^2) \quad \dots \dots (1.56)$$

I_{x1} = Moment of section (1) about its centroid

I_{x2} = Moment of section (2) about its centroid

A_1 = Area of section (1), and A_2 = Area of section (2)

h_1 = Distance between the centroid of the section (1) and the centroid of the given section

h_2 = Distance between the centroid of the section (2) and the centroid of the given section

$$I_{yy} = \frac{bD^3}{12} + \frac{Bd^3}{12} \quad \dots \dots (1.57)$$

[$\therefore$ B=Base, d= Height of section 1]

[$\therefore$ b=Base, D= Height of section 2]

Section modulus,

$$Z_{yy} = \frac{I_{yy}}{x_c} \quad \dots \dots (1.58)$$

Problem 1.6 Fig 1.25 depicts a T-section with dimensions $15 \times 15 \times 5$ mm. Determine the moment of inertia of the section about the vertical and horizontal axes passing through its centroid. Calculate the polar moment of inertia of the given T-section.

Solution:

The centroid of the T-section can be determined by considering the section is symmetric about the Y-Y axis. To calculate the centroid of the section, the T-section is divided into two rectangle sections, ABCD and IJKL.

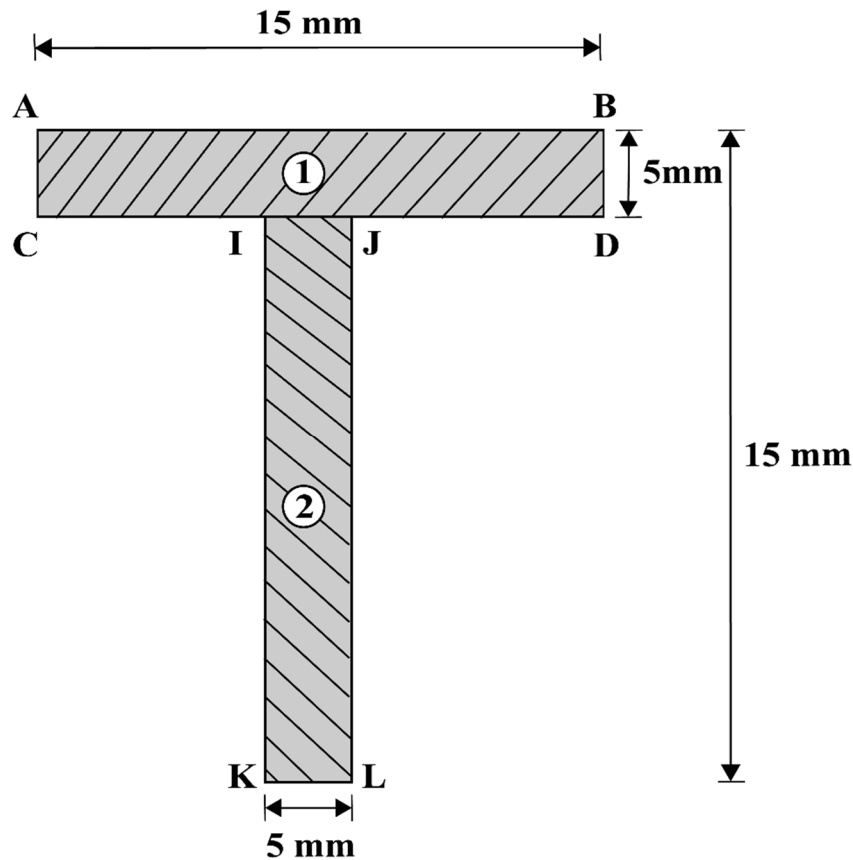


Fig. 1.25: Moment of inertia of the T-section

Let

$\bar{y}$ = Centroid distance of the section from the base-line KL.

A_1 = Area of the section (1) ABCD = $15 \times 5 = 75 \text{ mm}^2$

y_1 = Centroid distance of the area A_1 from the base-line KL = $\frac{5}{2} + 10$
 $= 2.5 + 10 = 12.5 \text{ mm}$

A_2 = Area of the section (2) IJKL = $10 \times 5 = 50 \text{ mm}^2$

y_2 = Centroid distance of the area A_2 from the base-line KL = $\frac{10}{2} = 5 \text{ mm}$

The centroid of the section is obtained by using (Eqn 1.2),

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(75 \times 12.5) + (50 \times 5)}{75 + 50} = \frac{1187.5}{125} = 9.5 \text{ mm}$$

Since the centroid of the T-section is 9.5 mm away from KL, Next, find the moment of inertia of the T-section.

Now,

$$h_1 = y_1 - \bar{y} = 12.5 - 9.5 = 3 \text{ mm}$$

$$h_2 = \bar{y} - y_2 = 9.5 - 5 = 4.5 \text{ mm}$$

Now,

$$I_{x1} = \frac{B d^3}{12} = \frac{15 \times 5^3}{12} = 156.25 \text{ mm}^4 \quad [\because B=\text{Base, } d=\text{Height of section 1}]$$

$$I_{x2} = \frac{b D^3}{12} = \frac{5 \times 10^3}{12} = 2500 \text{ mm}^4 \quad [\because b=\text{Base, } D=\text{Height of section 2}]$$

From the parallel axis theorem,

Moment of inertia section (1) about an X-X axis passes through the centroid of the given section.

$$= I_{x1} + A_1 h_1^2 = 156.25 + (75 \times 3^2) = 156.25 + 675 = 831.25 \text{ mm}^4$$

Moment of inertia section (2) about an X-X axis passes through the centroid of the given section.

$$= I_{x2} + A_2 h_2^2 = 2500 + (50 \times 4.5^2) = 2500 + 1012.5 = 3512.5 \text{ mm}^4$$

$\therefore$ The Moment of inertia about an X-X axis passes through the centroid of the given section,

$$I_{xx} = 831.25 + 3512.5 = 4343.75 \text{ mm}^4$$

The centroid and moment of inertia of the T-section about the Y-Y axis is given by

$$I_{yy} = \frac{b^3 D}{12} + \frac{B^3 d}{12}$$

$$I_{yy} = \left(\frac{5^3 \times 10}{12} + \frac{15^3 \times 5}{12} \right) = 104.166 + 1406.25 = 1510.41 \text{ mm}^4$$

The equation now gives the polar moment of inertia (I_{zz}) as

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = 4343.75 + 1510.41 = 5854.16 \text{ mm}^4$$

Problem 1.7 Determine the moment of inertia of the section shown in Fig 1.26 about the X-axis passing through the centroid.

Solution:

The centroid of the I-section can be determined by considering the section is symmetric about the Y-Y axis. Divide the I section into three rectangle sections: ABCD, IJKL, and EFGH

The centroid of the section is obtained by using (Eqn 1.2),

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

Let $\bar{y}$ = Centroid distance of the section from the base-line GH.

$$A_1 = \text{Area of the section (1) ABCD} = 12 \times 4 = 48 \text{ mm}^2$$

$$y_1 = \text{Centroid distance of the area } A_1 \text{ from the base-line GH} = \frac{4}{2} + 16 + 4$$

$$= 2 + 16 + 4 = 22 \text{ mm}$$

$$A_2 = \text{Area of the section (2) IJKL} = 16 \times 4 = 64 \text{ mm}^2$$

$$y_2 = \text{Centroid of the area } A_2 \text{ distance from the base-line GH} = \frac{16}{2} + 4$$

$$= 8 + 4 = 12 \text{ mm}$$

$$A_3 = \text{Area of the section (3) EFGH} = 25 \times 4 = 100 \text{ mm}^2$$

$$y_3 = \text{Centroid distance of the area } A_3 \text{ from the base-line GH} = \frac{4}{2} = 2 \text{ mm}$$

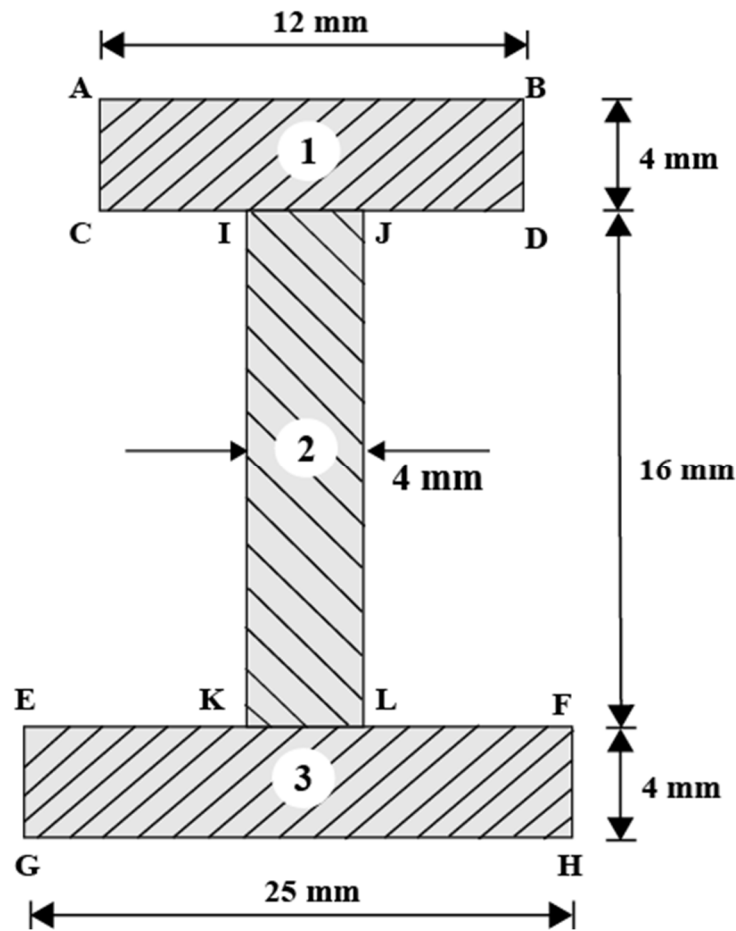


Fig. 1.26 Moment of inertia of the I-section

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = \frac{(48 \times 22) + (64 \times 12) + (100 \times 2)}{48 + 64 + 100} = \frac{2024}{212} = 9.547 \text{ mm}$$

Therefore, determine the centroid of the given section which is 9.547 mm away from GH and calculate the moment of inertia of the I-section.

Let,

$$h_1 = y_1 - \bar{y} = 22 - 9.547 = 12.453 \text{ mm}$$

$$h_2 = y_2 - \bar{y} = 12 - 9.547 = 2.453 \text{ mm}$$

$$h_3 = \bar{y} - y_3 = 9.547 - 2 = 7.547 \text{ mm}$$

Now,

$$I_{x1} = \frac{12 \times 4^3}{12} = 64 \text{ mm}^4$$

$$I_{x2} = \frac{4 \times 16^3}{12} = 1365.33 \text{ mm}^4$$

$$I_{x3} = \frac{25 \times 4^3}{12} = 133.33 \text{ mm}^4$$

The moment of inertia section (1) about an X-X axis passes through the centroid of the given section.

$$= I_{x1} + A_1 h_1^2 = 64 + (48 \times 12.453^2) = 64 + 7443.70 = 7507.70 \text{ mm}^4$$

The moment of inertia section (2) about an X-X axis passing through the centroid of the given section.

$$= I_{x2} + A_2 h_2^2 = 1365.33 + (64 \times 2.453^2) = 1365.33 + 385.10 = 1749.43 \text{ mm}^4$$

The moment of inertia section (3) about an X-X axis passing through the centroid of the given section.

$$= I_{x3} + A_3 h_3^2 = 133.33 + (100 \times 7.547^2) = 133.33 + 5695.72 = 5829.05 \text{ mm}^4$$

∴ The moment of inertia about an X-X axis passing through the centroid of the given section,

$$I_{xx} = 7507.70 + 1749.43 + 5829.05 = 15086.18 \text{ mm}^4$$

Problem 1.8 Determine the polar moment of inertia of the I-section as shown in Fig. 1.27.

Solution:

The centroid of the I-section can be determined by considering the section is symmetric about the Y-Y axis. Divide the given section into three rectangles: ABCD, IJKL, and EFGH.

The centroid of the section is obtained by using (Eqn 1.2),

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

Let $\bar{y}$ = Centroid distance of the section from the base-line GH.

$$A_1 = \text{Area of the section (1) ABCD} = 100 \times 12 = 1200 \text{ mm}^2$$

$$y_1 = \text{Centroid distance of the area } A_1 \text{ from the base-line KL} = \frac{12}{2} + 130 + 20 \\ = 156 \text{ mm}$$

$$A_2 = \text{Area of the section (2) IJKL} = 130 \times 15 = 1950 \text{ mm}^2$$

$$y_2 = \text{Centroid distance of the area } A_2 \text{ from the base-line GH} = \frac{130}{2} + 20 = 85 \text{ mm}$$

$$A_3 = \text{Area of the section (3) EFGH} = 120 \times 20 = 2800 \text{ mm}^2$$

$$y_3 = \text{Centroid distance of the area } A_3 \text{ from the base-line GH} = \frac{20}{2} = 10 \text{ mm}$$

$$\bar{y} = \frac{(1200 \times 156) + (1950 \times 85) + (2800 \times 10)}{1200 + 1950 + 2800} = 64.025 \text{ mm}$$

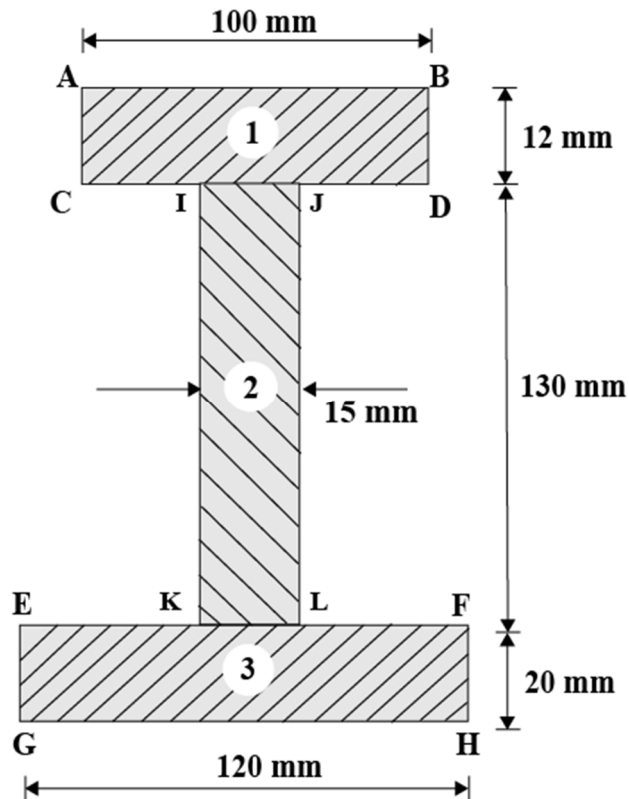


Fig. 1.27 Moment of inertia of the I-section

The centroid of the I section is 64.025 mm away from GH. Calculate the moment of inertia I-section.

Now, let

$$h_1 = y_1 - \bar{y} = 156 - 64.025 = 91.975 \text{ mm}$$

$$h_2 = y_2 - \bar{y} = 85 - 64.025 = 20.975 \text{ mm}$$

$$h_3 = \bar{y} - y_3 = 64.025 - 10 = 54.025 \text{ mm}$$

Now,

$$I_{x1} = \frac{100 \times 12^3}{12} = 14400 \text{ mm}^4$$

$$I_{x2} = \frac{15 \times 130^3}{12} = 2746250 \text{ mm}^4$$

$$I_{x3} = \frac{120 \times 20^3}{12} = 80000 \text{ mm}^4$$

Moment of inertia section (1) about an X-X axis passing through the centroid of the section.

$$= I_{x1} + A_1 h_1^2 = 14400 + (1200 \times 91.975^2) = 1.01 \times 10^7 \text{ mm}^4$$

Moment of inertia section (2) about an X-X axis passing through the centroid of the section.

$$= I_{x2} + A_2 h_2^2 = 2746250 + (1950 \times 20.975^2) = 3.60 \times 10^6 \text{ mm}^4$$

Moment of inertia section (3) about an X-X axis passing through the centroid of the section.

$$= I_{x3} + A_3 h_3^2 = 80000 + (2800 \times 54.025^2) = 8.2 \times 10^6 \text{ mm}^4$$

∴ The moment of inertia section about an X-X axis passing through the centroid of the section is,

$$I_{xx} = (1.01 \times 10^7) + (3.60 \times 10^6) + (8.2 \times 10^6) = 2.2 \times 10^7 \text{ mm}^4$$

The moment of inertia section about a Y-Y axis passing through the centroid of the section,

$$I_{yy} = \left(\frac{100^3 \times 12}{12} + \frac{15^3 \times 130}{12} + \frac{120^3 \times 20}{12} \right)$$

$$I_{yy} = (1 \times 10^6) + (3.6 \times 10^4) + (2.8 \times 10^6)$$

$$= 3916562.5 = 3.9 \times 10^6 \text{ mm}^4$$

The equation now gives the polar moment of inertia (I_{zz}) as

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = (2.2 \times 10^7) + (3.9 \times 10^6) = 2.5 \times 10^7 \text{ mm}^4$$

1.12.9 MOMENT OF INERTIA OF A COMPOSITE SECTION

To determine the moment of inertia of a composite section, the following steps can be followed.

- Divide the given section into different known sections (Triangular, Rectangular, Circular, etc.) and calculate the centroid of the composite section.
- Determine the moment of inertia of each section about its respective centroid.
- Using the parallel axis theorem for transfer the moment of inertia of each section about the required axis (XX axis).

$$I_{xx} = I_{x1} + A_1 h_1^2 + I_{x2} + A_2 h_2^2 + I_{x3} + A_3 h_3^2 \quad \dots \dots (1.59)$$

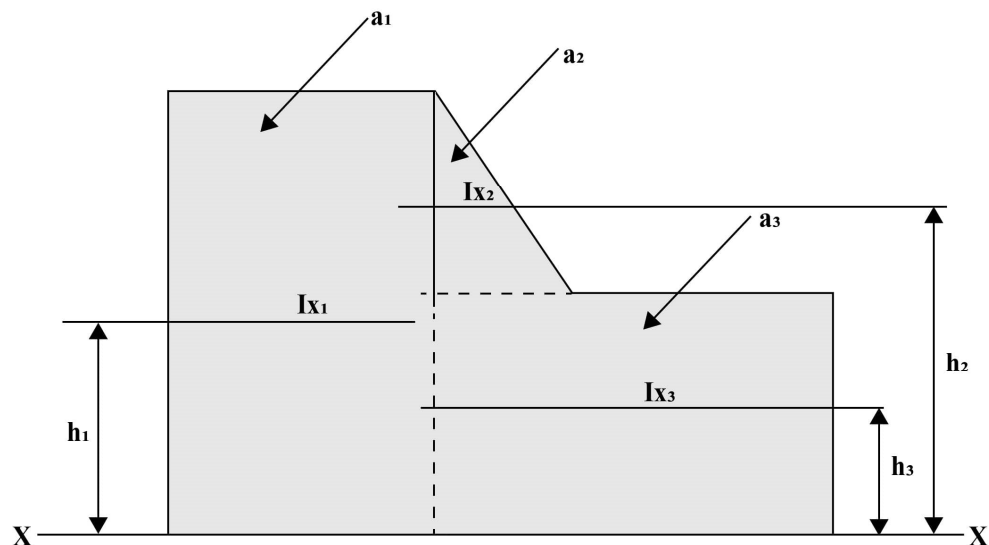


Fig. 1.28: Moment of inertia of the composite section

Where,

I_{xx} = Moment of inertia of the section about XX axis

A = Section area

h = Distance between the required axis and the centroid of the section.

- Therefore, the algebraic sum of the moment of inertia about the appropriate axis provides the moment of inertia of the specified section.

1.12.10 MOMENT OF INERTIA OF A BUILD-UP SECTION

To determine the moment of inertia of a build-up section, which consists of multiple sections like rectangular sections, channel sections, I-sections, etc., by the following steps:

- Calculate the moment of inertia of each section about their respective centre of gravity using the known dimensions of each section.
- Apply the parallel axis theorem to find the moment of inertia of each section to the appropriate Y-Y or X-X axis.

1.12.11 MOMENT OF INERTIA OF A PARABOLIC SECTION

Find the moment of inertia about the X and Y axis for the parabolic section as shown in Fig. 1.29.

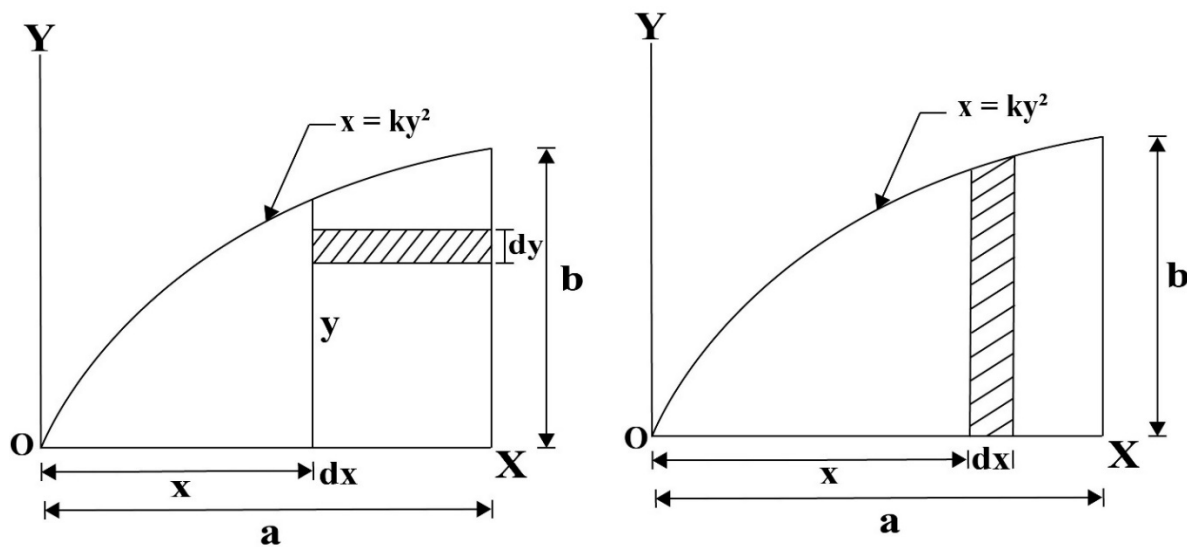


Figure 1.29(a)-(b): Moment of inertia of the parabolic section

Width of the elementary segment = $a - x = a - ky^2$

$$\text{Area} = (a - ky^2) dy$$

$$I_{xx} = (a - ky^2) dy y^2 \quad \dots \dots (1.60)$$

Integrating over the whole area,

$$\begin{aligned} I_{xx} &= \int_0^b (a - ky^2) y^2 dy \\ &= \left(\frac{ay^3}{3} - \frac{ky^5}{5} \right)_0^b \\ I_{xx} &= \left(\frac{ab^3}{3} - \frac{kb^5}{5} \right) \end{aligned}$$

Since $a = kb^2$, $b^2 = \frac{a}{k}$

$$\begin{aligned} I_{xx} &= \left(\frac{ab^3}{3} - \frac{ab^3}{5} \right) \\ I_{xx} &= \frac{2}{15} ab^3 \quad \dots \dots (1.61) \end{aligned}$$

To calculate I_{yy} , consider an elementary segment parallel to the Y-Y axis. For the elementary segment,

$$\begin{aligned} \text{area} &= y dx = y \times 2ky dy, \\ I_{yy} &= 2ky^2 dy x^2. \end{aligned}$$

Therefore,

$$I_{yy} = 2ky^2 dy k^2 y^4 \quad \dots \dots (1.62)$$

Integrating over the whole area,

$$\begin{aligned} I_{yy} &= \int_0^b 2k^3 y b dy = 2k^3 \left(\frac{y^2}{2} \right)_0^b = \frac{2k^3 b^3}{2} \\ I_{yy} &= \frac{2}{7} ba^3 \quad \dots \dots (1.63) \end{aligned}$$

The formula for calculating I_{xx} and I_{yy} is as follows:

$$I_{xx} = \frac{2}{15} ab^3, \text{ and } I_{yy} = \frac{2}{7} ba^3$$

1.13 PRODUCT OF INERTIA

Fig. 1.30 shows the area of the object (A). Consider the dA of a small area. The moment of the area about the x -axis is $(y \cdot dA)$. The moment of $(y \cdot dA)$ about the y -axis is now $(xy \cdot dA)$. Then, $xy \cdot dA$ is referred to as the product of inertia of the area element dA concerning the x -axis and y -axis.

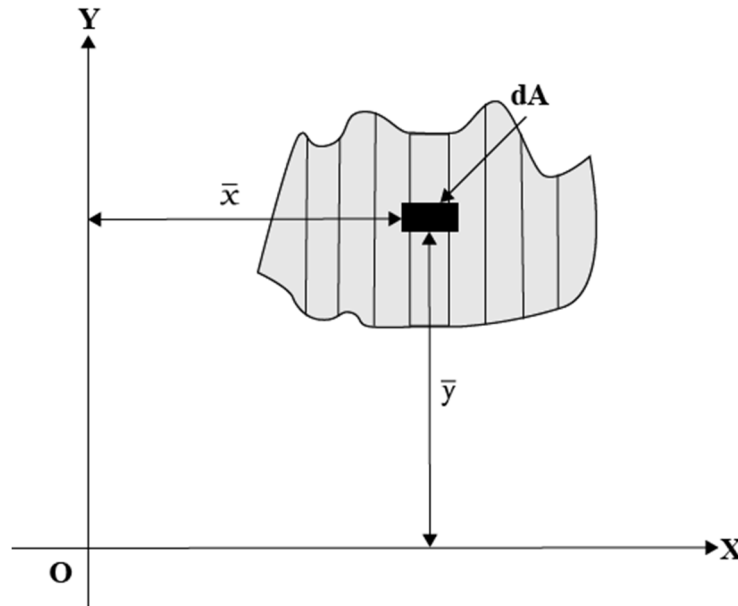


Fig. 1.30: Figure showing the product of inertia

The integral $\int xy \, dA$ is referred to as the product of inertia of the entire area A with respect to both the x and y axes. This product of inertia is typically denoted as I_{xy} .

$$\therefore I_{xy} = \int xy \, dA \quad \dots \dots (1.64)$$

Therefore, the product of inertia of a planar area can be obtained by multiplying an elemental area by the product of its coordinates and then integrating this product over the entire area.

The mathematical expression for the product of inertia (I_{xy}) is given by:

This formula represents how to calculate the product of inertia for a given area.

$$I_{xy} = \sum x_i y_i A_i = x_1 y_1 A_1 + x_2 y_2 A_2 + \dots \dots \dots \quad \dots \dots (1.65)$$

Where,

$x_i y_i$ = coordinates of the centroid of area A_i .

1.14 POLAR MOMENT OF INERTIA

The polar moment of inertia of a plane area is described as the moment of inertia about an axis which is perpendicular to the plane of the figure and passes through the centroid of the area. The polar moment of inertia is denoted by J .

Mathematically, we may express the polar moment of inertia, i.e., J , and is given by the expression,

$$J = \int r^2 dA \quad \dots \dots (1.66)$$

The above equation of the polar moment of inertia allows us to conclude that the polar moment of inertia can be determined by multiplying the area of an element by the square of its distance from the axis of reference. The polar moment of inertia is a significant parameter which is resisting the twisting or torsion. It indicates the strength of the object in its ability to withstand torsion loading.

Considering Fig. 1.31, if the X and Y axis in the plane area are mutually perpendicular to each other and passing through point O, then

$$J_{zz} = \int_0^A r^2 dA = \int_0^A (x^2 + y^2) dA$$

$$J_{zz} = \int_0^A x^2 dA + \int_0^A y^2 dA$$

$$J_{zz} = I_{xx} + I_{yy} \quad \dots \dots (1.67)$$

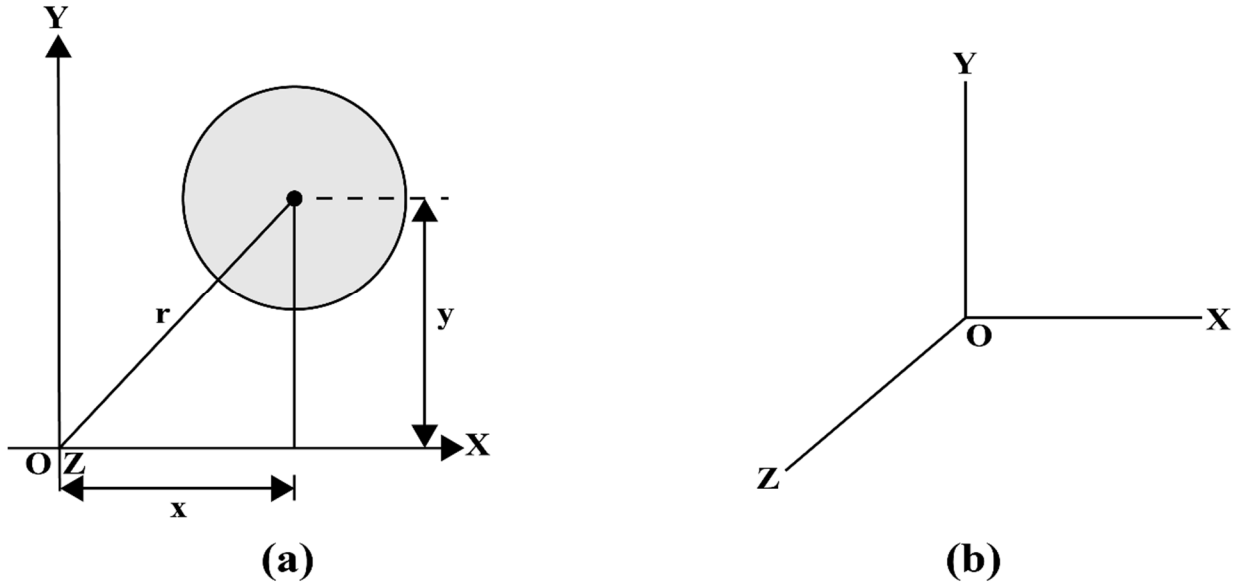


Fig. 1.31: Figure showing the Polar moment of inertia

It is understood that moment of inertia is employed in the design of structural components. However, when the structure is exposed to deflection or bending, the area moment of inertia is considered, but the polar moment of inertia is considered when the structure is subjected to torsional loading.

UNIT SUMMARY

- The chapter on Moment of Inertia delves into a fundamental concept in physics and engineering that describes an object's resistance to external moments.
- It begins by introducing the concept of rotational inertia, akin to mass in linear motion.
- The chapter elucidates the mathematical definition of moment of inertia for various geometric shapes, including point masses, rods, disks, and more complex objects.
- Moreover, the chapter explores the parallel axis theorem and perpendicular axis theorem, indispensable tools for determining moment of inertia for objects with non-axisymmetric shapes.
- Applications in engineering, such as in designing rotating machinery and analyzing the stability of structures, are also highlighted to illustrate the practical relevance of the concept of mass moment of inertia.
- In summary, the chapter on Moment of Inertia provides a comprehensive understanding of this crucial concept in mass moment of inertia, laying the groundwork for further exploration in physics and engineering disciplines.

EXERCISE

A. Multiple Choice Questions

- 1.1 The rotational inertia of a rigid body is referred to as its
(a) Moment of energy (c) Moment of inertia
(b) Moment of force (d) Moment of acceleration
- 1.2 If a body is rotating about an axis passing through its center of mass, the angular momentum of the body is directed along its
(a) Circumference (c) Axis of rotation
(b) Radius (d) None of the options
- 1.3 Linear velocities of all the particles of the body in rotational motion is
(a) 1 (c) Same
(b) 0 (d) Different
- 1.4 The center of mass of a body
(a) lies inside the body (c) lies on the surface of the body always
(b) lies outside the body always (d) None of the options
- 1.5 Radius of gyration is denoted by
(a) R (c) K
(b) G (d) I
- 1.6 The combination of rotational motion and the translational motion of a rigid body is known as
(a) Frictional motion (c) Angular motion
(b) Axis motion (d) Rolling motion
- 1.7 Moment of inertia depends on
(a) Shape and size of the body (c) Position of the axis of rotation
(b) Mass (d) All of these

1.8 The M.I. of a uniform circular disc about diameter is I . Its M.I. about an axis perpendicular to its plane passing through a point on its rim will be

- (a) $4I$
- (b) $6I$
- (c) $8I$
- (d) $9I$

1.9 The moment of inertia of a triangular section of base 'a' and height 'L' about an axis passing through its C.G and parallel to the base is

- (a) $aL^3/36$
- (b) $aL^3/6$
- (c) $aL^3/8$
- (d) $aL^3/12$

1.10 If the area is in mm^2 and distance in mm, then M.I is given as

- (a) mm^3
- (b) mm^4
- (c) mm^2
- (d) mm

1.11 The product of moment of inertia about the principal axes is

- (a) 0
- (b) Infinite
- (c) < 1
- (d) 1

1.12 Mass moment of inertia of a uniform thin rod of mass (m) and length (l) about its mid-point and perpendicular to its length is

- (a) $2/3ml^2$
- (b) $ml^2/12$
- (c) $ml^2/3$
- (d) $3/4ml^2$

1.13 Moment of inertia of a thin spherical shell of mass M and radius R, about its diameter is

- (a) MR^2
- (b) $1/2 MR^2$
- (c) $2/5 MR^2$
- (d) $2/3 MR^2$

1.14 Centre of gravity of a thin hollow cone lies on the axis of symmetry at a height of

- (a) one-half of the total height above base
- (b) one-third of the total height above base
- (c) one-fourth of the total height above base
- (d) None of these

- 1.15 Moment of inertia of a thin spherical shell of mass M and radius R about a diameter is
 (a) $\frac{2}{5} MR^2$ (c) $\frac{2}{3} MR^2$
 (b) $\frac{4}{5} MR^2$ (d) $\frac{3}{5} MR^2$
- 1.16 Moment of inertia about an axis perpendicular to the plane of an area is known as
 (a) First moment (c) Polar moment
 (b) Second moment (d) Axial moment
- 1.17 The moment of inertia of a circular section about an axis perpendicular to the section is
 (a) $\frac{\pi d^3}{32}$ (c) $\frac{\pi d^4}{32}$
 (b) $\frac{\pi d^4}{64}$ (d) $\frac{\pi d^3}{64}$
- 1.18 The moment of inertia of a body is always minimum with respect to its
 (a) Base (c) Vertical axis
 (b) Centroid axis (d) Horizontal axis

Answer to Multiple-Choice Questions

1.1 (c), 1.2 (c), 1.3 (d), 1.4 (d), 1.5 (c), 1.6 (d), 1.7 (d), 1.8 (d), 1.9 (a), 1.10 (b), 1.11 (a), 1.12 (b), 1.13 (d), 1.14 (b), 1.15 (c), 1.16 (c), 1.17 (c), 1.18 (b)

B. Subjective Questions

- 1.1 How would you find out the moment of inertia of a plane area?
- 1.2 How to calculate the moment of inertia about its vertical axis?
- 1.3 What are the factors on which the moment of inertia depends?
- 1.4 State and prove the theorem of parallel and perpendicular axis applied to moment of inertia
- 1.5 Describe the method of finding out the moment of inertia of a composite section
- 1.6 Derive an equation for moment of inertia of the following sections about centroids axis: (a) rectangular section, (b) circular section.

1.7 What is the difference between inertia and moment of inertia.

C. Numerical Problems

1. Determine the moment of inertia of the centroidal axes for the circular plate with a diameter R hole punched out, as illustrated in Fig. 1.32.

$$\left[\text{Ans: } I_{XX} = \frac{15 \pi R^4}{64}; I_{YY} = \frac{29 \pi R^4}{192} \right]$$

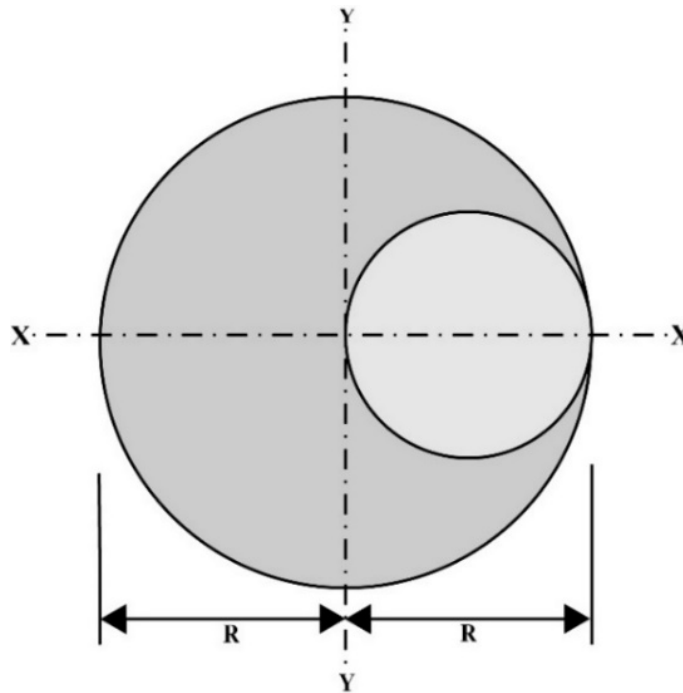


Fig. 1.32: Moment of inertia of the circular plate

2. Determine the Moment of inertia of a T-section having web and flange both $120 \text{ mm} \times 30 \text{ mm}$ about the X-X axis passes through the centre of gravity of the section. (Ans: $14715 \times 10^3 \text{ mm}^4$)
3. Determine the Moment of inertia of an I-section with equal sizes of web and flanges of $30 \text{ mm} \times 10 \text{ mm}$ about an axis that passes through its centre of gravity and is parallel to the Y-Y and X-X axes. (Ans: $267.5 \times 10^3 \text{ mm}^4$; $47 \times 10^3 \text{ mm}^4$)
4. Find the centre of gravity of a T-section as shown in Fig. 1.33 (Ans: 8.272 cm)

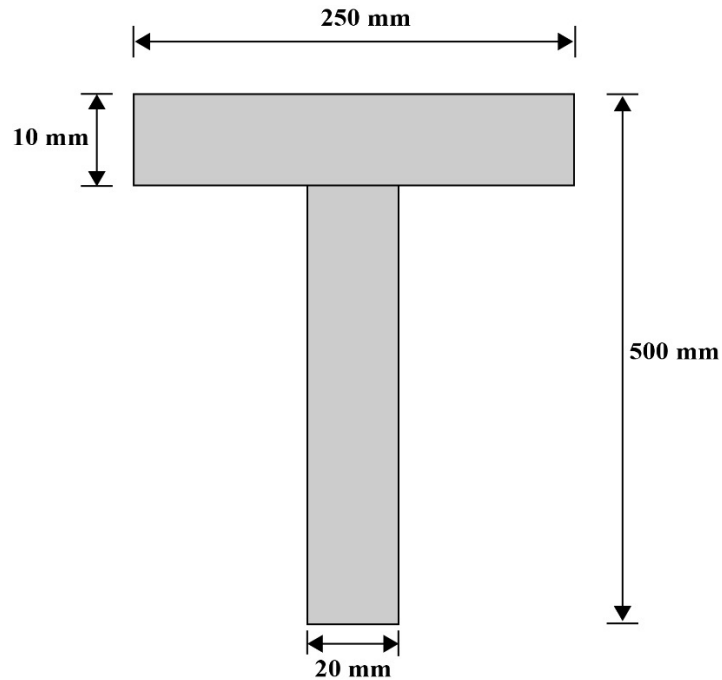


Fig. 1.33: Moment of inertia of a lamina with circular hole

5. Fig. 1.34 showing a lamina with a circular hole of 30 mm diameter about the axis XY. (Ans: $638.3 \times 10^3 \text{ mm}^4$)

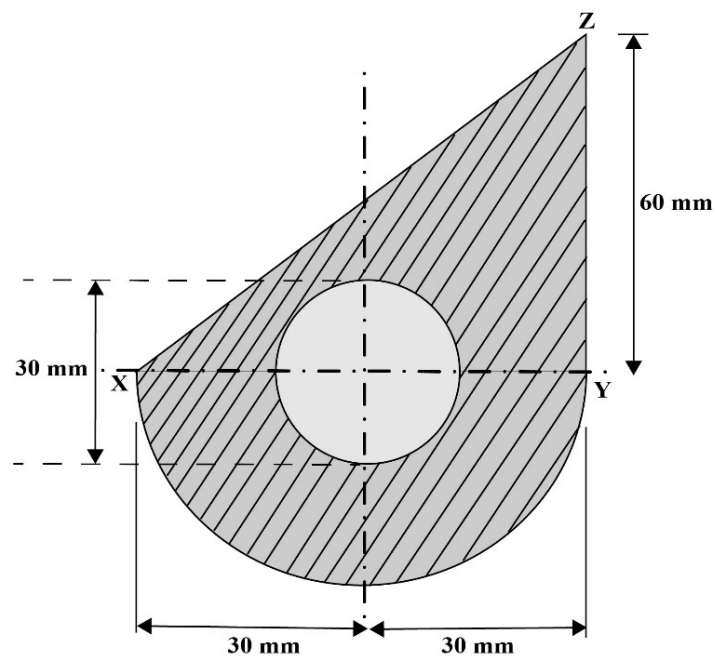


Fig. 1.34: Figure showing the centre of gravity of a T-section

KNOW MORE

- *Automotive Industry:* Moment of inertia is crucial in designing vehicle components such as wheels, axles, and drivetrains. Engineers use it to optimize acceleration, braking, and overall performance while ensuring stability and safety.
- *Aerospace Engineering:* In aerospace applications, moment of inertia plays a vital role in designing aircraft and spacecraft components. It affects factors like maneuverability, stability, and control systems, contributing to safe and efficient flight.
- *Robotics:* Moment of inertia is essential in robotics for designing robot arms, joints, and actuators. Engineers use it to optimize motion control, ensure precise positioning, and improve overall performance in robotic systems.
- *Industrial Machinery:* Moment of inertia is critical in various industrial applications, including conveyor systems, material handling equipment, and manufacturing machinery. Engineers use it to optimize machine performance, reduce energy consumption, and ensure smooth operation in industrial processes.
- *Sports Equipment:* Moment of inertia influences the performance of sports equipment such as golf clubs, tennis rackets, and bicycles. Designers use it to optimize power, control, and responsiveness, enhancing the performance and experience for athletes.
- *Architectural Engineering:* In architectural design, moment of inertia helps engineers analyze and design structural components such as beams, columns, and foundations. It ensures structural stability, strength, and resistance to loads and forces.

REFERENCE AND SUGGESTED READINGS

1. J.L. Meriam and L.G. Kraige, Engineering mechanics: Dynamics, Wiley Publisher, New York
2. D.S. Bedi, “Engineering Mechanics”; Khanna publications, New Delhi.
3. Khurmi, R.S., Strength of Materials, S Chand and Co. Ltd. New Delhi.
4. Bansal RK, “A text book of Engineering Mechanics”; Laxmi publications, New Delhi
5. Bedi D.S., Strength of Materials, Khanna Publishing House, Delhi, Ed. 2018
6. Punmia B C, Strength of Materials, Laxmi Publications (p) Ltd. New Delhi.
7. Rattan S.S., Strength of Materials, McGraw Hill Education; New Delhi.
8. Subramaniam R, Strength of Materials, Oxford University Press.
9. Timoshenko, S., Strength of Materials, Vol. I, CBS, New Delhi.

Dynamic QR Code for Further Reading

Moment of inertia
of I section



Moment of inertia
of Z section



Moment of inertia
of C section



Moment of inertia
of composite section



Theorem of perpendicular
and parallel axes



Experiments based on
moment of inertia



Formulas of moment of inertia
for all the shapes



Concept of Centroid



2

SIMPLE STRESSES AND STRAINS

UNIT SPECIFICS

Upon completion of this chapter, the reader will have acquired the following abilities:

- *Classification of stress.*
- *Express and utilize Hooke's law.*
- *Calculate stress intensities resulting from applied forces in both simple and complex sections.*
- *Determine stresses and strains due to temperature.*
- *Calculate the reactive forces in basic statically indeterminate systems.*
- *Deduce and articulate the relationships among elastic constants.*
- *Provide definitions and explanations for the elastic attributes of materials.*
- *Elaborate on the concepts of stress concentration, residual stress, and fatigue.*

This unit serves to acquaint readers with the fundamental concepts of stress and strain, pivotal in understanding the behavior of materials under mechanical loads. Stress, defined as the force per unit area, represents the internal resistance of a material to external forces, while strain measures the deformation of the material relative to its original dimensions. Through this unit, readers will grasp the basics of stress-strain relationships, including elastic, plastic, and failure behaviors, crucial for analyzing and designing structures in various engineering disciplines.

Apart from this at the end of the unit, a succinct recapitulation of the overall broad concepts is provided in the form of a unit summary. Besides, a large number of multiple-choice questions as well as descriptive type questions with Bloom's taxonomy action verbs are included. A list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge. Video resources along with QR codes are given for getting more information on various topics of interest which can be surfed or scanned through mobile phones for viewing.

RATIONALE

The concept of stress and strain forms the basis of understanding the behavior of materials under applied loads, and it's fundamental in various fields such as engineering, physics, and materials science. Stress refers to the force applied per unit area on a material, while strain represents the resulting deformation or change in shape of the material relative to its original size. The relationship between stress and strain is typically described by material properties such as Young's Modulus or the stress-strain curve. In civil engineering, for instance, stress and strain analysis is crucial for designing safe and durable infrastructure like bridges and buildings, ensuring that they can support expected loads without excessive deformation or failure. Similarly, in materials science, understanding stress and strain helps researchers develop new materials with desired mechanical properties for various applications, from aerospace to biomedical engineering. Overall, the simple yet powerful concepts of stress and strain provide a fundamental framework for analyzing engineering materials and structures across diverse disciplines.

PRE-REQUISITES

Before reading this unit, the students are advised to revisit the following:

Engineering Mechanics: Sem I

Physics: Mechanical Properties of Solids (Class XI)

UNIT OUTCOMES

After completing this unit, you can be able to:

U2-O1: Have comprehensive understanding of stress and strain in rigid, elastic, and plastic bodies.

U2-O2: Analysis of deformation of elastic bodies under different forces.

U2-O3: Knowledge of various types of stress, including normal stress (tensile, compressive), shear stress, torsional stress, and bending stress.

U2-O4: Understanding of elasticity principles such as Hooke's law, Young's modulus, and stress-strain relations for different materials.

U2-O5: Familiarity with concepts like true stress, nominal stress, factor of safety, thermal stress, strains types, and elastic moduli relationships.

Unit-2 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1-Weak Correction; 2- Medium Correction; 3- Strong Correction)				
	CO-1	CO-2	CO-3	CO-4	CO-5
U2-O1	3	1	2	1	2
U2-O2	3	1	2	1	2
U2-O3	3	1	2	1	2
U2-O4	3	1	2	1	2
U2-O5	3	1	2	1	2

2.1 INTRODUCTION

When an object is subjected to an external force, it tends to deform. This force is resisted due to the cohesion between the molecules within the object. The reaction to this force exhibited by the material is referred to as the stress in the material. Up to a certain threshold, typically in the elastic stage, the material's resistance is directly proportional to the deformation induced by the external force. Moreover, during this elastic phase, the resistance matches the magnitude of the applied force. However, once the elastic limit is surpassed, the material's resistance becomes less than the applied load, leading to continuous deformation until, eventually, failure occurs. The force resisting deformation per unit area is termed as stress.

2.2 RIGID BODIES

- A rigid body is characterized as a body that maintains its shape and size without going through any deformation in response to an external force. In such cases the distance between the two points in the body remains constant even when exposed to external forces.
- In reality, body is never perfectly rigid. When subjected to an external force, a member experiences some degree of deformation in its shape or size. However, if these deformations are sufficiently small to be negligible, the bodies are considered as rigid bodies.
- Nevertheless, if these deformations are finite in specific applications, it becomes critical to regard the body as deformable. Such bodies include bars, beams, columns, shafts, and pressure vessels.

2.3 ELASTIC BODIES

- Elasticity is the characteristic of material that allows it to undergo deformation when subjected to external forces and revert to its original shape upon the removal of external loads.
- Materials that exhibit this property of elasticity are referred to as elastic materials. Some examples of elastic materials include (metals and non-metallic materials like steel, etc).

2.4 PLASTIC BODIES

- Plasticity is the quality of materials that the bodies undergo permanent deformation and does not regain the original shape once the deforming force is no longer applied.
- Materials that exhibit characteristic of plasticity are referred to as plastic materials. Such materials include wet clay, paraffin wax, etc.

2.5 DEFORMATION OF AN ELASTIC BODY UNDER VARIOUS FORCES

The forces exerted on a body are referred to as loads. When this load is concentrated at a single point, it is known as a point load, whereas if it is distributed over a length, it is termed as distributed load. When the distributed load remains constant across its length, it is called a uniformly distributed load.

When the entire structure is in a state of equilibrium, it implies that its members are also in equilibrium, meaning that the sum of all the forces acting on a member must be zero. However, these forces applied to the body tend to cause deformation or tearing of the body. For instance, a load denoted as "P" acting on a body tends to pull it apart (as shown in Fig. 2.1a). This pulling force can also be applied if one

end of the body is fixed (as shown in Fig. 2.1b). In this case, the balancing force is provided by the reaction at the fixed end. Such a pulling force is known as a tension force. A tensile force tends to increase the length of the body while decreasing its cross-sectional area.

Similarly, a compression force tends to push or compress a body, causing the body to reduce in length (as illustrated in Fig. 2.1c).

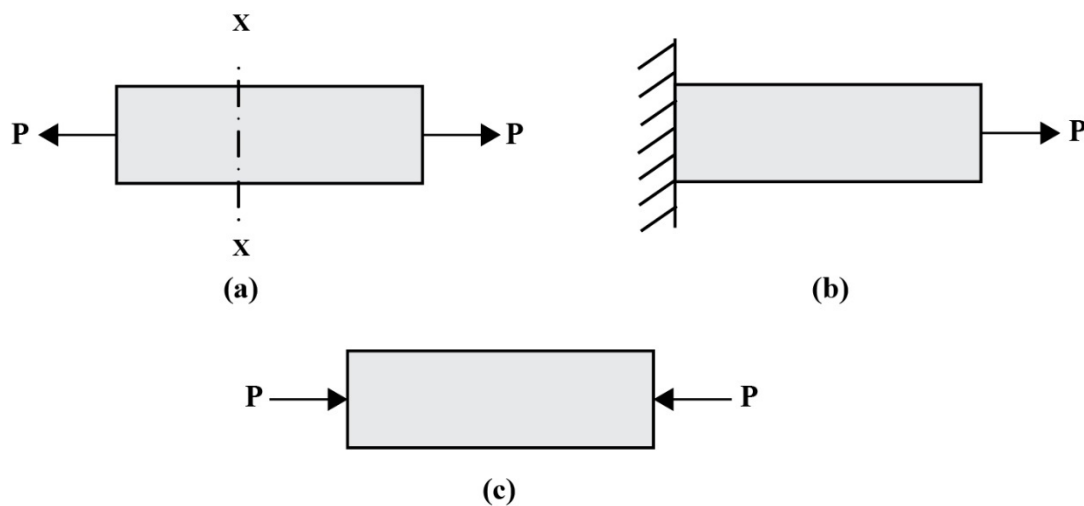


Fig. 2.1(a)-(c): Deformation of an elastic body under various forces

Generally, forces that act along the longitudinal axis of a body are referred to as direct or axial forces, while forces acting perpendicular to the longitudinal axis are termed transverse force.

In the basic principles of structural analysis, it is often considered that a material subjected to external forces behaves as perfectly elastic. It means any deformation induced in the material get completely recovered once the applied forces are removed. The materials are isotropic possessing uniform properties in all directions and homogeneous with consistent properties throughout the body.

2.6 STRESS

Every material exhibits some degree of elasticity by its nature. Therefore, when external forces are applied to the body, it experiences a certain degree of deformation. As the body is loaded, its molecules produce resistance. This resistance is commonly referred to as stress since it is determined as load per unit deformation area. Stress may be described mathematically as the force operating on a body divided by the area over which it is distributed. Thus,

$$\sigma = \frac{P}{A} \quad \dots \dots (2.1)$$

Stress (σ) = Force (P) / Area (A),

Where P represents the load acting on the body, and A is the cross-sectional area of the body. The stress unit in the S.I system is the pascal (Pa), equivalent to one newton per square meter (1 N/m²).

However, in practical applications, larger units of stress are commonly used, such as the megapascal (MPa), which is equivalent to 1 million pascals (1 N/mm²), or the gigapascal (GPa), which is equivalent to 1 billion pascals (1 kN/mm²). These larger units are more convenient when dealing with actual problems related to materials and structures.

2.6.1 UNIT OF STRESS

The stress unit is determined by the units used for the applied force and the area. In the MKS (meter-kilogram-second) system, if the force is in (kgf) and the area is in (m²), then the unit of stress is in (kgf/m²).

Conversely, if the area is in (cm²), the stress unit is (kgf/cm²). The choice of units depends on the specific requirements and conventions used in a particular context or industry. In the S.I units, force in newtons (N) and area is m².

2.7 STRAIN

When force is applied to a body, it undergoes some degree of deformation as shown in Fig. 2.2. Strain is defined as deformation measured per unit length. Strain may be described mathematically as the amount of deformation divided by the initial length of the body.

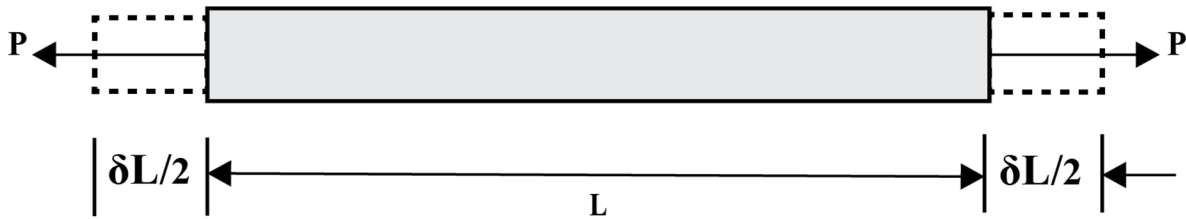


Fig. 2.2 Figure showing the normal strain

$$\varepsilon = \frac{\delta L}{L} \quad \dots \dots (2.2)$$

" δL " represents the change in length, and " L " represents the initial length of the body.

2.8 TYPES OF STRESSES

There are different types of stresses given in Fig. 2.3. In this section, we will delve into each type of stress in detail. Fundamentally, stresses can be categorized into three main types.

- Simple stress or Direct stress

- Combined stress
- Indirect stress

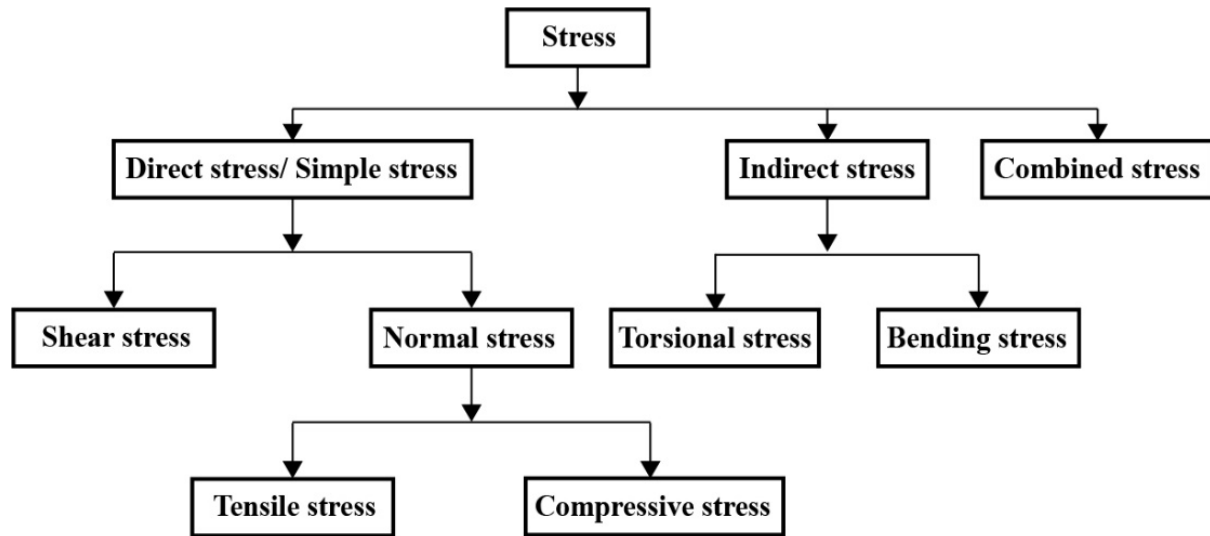


Fig. 2.3: Figure showing the Types of stresses

Additionally, direct stress or simple stress can be further classified into two types:

1. Normal Stress

- Tensile Stress
- Compressive Stress

2. Shear Stress

Similarly, indirect stress is also classified into two main types:

- Torsion Stress
- Bending Stress

2.8.1 NORMAL STRESS

Normal stress is fundamentally characterized as the stress that acts in a direction perpendicular to the area under consideration. As mentioned earlier, this normal stress can be either compressive or tensile.

2.8.1.1 TENSILE STRESS

The stress that is developed in a body when subjected to two equal and opposite forces, as shown in Fig. 2.4, increasing its length, is termed "tensile stress." Tensile stress occurs in a direction perpendicular to the area under consideration and exerts a pulling force on the area. Tensile strain is defined as the ratio of the increase in length to the initial length. The relationship between these two helps to quantify how a material responds to tensile stress and its elongation due to the load.

Let

P – represents the force exerted on the body.

A – is the area of the cross-section of the body.

L – the initial length of the body.

dL – increase in length of the body.

σ – stress on the body

ϵ – strain in the body.

Fig. 2.4(a) shows a bar with P as the tensile force applied at its ends. Consider the section X-X, which separates the bar into two sections.

- If the applied force P matches the resisting force (R), the left part of the X-X portion will be in equilibrium, as depicted in Fig. 2.4(b).
- Similarly, if the resisting force balances the applied force P , the right part of the X-X portion will be in equilibrium, as depicted in Fig. 2.4(c).

- The resisting force per unit area that ensures material balance is known as stress intensity. It represents the internal force within the material that counters the applied force and maintains equilibrium, as depicted in Fig. 2.4(d).

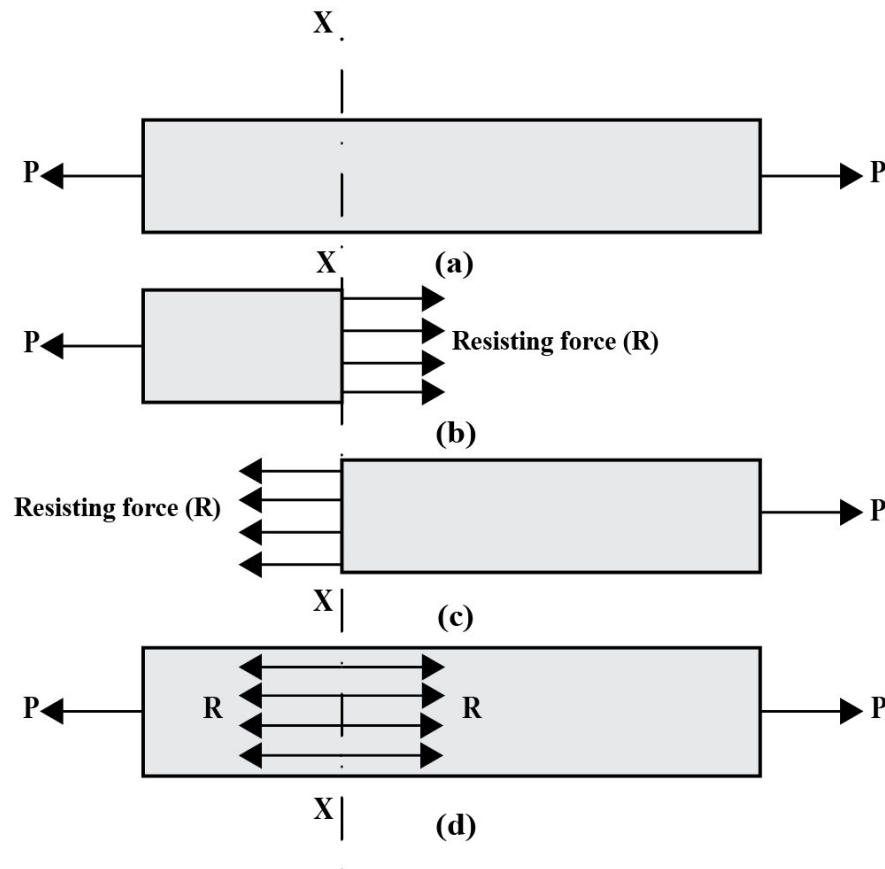


Fig. 2.4(a)-(d): Figure showing the Tensile stress

$$\therefore \text{Tensile stress } (\sigma) = \frac{\text{Resisting force @}}{\text{Cross-sectional area}} = \frac{\text{Tensile Load (P)}}{A} \quad \because P = R$$

$$\sigma = \frac{P}{A}$$

$$\therefore \text{Tensile strain} = \varepsilon = \frac{\text{Increase in length}}{\text{Original length}}$$

$$\varepsilon = \frac{\delta l}{l} \quad \dots \dots (2.3)$$

2.8.1.2 COMPRESSIVE STRESS

Compressive stress occurs in a body when two equal and opposing forces push toward each other, as shown in Fig. 2.5(a-d). Due to the compressive force, there is a reduction in the length of the body. Compressive strain is the ratio of the reduction in length to the initial length of the body. The stress-strain relations will be useful in understanding the material behaviour for different load conditions.

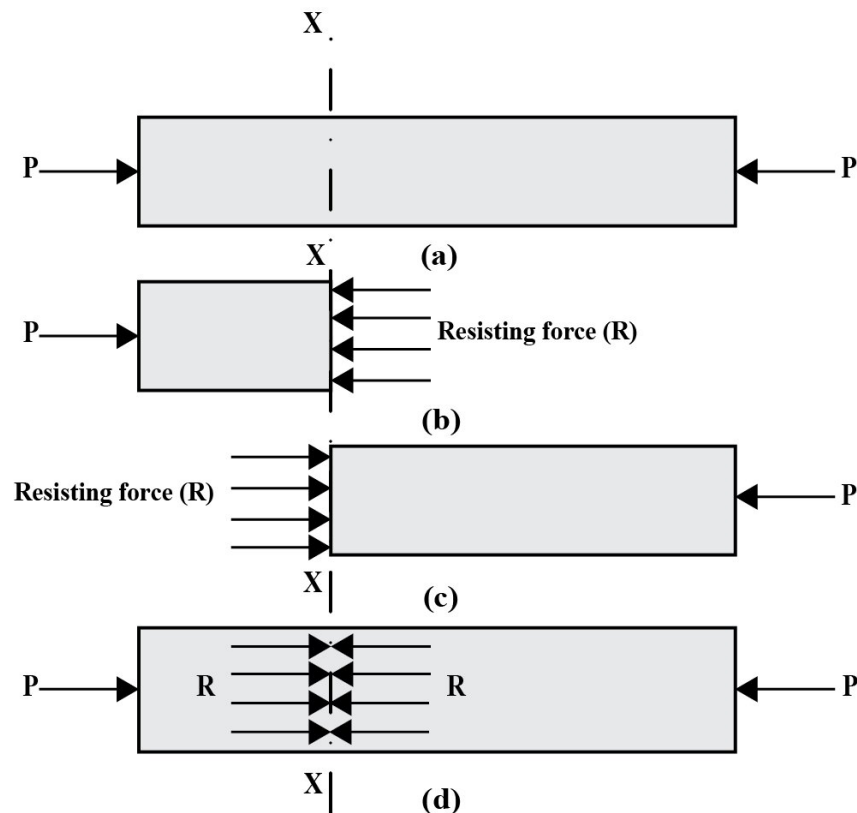


Fig. 2.5(a)-(d): Figure showing the Compressive stress

Compressive stress acts perpendicular to the area under consideration and exerts a pushing force.

Let

P : Represents the axial compressive force acting on the body.

A : The area of cross-sectional of the body.

L: The initial length of the body.

dL: The decrease in length of the body.

The compressive stress formula (σ) is as follows:

$$\sigma = \frac{P}{A}$$

The expression for compressive strain (ϵ) is:

$$\epsilon = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{dL}{L} \quad \dots \dots (2.4)$$

This equation reflects the ratio of reduction length (dL) to the initial length (L) of the body which is known as compressive stress.

2.8.2 SHEAR STRESS

The shear stress arises within a body when two equal and opposing forces act tangentially on a resisting section, as shown in Fig. 2.6. This form of stress leads to shear or slide apart across the section of the body.

Shear stress is specifically characterized by its direction, tangential to the area under consideration, and is denoted by τ .

The deformation or change in the shape due to the shear stress is called shear strain. The ratio of shear stress to shear strain is known as shear modulus.

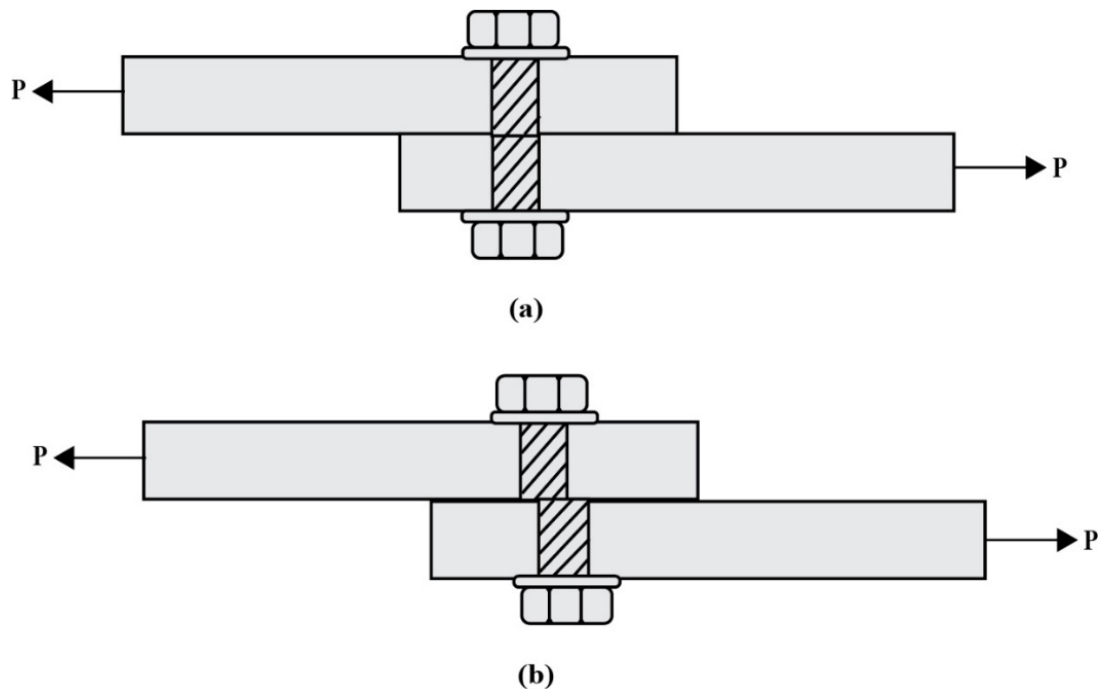


Fig. 2.6(a)-(b): Figure showing the Shear stress

In Fig. 2.7(a), the lower surface of the block is fixed. A tangential force denoted as P is applied on the upper face of the block, and the shear force is applied tangentially on the surface. To maintain equilibrium for the block, the surface EF will exert a tangential reaction force, which matches the magnitude of the applied shear force P but acts in the opposite direction. This reaction force from surface EF prevents the block from experiencing any translational motion in response to the shear force and maintains it in equilibrium.

Consider the section $X-X$, which is parallel to a force applied. This section splits the block into two parts.

- The upper section of the block remains in equilibrium when the applied shear force P is balanced by a resisting force R , as shown in Fig. 2.7(b).

- The lower part of the block is also in equilibrium if the applied shear force P is countered by an equal resisting force R , as shown in Fig. 2.7(c).
- The shear resistance is the internal force in the material which resists the applied force to maintain equilibrium. Shear stress is generally expressed as shear resistance per unit area. Shear stress quantifies the intensity of the internal forces acting within the material in response to the applied shear force.

$$\therefore \text{Shear Stress } (\tau) = \frac{\text{Shear resistance}}{\text{Shear area}} = \frac{R}{A} \quad \dots \dots (2.5)$$

$$\therefore \text{Shear Stress, } \tau = \frac{P}{A} \quad \because (R = P)$$

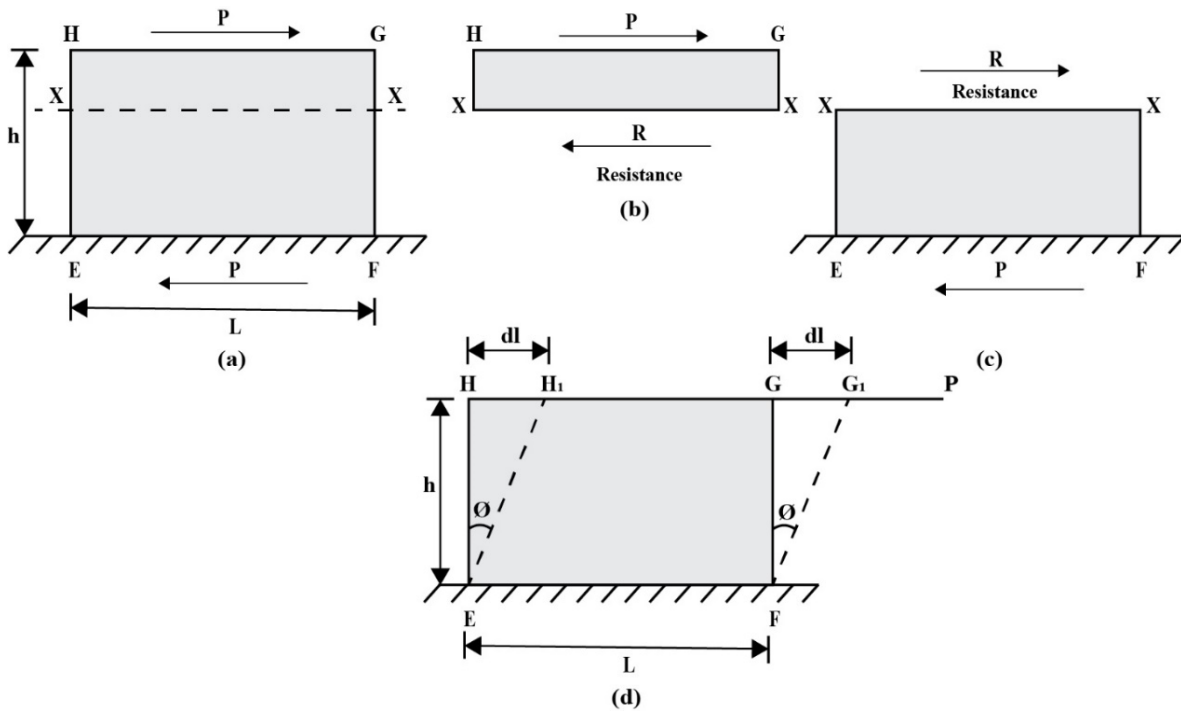


Figure 2.7(a)-(d): Strain energy due to shear loading

Consider that the lower face of the block is fixed when the shear force P is applied, the face $EFGH$ will undergo distortion and rotate to a new position, EFG_1H_1 , through an angle represented by ϕ , as illustrated in Fig. 2.7(d). This rotation or angular deformation is a result of the applied shear force and signifies the response of the material to the shearing action. The magnitude of this angular deformation is an important factor in understanding how the material behaves under shear stress.

The shear strain (ϕ) is given by,

$$\phi = \frac{\text{Transversal displacement}}{\text{Distance EH}}$$

$$\phi = \frac{HH_1}{EH} = \frac{dl}{h} \quad \dots \dots (2.6)$$

2.8.3 TORSIONAL STRESS

Torsional stress is a different form of shear stress that occurs in a structural member when one end is fixed while the other is subjected to twisting or torsional action.

For example, consider a shaft in which one end is fixed while the other end is free to rotate, as shown in Fig. 2.8. When a twisting force is applied at the free end of the shaft, it develops torsional stress along the length of the shaft. This type of stress is important in the design and analysis of components like shafts and axles, which experience torsional loads during operation.

As one end is fixed, it is not allowed to rotate freely. In this situation, the other end of the shaft, which is still attempting to rotate, creates a twisting mechanism within the shaft. This twisting action induces stress within the shaft, which is termed

torsional stress. Torsional stress is the result of the opposing rotational forces at the two ends of the shaft.

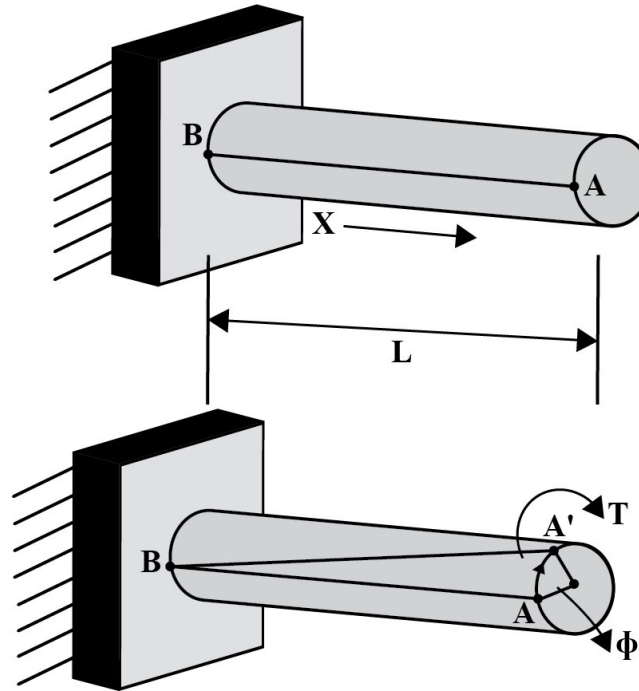


Fig. 2.8: Figure showing the Torsional stress

Torsional stress is a significant factor in rotating members, particularly in components like rotating shafts. This type of stress is indicative of the twisting or torsional forces experienced by such members during operation. Torsional stress is the shear stress which is represented by the " τ ".

2.8.4 BENDING STRESS

Consider a beam supported or fixed at one end A , and the other end is free. Force P is applied at the free end of the beam, as shown in Fig. 2.9. The beam deflects or bends due to the applied load.

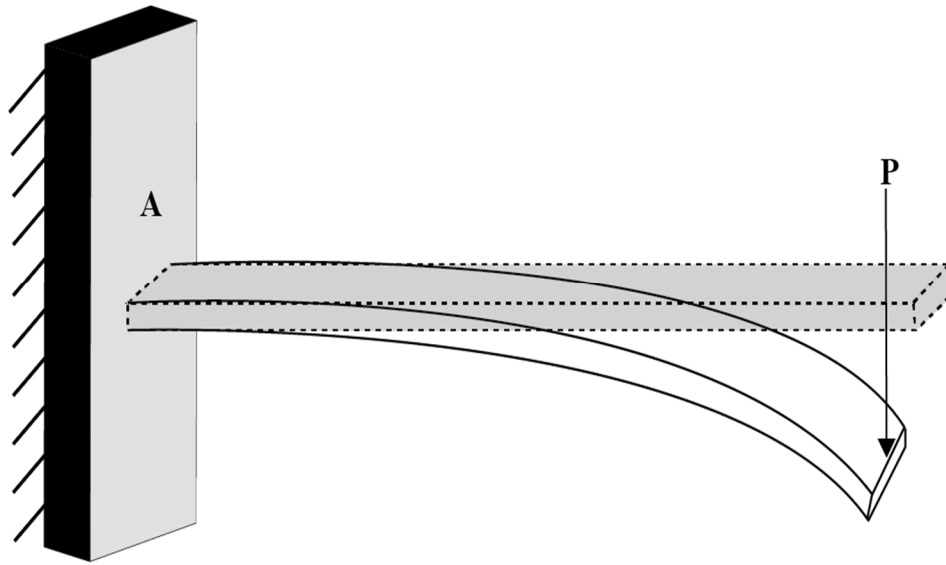


Fig. 2.9: Figure showing the bending stress

The expression for calculating bending stress σ_b in a beam is

$$\sigma_b = \frac{My}{I} \quad \dots \dots (2.7)$$

σ_b represents the bending stress.

M is applied bending moment (BM) in the beam.

y is the vertical distance away from the neutral axis (the point in the beam that experiences neither tension nor compression during bending).

I represents the moment of inertia (MI) of the cross-section. The detailed derivation will be discussed later.

2.9 ELASTIC LIMIT AND ELASTICITY

Elastic Limit: When a material is subjected to an external force, it deforms. However, when the applied force is removed, the material can return to its initial shape when the force applied is within a limit which is known as elastic limit. Elasticity refers to the ability of the materials to return to their initial position once

the external force is removed. The elastic limit of the material refers to the limiting magnitude of stress beyond which permanent deformation occurs.

Loss of Elasticity: When an external force reaches the elastic limit, the material experiences a partial or complete loss of elasticity. When the force is removed, the material will have residual deformation and won't return entirely to its original state. There is some loss in the elastic property of the material.

2.10 HOOKE'S LAW

It states that when a material is subjected to some external load within its elastic limit, the resulting stress is linearly proportional to the strain. In simpler terms, if the applied load is within the elastic limit, the stress-to-strain ratio remains constant. This constant is known as the modulus of elasticity, or elastic modulus.

$$\frac{\text{Stress}}{\text{Strain}} = E = \text{Constant} \quad \dots \dots (2.8)$$

2.11 YOUNG'S MODULUS

Young's Modulus (E) is a constant that represents the ratio of tensile or compressive stress to the corresponding strain within the elastic limit of the material. This modulus quantifies a material's stiffness and ability to resist deformation when subjected to stretching (tensile) or compressing compressive forces.

$$E = \frac{\text{Tensile or compressive stress}}{\text{Tensile or compressive strain}}$$

$$E = \frac{\sigma}{\epsilon} \quad \dots \dots (2.9)$$

Here,

σ is stress, and ϵ represents strain.

The proportionality constant E (Young's Modulus) quantifies the material's sensitivity to stress and strain.

2.12 STRESS-STRAIN RELATIONSHIP

The stress-strain relationship of a material is established through experimental tests, typically tension tests, conducted on standard specimens. These tests help to characterize how a specific material responds to applied loads, specifically in tensile or compressive loading.

2.12.1 TENSILE BEHAVIOUR

Fig. 2.10 shows a typical tensile test specimen of mild steel as per the ASTM standard. The specimen is fixed at its ends within a universal testing machine, and an extensometer is attached to measure its extension over a specific length denoted as L_1 (as seen in Fig. 2.10(a)). This designated length is referred to as the gauge length. The test involves gradually applying load at regular intervals, and measuring the corresponding deformation.

Nominal stress, also known as stress, is the measure of the applied load divided by the initial cross-sectional area of the material.

$$\text{Strain } (\epsilon) = (\text{extensometer readings} / \text{Gauge Length } (L_1)) \quad \dots \dots (2.10)$$

Fig. 2.11 shows the stress-versus-strain diagram for a standard mild steel sample, highlighting important points along the stress-strain curve. The stress-strain curve reveals the presence of the following points:

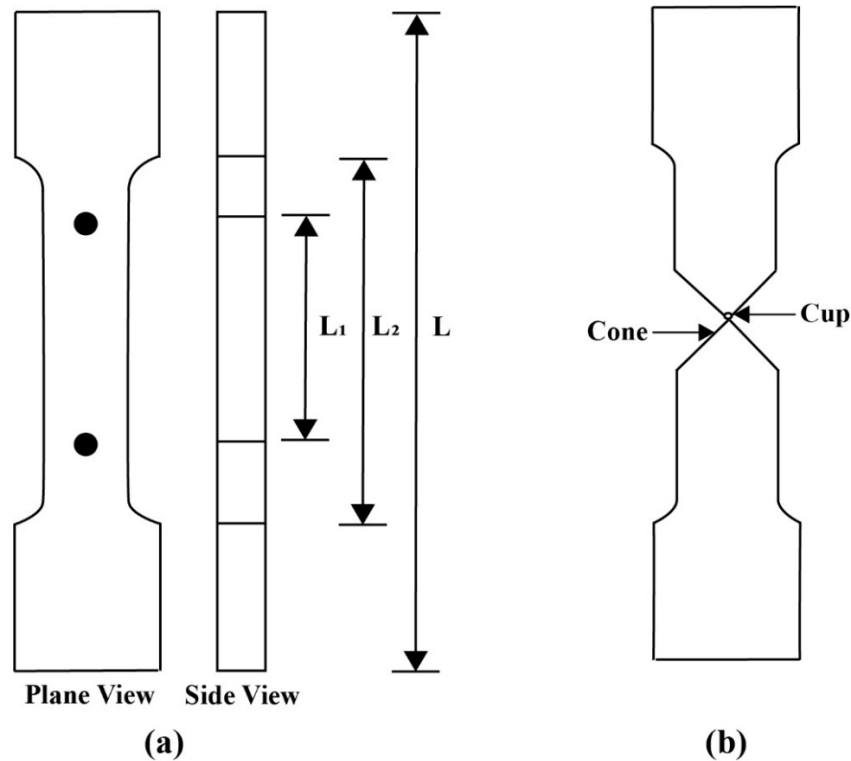


Fig. 2.10: (a) Extension due to a specific length (b) Neck formation

- **Proportionality Limit (A):** This refers to the maximum stress value at which stress exhibits a linear and proportional relationship with strain.
- **Upper Yield Point (B):** The upper yield point marks the stress level at which the load begins to decrease and the material starts undergoing yielding. At this stage, the strain is approximately 0.125%, and the stress is around 250 N/mm^2 for mild steel.
- **Lower Yield Point (C):** When the applied load exceeds the upper yield point, there is a reduction in the load-carrying capacity and then the stress remains constant at the lower yield point, but the strain increases.
- **Ultimate Stress (D):** Ultimate stress denotes the highest value of stress that a material can sustain without breaking. Typically, the value is about 370-400

N/mm². During this stage, the cross-sectional area in the gauge section starts decreasing rapidly, resulting in neck formation (Fig. 2.10(b)). After this point, the load resistance and developed stress begin to decrease.

- **Breaking Point (E):** It is the stress at which the sample eventually fails, typically occurring at 20 to 25% strain. It is the point at which the material fractures.
- The observations of the stress-strain curve provide valuable insights into the material's behavior under different levels of stress and strain, helping engineers and scientists understand its mechanical properties and performance characteristics.
- When unloading is performed within the elastic limit of a material, the material will fully regain its original length. In this case, the stress-strain curve will follow the same path as during loading, essentially retracing the loading curve shown in Fig. 2.11.
- However, the stress-strain curve will exhibit a different path if the sample is unloaded after being subjected to stresses beyond its elastic limit. More precisely, it will trace a straight line that is parallel to the original linear segment of the curve, denoted by the line FF' in Fig. 2.11. The material experiences permanent deformation or strain (represented by the distance OF'). This permanent deformation is referred to as a "permanent set."
- A permanent set indicates that the material has undergone plastic deformation and will not return to its initial position after unloading.

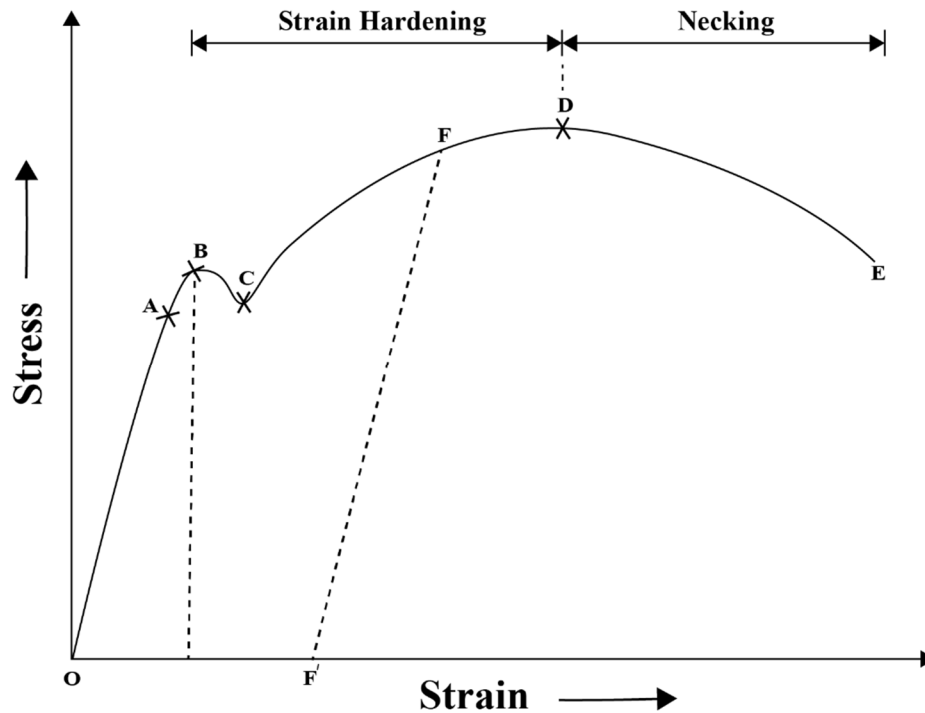


Fig. 2.11: Stress-strain curve for mild steel

2.12.2 STRESS-STRAIN RELATION IN ALUMINUM AND HIGH-STRENGTH STEEL

For aluminum alloys, the stress-strain curve may not exhibit a distinct yield point in as in steel. In contrast, necking typically occurs at the ultimate tension point, and the breaking point is lower than the ultimate stress. This behavior is illustrated in Fig. 2.12, where the stress-strain curve shows a gradual transition of elastic region to plastic region without a well-defined yield point.

To account this, engineers and scientists often use the 0.2% proof stress as a practical substitute for the yield point. The 0.2% proof stress (p_y) is the stress level at which, if unloading is performed, a permanent deformation of 0.2% occurs. It is the practical yield point for many engineering materials. It provides a consistent and

measurable reference point for determining the yield strength of the material, even in cases where the stress-strain curve lacks a clear-cut yield point.

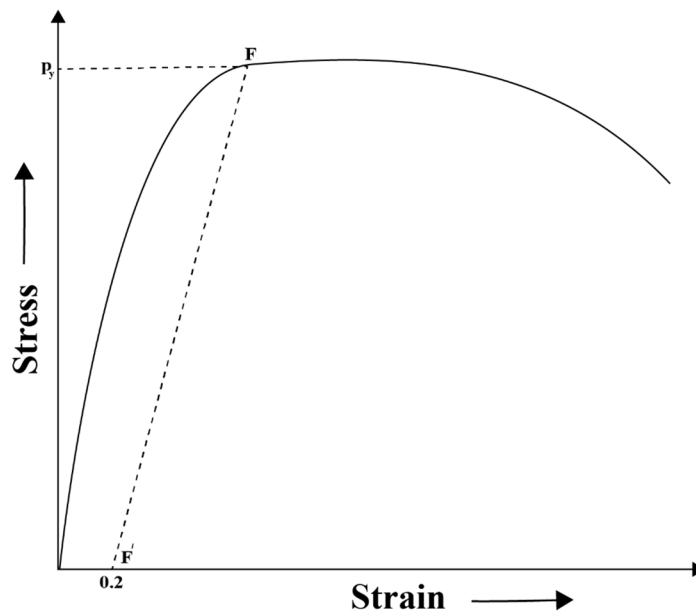


Fig. 2.12: Stress-strain curve for aluminum alloys and high-strength steel

2.12.3 STRESS-STRAIN RELATION IN BRITTLE MATERIAL

The stress-strain behaviour in brittle materials such as cast-iron exhibits distinctive characteristics, as shown in Fig. 2.13. The amount of plastic deformation in brittle material is very less or does not exist.

No Change in Strain Rate: As stress increases in brittle materials, there is limited strain value with loading. Unlike ductile materials exhibiting yielding and necking, brittle materials maintain a constant strain rate until failure.

No Yield Point: Unlike some ductile materials with a yield point, it is difficult to find the yield point in brittle materials. There is no well-defined point on the stress-strain curve where plastic deformation begins.

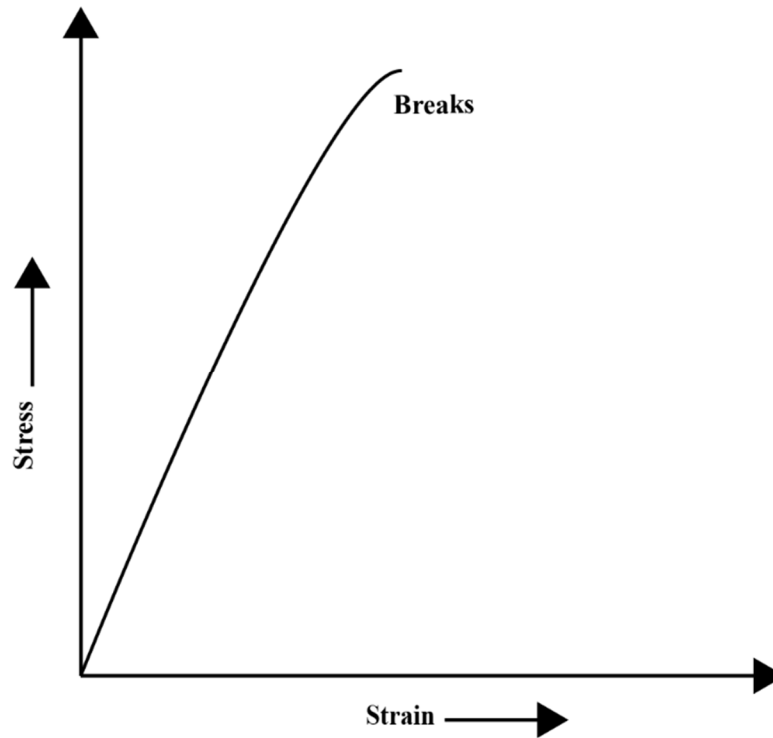


Fig. 2.13: Figure showing the Stress-strain curve

No Necking: Brittle materials do not experience necking, and the localized reduction in cross-sectional area is observed only in ductile materials.

Ultimate and Breaking Point: The ultimate point (maximum stress) and the breaking point are the same in brittle materials. Failure occurs abruptly and without warning, often with very little plastic deformation.

Small Strain at Failure: The strain at failure in brittle materials is typically very small. Failure occurs suddenly and without significant deformation, which is a typical characteristic of brittle fracture.

2.12.4 PERCENTAGE ELONGATION (%) AND PERCENTAGE REDUCTION IN AREA (%)

The ductility of a material is commonly evaluated using two key parameters:

(a) Percentage Elongation: This metric quantifies a material's ability to stretch before rupturing. It is the percentage ratio of the final extension at rupture ($L' - L$) to the initial length (L). The formula is:

$$\text{Percentage elongation (\%)} = \frac{L' - L}{L} \times 100 \quad \dots \dots (2.11)$$

Where L' – Length at rupture, L – Initial length

Industrial standards often require that the initial length be a minimum of five times the diameter, and measurements should include any necking during testing. For ductile materials, the percentage elongation often falls within 20 to 25%.

(b) Percentage Reduction in Area: This parameter assesses how much the cross-sectional area of a material is reduced at the point of rupture compared to its original cross-sectional area. It is expressed as a percentage and is calculated using the formula:

$$\text{Percentage reduction in area (\%RA)} = \frac{A - A'}{A} \times 100 \quad \dots \dots (2.12)$$

Where, A represents the original cross-sectional area, and A' represents the reduced cross-sectional area, typically measured after necking. For steel, the percentage reduction in area typically falls within the range of 60% to 70%.

These measurements are essential for evaluating a material's ductility, especially in cases where the material needs to exhibit plastic deformation and energy absorption before failing, as seen in many engineering and structural applications.

2.12.5 BEHAVIOUR OF MATERIAL UNDER COMPRESSION

The behavior of materials under compression testing differs from tensile testing in several ways:

- (a) Use of Short Specimens:** Compression tests typically employ shorter specimens to avoid the possibility of buckling (lateral bending) in long specimens. However, this results in measuring smaller changes in length, can lead to reduced accuracy compared to tensile tests.
- (b) Stress-Strain Curve Up to Yield Point:** In ductile materials, the stress-strain curve under compression follows a path nearly identical to the curve observed in tensile tests, up to and even slightly beyond the yield point. However, for larger stress values, the curves diverge. Unlike tensile tests, there is no occurrence of necking in compression tests.
- (c) Ultimate Compressive Stress in Brittle Materials:** For most brittle materials, the ultimate compressive stress measured during compression tests is notably greater than in tension. This phenomenon can be attributed to the presence of flaws and cracks within brittle materials. These imperfections tend to weaken the material when subjected to tension, but their influence on its strength in compression is not as pronounced.

2.13 TRUE STRESS AND NOMINAL STRESS

Direct stress relies on dividing the applied load by the original area of cross-sectional of a material. In the case of mild steel or any ductile material, the stress starts to decrease after neck formation, as shown in Fig. 2.14. However, it's essential to recognize that its cross-sectional area changes as the load is increasing. To differentiate these two values, we define the terms "nominal stress" and "true stress," as follows:

Nominal Stress: Nominal stress is defined as the load divided by the original cross-sectional area of the material.

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Original cross sectional area}} \quad \dots \dots (2.13)$$

True Stress: True stress is defined as the load divided by the actual cross-sectional area of the material.

$$\text{True stress} = \frac{\text{Load}}{\text{Actual cross sectional area}} \quad \dots \dots (2.14)$$

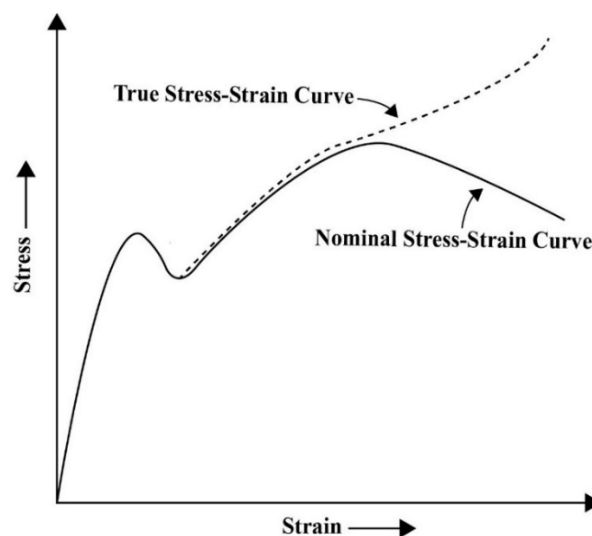


Fig. 2.14: Figure showing the True stress and strain curve

After neck formation (beyond the ultimate stress point), the stress-strain curve shows a downward slope, and fracture ultimately occurs at a lower stress value (nominal stress). True stress, on the other hand, continues to rise when strain rises, as shown in Fig. 2.14.

2.14 FACTOR OF SAFETY

It is defined as the ratio of the maximum load a material can withstand to the actual applied load that is deemed safe for a given application. Mathematically, it is expressed as:

$$\text{Factor of safety (FOS)} = \frac{\text{Ultimate load}}{\text{Actual load}} \quad \dots \dots (2.15)$$

The factor of safety is a measure of how much stronger a component or structure is compared to the stresses it is expected to experience during normal operation. A higher factor of safety indicates a greater margin of safety.

The specific factor of safety values used in design depends on the material being used and the desired level of reliability. Commonly used factor of safety values in practice are:

For steel: Typically, around 1.85

For concrete: Generally, around 3

For timber: Typically falls in the range of 4 to 6

These values are chosen to ensure that the structure or component can safely withstand the expected loads and environmental conditions, accounting for uncertainties and variations in materials and construction. A higher factor of safety provides greater safety margins but may result in more material usage and increased costs.

2.15 SHORTENING/ EXTENSION OF A BAR

Consider the bar shown in Fig. 2.15

$$\text{Stress} \quad \sigma = \frac{P}{A}$$

$$\text{Strain} \quad \epsilon = \frac{\delta L}{L}$$

From Hooke's law.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon} = \frac{\frac{P}{A}}{\frac{\delta L}{L}} = \frac{PL}{\delta L \cdot A}$$

$$\delta L = \frac{PL}{AE} \quad \dots \dots (2.16)$$

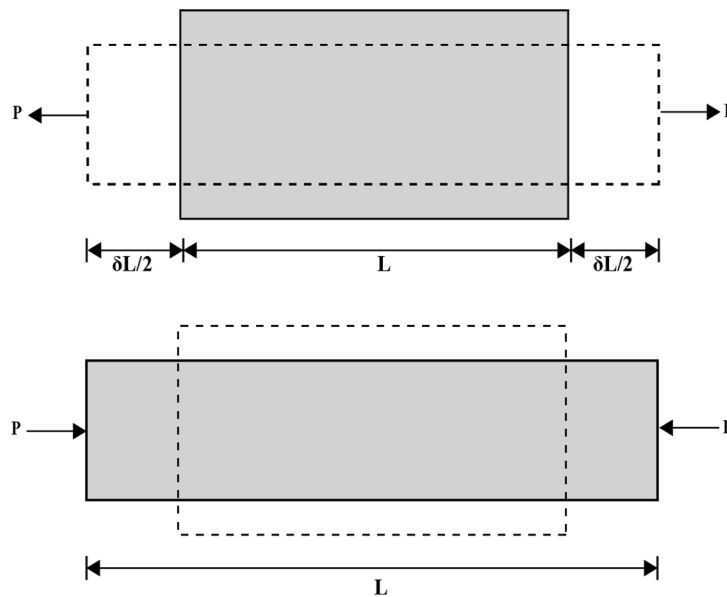


Fig. 2.15: Figure showing the shortening/ extension of a bar

Problem 2.1

A rod with a 2000 mm length and a 30 mm diameter experiences an axial pull of 40 kN. Given the material's modulus of elasticity is $2 \times 10^5 \text{ N/mm}^2$, calculate the following:

- (i) Stress,
- (ii) Strain

(iii) Elongation of the rod

Solution:

Rod length, $L = 2000\text{mm}$

Rod diameter, $D = 30\text{mm}$

$$\therefore \text{Area, } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times (30)^2 = 225\pi \text{ mm}^2$$

Axial pull, $P = 40\text{kN} = 40,000 \text{ N}$

Modulus of elasticity, $E = 2.0 \times 10^5 \text{ N/mm}^2$

(i) The stress (σ) is given by $\sigma = \frac{P}{A}$ as

$$\sigma = \frac{P}{A} = \frac{40000}{225\pi} = 56.588 \text{ N/mm}^2$$

(ii) Using equation $E = \frac{\sigma}{\epsilon}$ The strain is determined as

$$\therefore \text{Strain, } \epsilon = \frac{\sigma}{E} = \frac{56.588}{2 \times 10^5} = 0.00028294$$

(iii) Elongation is obtained by using the equation $\epsilon = \frac{dL}{L}$

$$\therefore \text{Elongation, } \delta L = \epsilon \times L = 0.00028294 \times 2000 = 0.5685 \text{ mm.}$$

Problem 2.2

Determine the smallest diameter for a steel wire employed in lifting a 5000N load while ensuring that the stress within the wire does not surpass 100MN/m^2 .

Solution:

$P = 5000 \text{ N}$

$$\sigma = 100\text{MN/m}^2 = 100 \times 10^6 \text{ N/m}^2 \quad (\because \text{M} = \text{Mega} = 10^6)$$

$$\sigma = 100 \text{ N/mm}^2 \quad (\because 10^6 \text{ N/m}^2 = 1\text{N/mm}^2)$$

$D = \text{Wire diameter in mm}$

$$\therefore \text{Area, } A = \frac{\pi}{4} \times D^2$$

$$\text{Stress } (\sigma) = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$100 = \frac{5000}{\frac{\pi}{4} \times D^2} = \frac{5000 \times 4}{\pi \times D^2}$$

$$D^2 = \frac{5000 \times 4}{\pi \times 100} = 63.66$$

$$D = 7.978 \text{ mm}$$

Problem 2.3

Calculate the Young's modulus for a brass rod with a diameter of 30mm and a length of 300mm. The rod is under a tensile load of 60kN, resulting in an extension of 0.5mm.

Solution:

Diameter of the rod, $D = 30\text{mm}$

$$\therefore \text{Area, } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 30^2 = 706.85 \text{ mm}^2$$

Tensile load, $P = 60\text{kN} = 60 \times 1000 = 60,000 \text{ N}$

Rod extension, $\delta L = 0.5\text{mm}$

Rod length, $L = 300\text{mm}$

Stress (σ) is provided by the equation

$$\sigma = \frac{P}{A} = \frac{60000}{706.85} = 84.882 \text{ N/mm}^2$$

The equation provides strain (ϵ)

$$\epsilon = \frac{\delta L}{L} = \frac{0.5}{300} = 0.0016$$

Using equation $E = \frac{\text{Stress}}{\text{Strain}}$, the (E) is calculated as

$$\begin{aligned} E &= \frac{\text{Stress}}{\text{Strain}} = \frac{84.882}{0.0016} = 53051.25 \text{ N/mm}^2 = 53051.25 \times 10^5 \text{ N/m}^2 \\ &= 53.051 \times 10^9 \text{ N/m}^2 \\ &= 53.051 \text{ GPa} \end{aligned}$$

Problem 2.4

A tensile test was conducted on a steel bar, and the subsequent test data was recorded:

- (i) Rod diameter: 50 mm
- (ii) The gauge length of the bar is 250mm
- (iii) Load: 300 kN
- (iv) Extension at a load of 200 kN: 0.25 mm
- (v) Maximum load observed: 400 kN
- (vi) Total extension during the test: 70mm
- (vii) Rod diameter at failure: 25 mm

Calculate the following:

- (a) Young's modulus
- (b) Stress at the elastic limit
- (c) Percentage elongation
- (d) Percentage reduction in area

Solution:

$$\text{Rod area, } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 50^2 = 19.634 \times 10^{-4} \text{ m}^2$$

- (a) Calculate the stress and strain values within the elastic limit to determine Young's modulus.

The load of 200kN (within the elastic limit) and a corresponding extension of 0.25 mm are given. Hence, these values are used for stress and strain within elastic limits.

$$\therefore \text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{200 \times 1000}{19.634 \times 10^{-4}} \text{ N/m}^2$$

$$\sigma = 10186.41 \times 10^4 \text{ N/m}^2$$

$$\text{and Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

$$= \frac{0.25}{250} = 0.001$$

$\therefore$ Young's Modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{10186.4 \times 10^4}{0.001} = 101.86400 \times 10^9 \text{ N/m}^2$$

$$E = 101.864 \text{ GPa}$$

(b) The elastic limit stress is represented as,

$$\text{Stress} = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{300 \times 1000}{19.634 \times 10^{-4}} = 152.79 \times 10^6 \text{ N/m}^2 = 152.79 \text{ MPa}$$

(c) The percentage of elongation is calculated as:

$$\text{Percentage elongation} = \frac{\text{Total increase in length}}{\text{Original length}} \times 100$$

$$= \frac{70}{250} \times 100 = 28\%$$

(d) The percentage of reduction in area is calculated as:

$$\text{Percentage reduction in area} = \frac{(\text{Original area} - \text{area at the failure})}{\text{Original area}} \times 100$$

$$= \frac{\left(\frac{\pi}{4} \times 5^2 - \frac{\pi}{4} \times 2.5^2\right)}{\frac{\pi}{4} \times 5^2} \times 100 = \frac{19.634 - 4.9087}{19.634} \times 100 = 74.99\%$$

Problem 2.5

Determine the internal diameter of the hollow steel tube that supports a 2.0 MN axial load while ensuring a factor of safety is 4. Given that the ultimate stress is 500 N/mm² and the external diameter of the column member is 300mm.

Solution:

Ultimate stress = 500 N/mm²

Axial load, $P = 2.0 \text{ MN} = 2 \times 10^6 = 2000000 \text{ N}$

External diameter $D = 300\text{mm}$

Factor of safety = 4

Let d = Internal diameter in mm

∴ Column cross-sectional area,

$$A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (300^2 - d^2) \text{ mm}^2$$

Using the factor of safety equation, we get

$$\text{The factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

$$4 = \frac{500}{\text{Working stress}}$$

$$\text{Working stress} = \frac{500}{4} = 125 \text{ N/mm}^2$$

$$\sigma = 125 \text{ N/mm}^2$$

Now, using the (σ) equation, we get,

$$\sigma = \frac{P}{A}$$

$$125 = \frac{2000000}{\frac{\pi}{4} \times (300^2 - d^2)}$$

$$(300^2 - d^2) = \frac{2000000 \times 4}{\pi \times 125} = 20371.83$$

$$(300^2 - d^2) = 20371.83$$

$$d^2 = 900000 - 20371.83 = 69628.17$$

$$d = 263.87 \text{ mm}$$

Problem 2.6

Determine the minimum and maximum stresses generated in a stepped bar, as shown in Fig. 2.16, when subjected to an axially compressive force of 40kN.

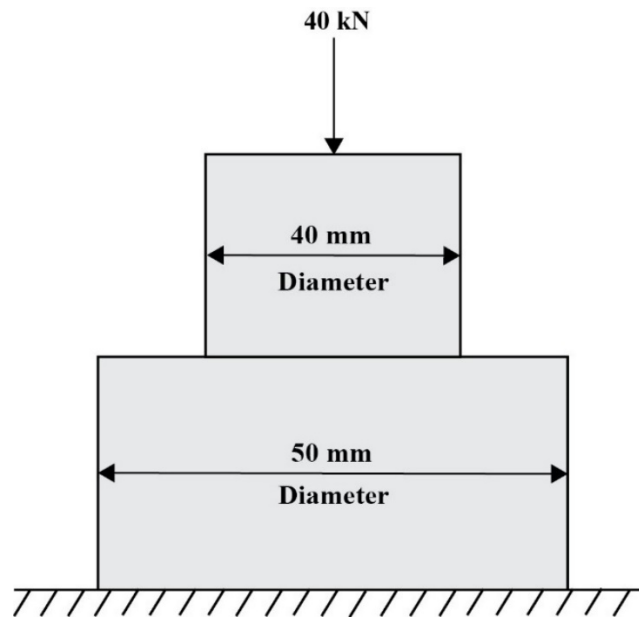


Fig. 2.16 Stepped Bar Subjected to Axial Compression

Solution:

Axial load, $P = 40\text{kN} = 40 \times 10^3 \text{ N}$

Upper part diameter $D_1 = 40 \text{ mm}$

Lower part diameter $D_2 = 50\text{mm}$

$$\therefore \text{Area of the upper part, } A_1 = \frac{\pi}{4} \times (40)^2 = 400\pi \text{ mm}^2$$

$$\therefore \text{Area of the lower part, } A_2 = \frac{\pi}{4} \times (50)^2 = 625 \pi \text{ mm}^2$$

Stress is calculated as the load by area. Therefore, stress reaches the maximum value where the area is minimum. Consequently, the upper part will experience the maximum stress, while the lower part will experience the minimum stress.

$$\therefore \text{Maximum stress} = \frac{\text{Load}}{A_1} = \frac{40 \times 10^3}{400\pi} = 31.830 \text{ N/mm}^2$$

$$\therefore \text{Minimum stress} = \frac{\text{Load}}{A_2} = \frac{40 \times 10^3}{625\pi} = 20.371 \text{ N/mm}^2$$

2.16 DETERMINATION OF STRESS IN BARS OF VARYING CROSS-SECTION

Consider a bar with varying diameters and lengths, resulting in distinct cross-sectional areas, as illustrated in Fig. 2.17.

When an axial load P is applied to this bar, each section undergoes with identical axial load P . However, the strains, stresses, and changes in length will vary across the sections. To determine the overall change in length, the individual changes in length of each section must be obtained and added.

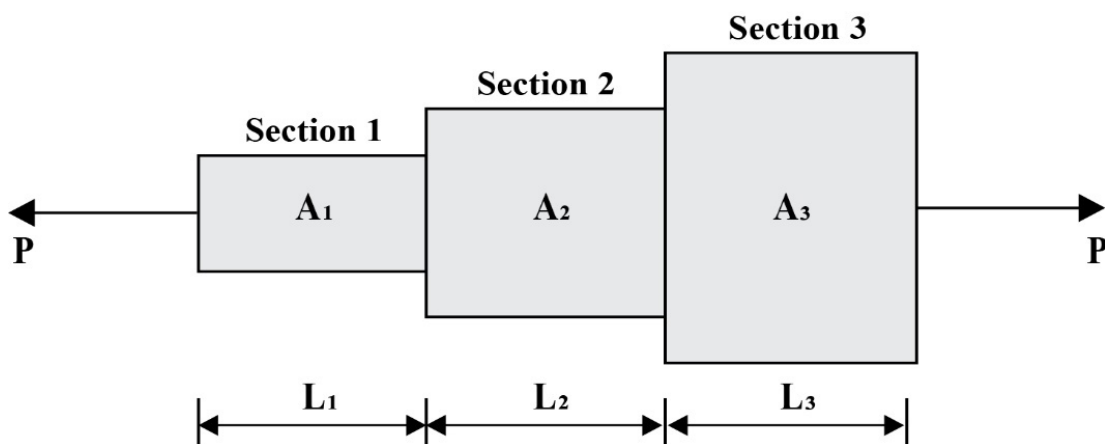


Fig. 2.17: Bars with Varying Cross-Sections

Let

The bar is made of the same material

P be the axial load is applied to the bar.

$L_1, A_1, L_2, A_2, L_3, A_3$ be the corresponding length and area of cross section of sections 1, 2, and 3, respectively.

E be the Young's modulus of the bar.

Now, calculate the stress for section 1:

$$\sigma_1 = \frac{\text{Load}}{\text{Area of section 1}} = \frac{P}{A_1}$$

Similarly, the stress for sections 2 and 3 is given as,

$$\sigma_2 = \frac{P}{A_2} \text{ and } \sigma_3 = \frac{P}{A_3}$$

The strain for the section 1,

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E} \quad \left(\because \sigma_1 = \frac{P}{A_1} \right)$$

Similarly, the strain for sections 2 and 3 is calculated as,

$$\epsilon_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E} \text{ and } \epsilon_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E}$$

But strain in section 1 = $\frac{\text{Change in section 1 length}}{\text{Length of section 1}}$ or $\epsilon_1 = \frac{dL_1}{L_1}$

Where

dL_1 = Change in length of section 1

$\therefore$ Change in length of section 1,

$$\delta L_1 = \epsilon_1 L_1$$

$$\delta L_1 = \frac{PL_1}{A_1 E} \quad \left[\because \epsilon_1 = \left(\frac{P}{A_1 E} \right) \right]$$

Likewise, changes in the length of sections 2 and 3 are determined as follows:

$\therefore$ Change in length of section 2,

$$\begin{aligned} \delta L_2 &= \epsilon_2 L_2 \\ \delta L_2 &= \frac{PL_2}{A_2 E} \quad \left[\because \epsilon_2 = \left(\frac{P}{A_2 E} \right) \right] \end{aligned}$$

and change in length of section 3,

$$\begin{aligned} \delta L_3 &= \epsilon_3 L_3 \\ \delta L_3 &= \frac{PL_3}{A_3 E} \quad \left[\because \epsilon_3 = \left(\frac{P}{A_3 E} \right) \right] \end{aligned}$$

$\therefore$ Calculating the overall change in the length of the entire bar:

$$\begin{aligned} \delta L &= \delta L_1 + \delta L_2 + \delta L_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E} \\ \delta L &= \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] \quad \dots (2.17) \end{aligned}$$

Eqn. (2.17) is applied when the modulus of elasticity is the same for all the sections. However, if the modulus of elasticity is different for the sections, the overall change in length of the bar can be expressed as:

$$\delta L = P \left[\frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3} \right] \quad \dots (2.18)$$

Problem 2.7

Calculate the stresses in each section and the overall bar expansion when an axial pull of 40000 N is applied to the bar composed of different sections, as shown in Fig. 2.18, with $E = 2.1 \times 10^5 \text{ N/mm}^2$.

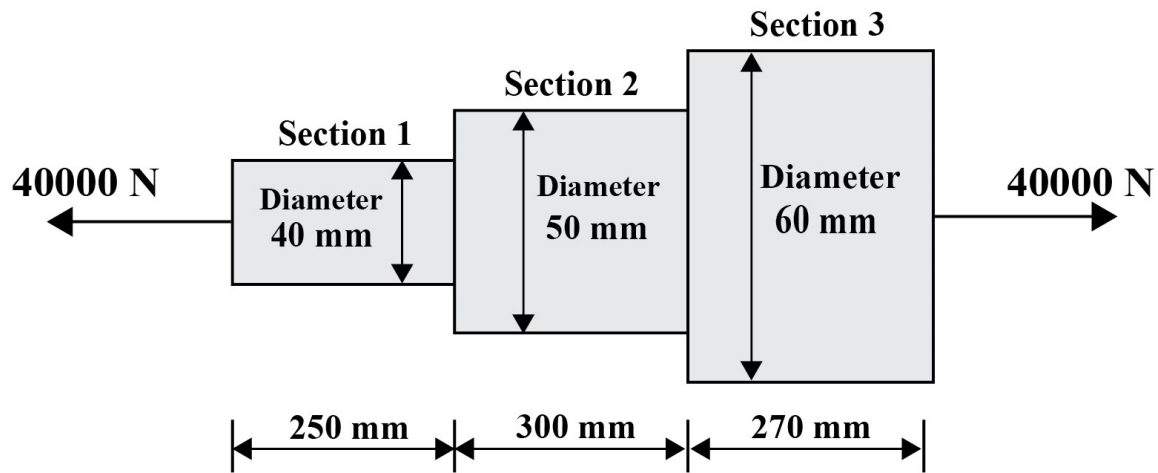


Fig. 2.18 Bar of Different Cross Section Subjected to Axial Pull

Solution:

Axial load $P = 40000 \text{ N}$

Section 1:

Length, $L_1 = 250 \text{ mm}$

Diameter, $D_1 = 40 \text{ mm}$

$$\therefore \text{Area, } A_1 = \frac{\pi}{4} (40)^2 = 400\pi \text{ mm}^2$$

Section 2:

Length, $L_2 = 300 \text{ mm}$

Diameter, $D_2 = 50 \text{ mm}$

$$\therefore \text{Area, } A_2 = \frac{\pi}{4} (50)^2 = 625\pi \text{ mm}^2$$

Section 3:

Length, $L_3 = 270 \text{ mm}$

Diameter, $D_3 = 60 \text{ mm}$

$$\therefore \text{Area, } A_3 = \frac{\pi}{4} (60)^2 = 900\pi \text{ mm}^2$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

(i) Calculate the stress in individual sections.

$$\begin{aligned}\text{Stress in Section 1, } \sigma_1 &= \frac{\text{Axial Load}}{\text{Area of section 1}} \\ &= \frac{P}{A_1} = \frac{40000}{400\pi} = 31.830 \text{ N/mm}^2\end{aligned}$$

$$\text{Stress in Section 2, } \sigma_2 = \frac{P}{A_2} = \frac{40000}{625\pi} = 20.371 \text{ N/mm}^2$$

$$\text{Stress in Section 3, } \sigma_3 = \frac{P}{A_3} = \frac{40000}{900\pi} = 14.147 \text{ N/mm}^2$$

(ii) Determine the overall bar expansion.

Using the formula, we obtain

$$\begin{aligned}\text{Total extension } \delta L &= \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] \\ &= \frac{40000}{2.1 \times 10^5} \left[\frac{250}{400\pi} + \frac{300}{625\pi} + \frac{270}{900\pi} \right] \\ &= \frac{40000}{2.1 \times 10^5} [0.1989 + 0.1527 + 0.0955] \\ &= [0.1904] \times [0.4471] \\ \delta L &= 0.8519 \text{ mm}\end{aligned}$$

2.17 PRINCIPLE OF SUPERPOSITION

When the structural members are subjected to both thermal and mechanical loading simultaneously and if the load levels are within the elastic limit, the principle of superposition is applied. Stress due to both mechanical and thermal loading is equal to the sum of the stress due to mechanical loading and stress due to thermal loading, which is known as the principle of superposition. This principle is valid for all the load conditions of a structure which behaves within the elastic limit.

Problem 2.8

Determine the overall elongation of a brass bar with a 1500 mm^2 cross-sectional area when exposed to axial forces, as shown in Fig. 2.19. Young's modulus (E) = $1.05 \times 10^5 \text{ N/mm}^2$.

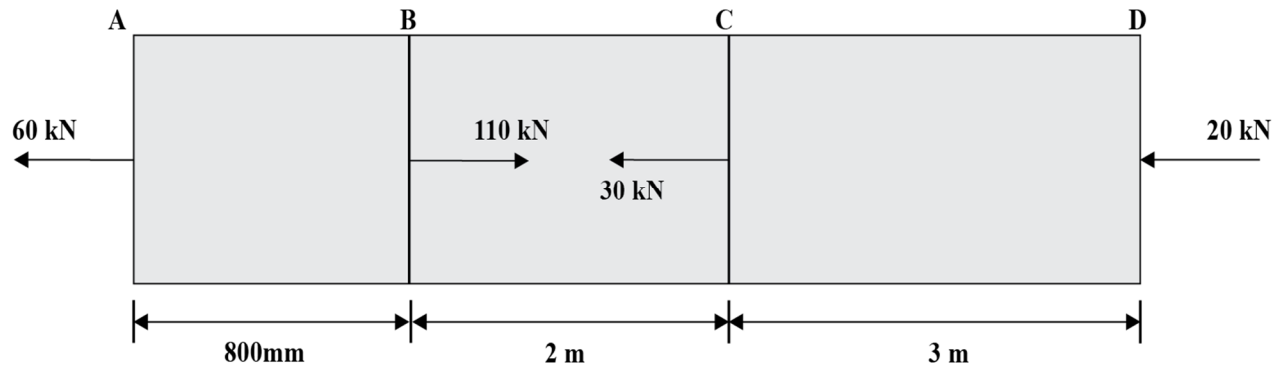


Fig. 2.19: Brass bar subjected to axial forces

Solution:

Area of a cross-section of the brass bar (A) = 1500 mm^2

Young's modulus of brass bar (E) = $1.05 \times 10^5 \text{ N/mm}^2$

Let's consider all the forces acting on the given points (A, B, C, D). The highest force is acting on point B which is 110 kN. Let's divide this force into 3 parts such as 60 kN, 50 kN and 20 kN to balance the other forces acting on the other points such that each part is in equilibrium. Now section AB is under tensile loading, section BC is under compressive loading, and section CD is under compressive loading.

We know that elongation $E = \frac{PL}{AE}$

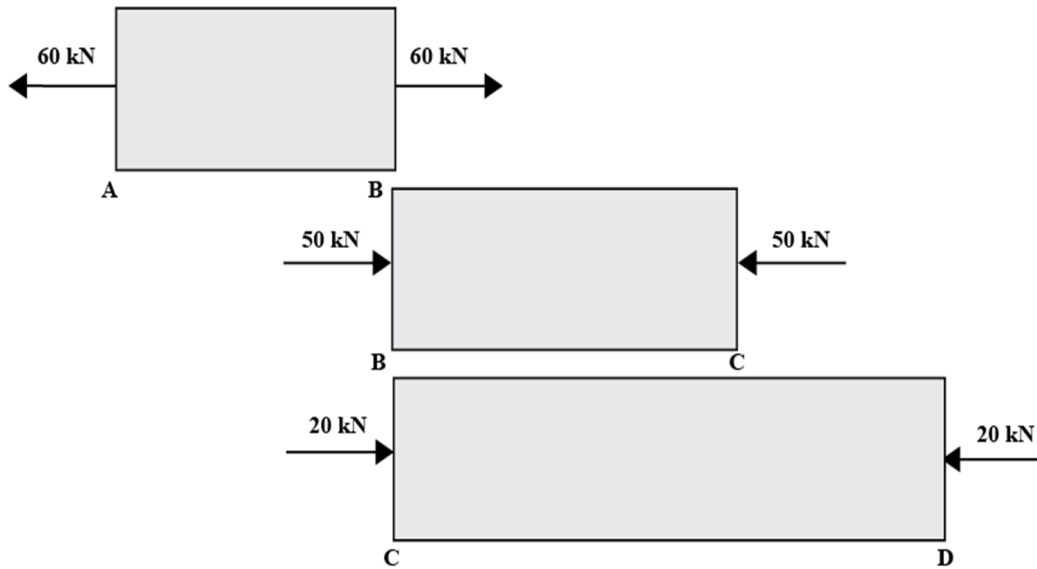


Fig. 2.20: Distribution of forces on bar segments

Free body diagram of Part AB: This section is experiencing a tensile load of 60 kN, increasing its length.

$$\begin{aligned}
 \therefore \text{Increase in the length AB} &= \frac{P_1}{AE} \times L_1 \\
 &= \frac{60000}{1500 \times 1.05 \times 10^5} \times 800 \quad (\because P_1 = 60,000 \text{ N}, L_1 = 800\text{mm}) \\
 &= 0.3047 \text{ mm}
 \end{aligned}$$

Free body diagram of Part BC: This section is under 50 kN compressive load, decreasing length.

$$\begin{aligned}
 \therefore \text{Decrease in the length BC} &= \frac{P_2}{AE} \times L_2 \\
 &= \frac{50000}{1500 \times 1.05 \times 10^5} \times 2000 \quad (\because P_2 = 50,000 \text{ N}, L_2 = 2000\text{mm}) \\
 &= 0.6349 \text{ mm}
 \end{aligned}$$

Free body diagram of Part CD: This section experiences a 20 kN compressive load, reducing its length.

$$\begin{aligned}
 \therefore \text{Decrease in the length CD} &= \frac{P_3}{AE} \times L_3 \\
 &= \frac{20000}{1500 \times 1.05 \times 10^5} \times 3000 \quad (\because P_3 = 20,000 \text{ N}, L_3 = 3000 = 3000 \text{ mm}) \\
 &= 0.3809 \text{ mm}
 \end{aligned}$$

$$\therefore \text{Total elongation of bar} = 0.3047 - 0.6349 - 0.3809 = -0.7111 \text{ mm}$$

(Considering a positive sign for an increase in length and a negative sign for a decrease in length)

The negative sign indicates that there will be a decrease in the length of the bar.

Problem 2.9

If a solid rod with a 50 mm diameter and 5 m length, subjected to a 50 kN tensile load, has a centrally bored hole with a 30 mm diameter, how much length should be removed from the rod to increase the total extension by 40% for the same tensile load? Young's Modulus (E) of the rod is $2 \times 10^5 \text{ N/mm}^2$.

Solution:

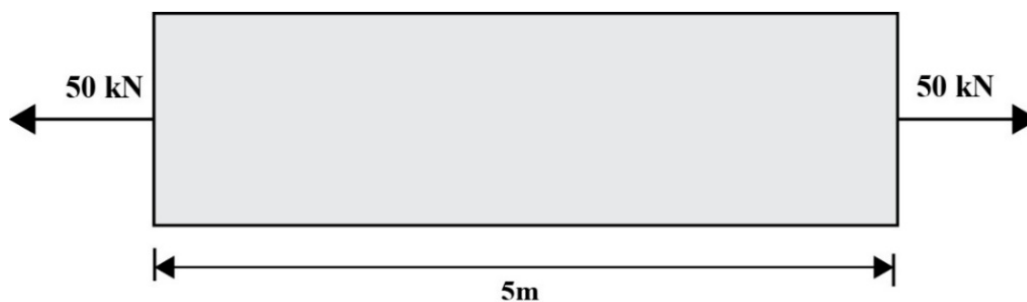


Fig. 2.21 Solid rod under tensile load

Tensile load applied, $P = 50 \text{ kN} = 50000$

Rod diameter, $D = 50 \text{ mm}$

$$\therefore \text{Rod area } A = \frac{\pi}{4} (50)^2 = 625\pi \text{ mm}^2$$

Rod length, $L = 5\text{m} = 5000\text{mm}$

Bore diameter, $d = 30\text{mm}$

$\therefore$ Rod area, $A = \frac{\pi}{4} (30)^2 = 225\pi \text{ mm}^2$

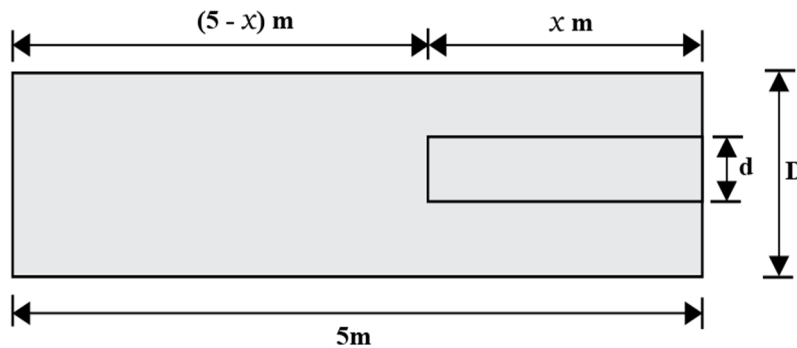


Fig. 2.22 Solid rod with bored hole

Extension of the rod with bore = $1.4 \times$ Extension of the rod without bore

After bore extension = $1.4 \times$ Before bore extension

$$E = 2 \times 10^5 \text{ N/mm}^2$$

Let the rod be bored to x m or $(x \times 1000\text{mm})$ in length.

Then the unbored section length = $(5 - x) \text{ m} = (5 - x) \times 1000\text{mm}$.

Extension of the rod without bore

Extension (δL) is provided by,

$$\delta L = \frac{P}{AE} \times L = \frac{50000 \times 5000}{625\pi \times 2 \times 10^5} = \frac{2}{\pi}$$

Extension of the rod with bore = $1.4 \times$ Extension of the rod without bore

$$= 1.4 \times \frac{2}{\pi} = \frac{2.8}{\pi} \text{ m} \quad \dots\dots (2.19)$$

The elongation of the rod with bore can be determined by calculating the elongation of the unbored and the bored sections separately.

For this, find the stresses in the unbored and bored sections.

$$\text{Stress in the unbored section} = \frac{\text{Load}}{\text{Area}} = \frac{50000}{625\pi} = \frac{80}{\pi} \text{ N/mm}^2$$

$$\begin{aligned} \therefore \text{Extension of unbored section} &= \frac{\text{Stress}}{E} \times \text{Length of unbored section} \\ &= \frac{80}{\pi \times 2 \times 10^5} \times (5 - x) \times 1000 = \frac{2(5-x)}{5\pi} \text{ m} \end{aligned}$$

$$\text{Stress in the bored section} = \frac{\text{Load}}{\text{Area}} = \frac{P}{(A-a)} = \frac{50000}{(625\pi - 225\pi)} = \frac{50000}{400\pi}$$

$$\therefore \text{Extension of bored section} = \frac{\text{Stress}}{E} \times \text{Length of bored section}$$

$$= \frac{50000}{400\pi \times 2 \times 10^5} \times 1000x = \frac{5x}{8\pi} \text{ m}$$

$$\therefore \text{Extension of the rod with bore} = \frac{2(5-x)}{5\pi} + \frac{5x}{8\pi} \quad \dots (2.20)$$

Equating the Equation (2.19) and (2.20), we get

$$\frac{2.8}{\pi} = \frac{2(5-x)}{5\pi} + \frac{5x}{8\pi}$$

$$2.8 = \frac{2(5-x)}{5} + \frac{5x}{8}$$

$$2.8 \times 40 = 2 \times 8(5-x) + (5 \times 5x)$$

$$112 = 16(5-x) + (25x)$$

$$112 = 80 - 16x + 25x$$

$$112 - 80 = 9x$$

$$32 = 9x$$

$$x = 3.5 \text{ m}$$

2.18 ANALYSIS OF UNIFORMLY TAPERED CIRCULAR ROD

Consider a bar that gradually changes its diameter from D_1 at one end to D_2 at the other end as shown in Fig. 2.23.

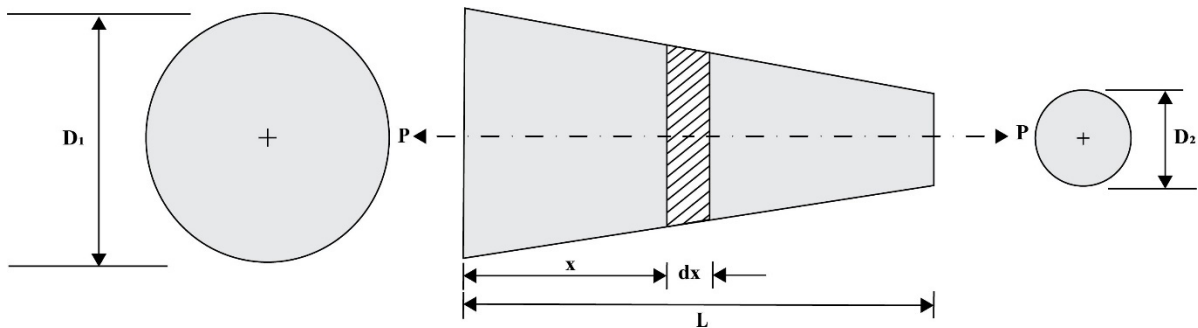


Fig. 2.23: Uniformly tapered circular rod

Let

P = Axial tensile load applied to the bar

L = Overall length of the bar

E = Young's modulus.

Now, consider a small element with a length dx of the bar at a distance ' x ' from the left end. Let diameter of this element be D_x , which varies with the distance x from the left end.

Then

$$D_x = D_1 - \left(\frac{D_1 - D_2}{L} \right) x$$

$$D_x = D_1 - k \cdot x \quad \text{where } k = \left(\frac{D_1 - D_2}{L} \right)$$

The area of the cross-section of the element

$$A_x = \frac{\pi}{4} D_x^2 = \frac{\pi}{4} (D_1 - k.x)^2$$

Stress on the element

$$\sigma_x = \frac{\text{Load}}{A_x}$$

$$\sigma_x = \frac{P}{\left[\frac{\pi}{4} (D_1 - k.x)^2\right]} = \frac{4P}{\pi(D_1 - k.x)^2}$$

Strain of the element ϵ_x is

$$\therefore \epsilon_x = \frac{\text{Stress}}{E} = \frac{\sigma_x}{E}$$

$$\epsilon_x = \frac{4P}{\pi(D_1 - k.x)^2} \times \frac{1}{E}$$

$\therefore$ The elongation of the elemental length dx is:

$$= \text{Strain} \times dx = (\epsilon_x \times dx)$$

$$= \frac{4P}{\pi E (D_1 - k.x)^2} \cdot dx \quad \dots\dots (2.21)$$

The overall elongation of the bar is determined by performing an integration of equation (2.21) over the length L .

$\therefore$ Total extension,

$$\delta L = \int_0^L \frac{4P}{\pi E (D_1 - k.x)^2} \cdot dx$$

$$\delta L = \frac{4P}{\pi E} \int_0^L (D_1 - k.x)^{-2} \cdot dx$$

$$\delta L = \frac{4P}{\pi E} \left[\frac{(D_1 - k.x)^{-1}}{(k)} \right]_0^L$$

$$\delta L = \frac{4P}{\pi E k} \left[\frac{1}{(D_1 - k.x)} \right]_0^L$$

$$\delta L = \frac{4P}{\pi E k} \left[\frac{1}{(D_1 - k.L)} - \frac{1}{(D_1 - k.(0))} \right]$$

$$\delta L = \frac{4P}{\pi E k} \left[\frac{1}{(D_1 - k.L)} - \frac{1}{D_1} \right]$$

By substituting the expression $k = \frac{D_1 - D_2}{L}$ into the preceding equation, we obtain the following expression

Total extension,

$$\delta L = \frac{4PL}{\pi E D_1 D_2} \quad \dots\dots (2.22)$$

If the rod is of uniform cross-section, then $D_1 = D_2 = D$

Total extension,

$$\delta L = \frac{4PL}{\pi E D^2} \quad \dots\dots (2.23)$$

Problem 2.10

Determine the elongation of a uniformly tapered rod, with diameter varying from a 50 mm to 25 mm diameter over a length 500 mm, when exposed to an axial load of 6000 N. The material has Young's modulus (E) of $2.1 \times 10^5 \text{ N/mm}^2$.

Solution:

Diameter of one end (D_1) = 50mm

Diameter of other end (D_2) = 25mm

Length of the rod (L) = 500mm

Axial load (P) = 6000N

$E = 2.1 \times 10^5 \text{ N/mm}^2$

Let

dL = Overall extension of the rod

From the Equation (2.22)

$$\text{The elongation } \delta L = \frac{4PL}{\pi E D_1 D_2} = \frac{4 \times 6000 \times 500}{\pi \times 2.1 \times 10^5 \times 50 \times 25} = 0.01455 \text{ mm.}$$

Problem 2.11

Determine the modulus of elasticity of a rod that exhibits uniform tapering from a 50mm to 25mm diameter over a 400mm length. The rod is under an axial load of 6kN, and its elongation measures 0.050mm.

Solution:

Diameter of one end (D_1) = 50mm

Diameter of other end (D_2) = 25mm

Length of the rod (L) = 400 mm

Axial load (P) = 6kN = 6000N

δL = 0.050mm

Using the Eqn (2.22),

$$\delta L = \frac{4PL}{\pi E D_1 D_2}$$

$$E = \frac{4PL}{\pi D_1 D_2 \delta L} = \frac{4 \times 6000 \times 400}{\pi \times 50 \times 25 \times 0.050} = 48892.398 \frac{\text{N}}{\text{mm}^2} = 48.9 \text{ GPa}$$

2.19 ANALYSIS OF UNIFORMLY TAPERED RECTANGULAR BAR

Consider a bar with a uniform thickness that tapers gradually in width from one end to the other as shown in Fig. 2.24.

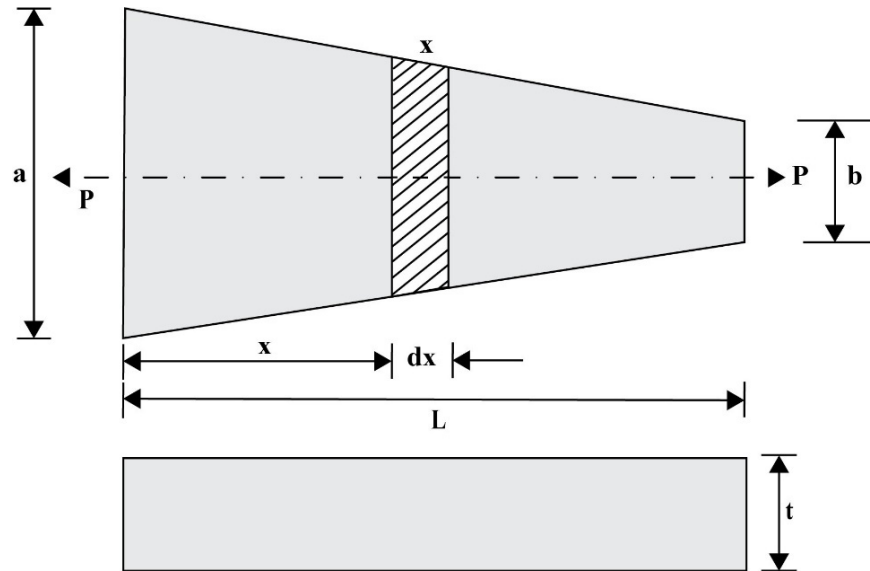


Fig. 2.24: Uniformly tapered rectangular bar

Let

P - represents the axial tensile load applied to the bar,

L - denotes the length of the bar.

E - Young's modulus,

a - Width at the larger end,

b - Width at the smaller end.

t - Thickness of the bar.

Consider a section X-X at distance x from the largest end.

Bar width at section X-X

$$= a - \frac{(a-b)x}{L}$$

Thus

$$\text{Width} = a - kx$$

$$\text{where } k = \frac{(a-b)}{L}$$

Bar thickness at section X-X = t

$$\begin{aligned}\therefore \text{Area of section X-X} &= \text{Thickness} \times \text{Width} \\ &= (a - kx)t\end{aligned}$$

$$\begin{aligned}\therefore \text{Stress on the section X-X} &= \frac{\text{Load}}{\text{Area}} \\ &= \frac{P}{(a - kx)t}\end{aligned}$$

The elongation or extension of the small elemental length 'dx'

$$\begin{aligned}&= \text{Length} \times \text{Strain} && \left(\because \text{Strain} = \frac{\text{Stress}}{E} \right) \\ &= \frac{\text{Stress}}{E} \times dx && \left(\because \text{Stress} = \frac{P}{(a - kx)t} \right) \\ &= \frac{\left(\frac{P}{(a - kx)t} \right)}{E} \times dx \\ &= \frac{\left(\frac{P}{(a - kx)t} \right)}{E} dx && \text{..... (2.24)}\end{aligned}$$

Total elongation of the bar is calculated by performing an integration of the equation (2.24) over the length L.

$$= \frac{P}{E(a - kx)t} \cdot dx$$

$\therefore$ Total extension,

$$\begin{aligned}\delta L &= \int_0^L \frac{P}{E(a - kx)t} dx \\ \delta L &= \frac{P}{Et} \int_0^L \frac{dx}{(a - kx)} \\ \delta L &= \frac{P}{Et} \cdot \log_e [(a - kx)]_0^L \times \left(-\frac{1}{k} \right) \\ \delta L &= -\frac{P}{Etk} \cdot [\log_e(a - kL) - \log_e a] \\ \delta L &= \frac{P}{Etk} \cdot [\log_e a - \log_e(a - kL)]\end{aligned}$$

$$\delta L = \frac{P}{Et k} \cdot \left[\log_e \frac{a}{(a - kL)} \right]$$

$$\delta L = \frac{P}{Et \left(\frac{a-b}{L} \right)} \cdot \left[\log_e \frac{a}{\left(a - \left(\frac{a-b}{L} \right) L \right)} \right] \quad \because \left(k = \frac{a-b}{L} \right)$$

$$\delta L = \frac{PL}{Et (a-b)} \cdot \log_e \frac{a}{b} \quad \dots\dots (2.25)$$

Problem 2.12

Determine the elongation of a steel rectangular bar with a length of 3.0 meters and a thickness of 20mm, subjected to a 50kN axial load. The width of the rod changes from 80mm at one end to 40mm at the other end. Given that Young's modulus is $2 \times 10^5 \text{ N/mm}^2$. Calculate the extension of the rod.

Solution:

Length of the bar (L) = 3m = 3000mm

Thickness of the bar (t) = 20mm

Axial load (P) = 50kN = 50000

Width at one end (a) = 80mm

Width at other end (b) = 40mm

Young's modulus (E) = $2 \times 10^5 \text{ N/mm}^2$

Let

δL = Extension of the rod.

Using the provided equation (2.25),

$$\delta L = \frac{PL}{Et (a-b)} \cdot \log_e \frac{a}{b}$$

$$\delta L = \frac{50000 \times 3000}{2 \times 10^5 \times 20 (80-40)} \cdot \log_e \left(\frac{80}{40} \right)$$

$$\delta L = 0.9375 \times 0.693 = 0.6496 \text{ mm}$$

$$\delta L = 0.6496 \text{ mm}$$

Problem 2.13

Calculate the axial load on a steel rectangular bar, 500mm in length and 15mm in thickness, which experiences a 0.25mm extension. This bar uniformly tapers in width from 150mm to 100mm. Young's modulus (E) is $2 \times 10^5 \text{ N/mm}^2$. Find the applied axial force.

Solution:

Extension of the bar (δL) = 0.25mm

Length of the bar (L) = 500mm

Thickness of the bar (t) = 15mm

Width at one end (a) = 150mm

Width at other end (b) = 100mm

Young's modulus (E) = $2 \times 10^5 \text{ N/mm}^2$

Let,

P = Axial load.

Using the provided equation (2.25),

$$\delta L = \frac{PL}{Et(a-b)} \cdot \log_e \frac{a}{b}$$

$$0.25 = \frac{P \times 500}{2 \times 10^5 \times 15 (150-100)} \cdot \log_e \left(\frac{150}{100} \right)$$

$$P = \frac{0.25 \times 2 \times 10^5 \times 15 (50)}{500 \times 0.4054}$$

$$P = 184972.759 = 184.972 \text{ kN}$$

2.20 BARS OF COMPOSITE SECTIONS

A composite bar is formed by combining two or more bars, each of the same length and made of different materials. When exposed to axial compressive or tensile loads, these bars are securely joined and act as a single unit. Two key considerations for composite bars are as follows:

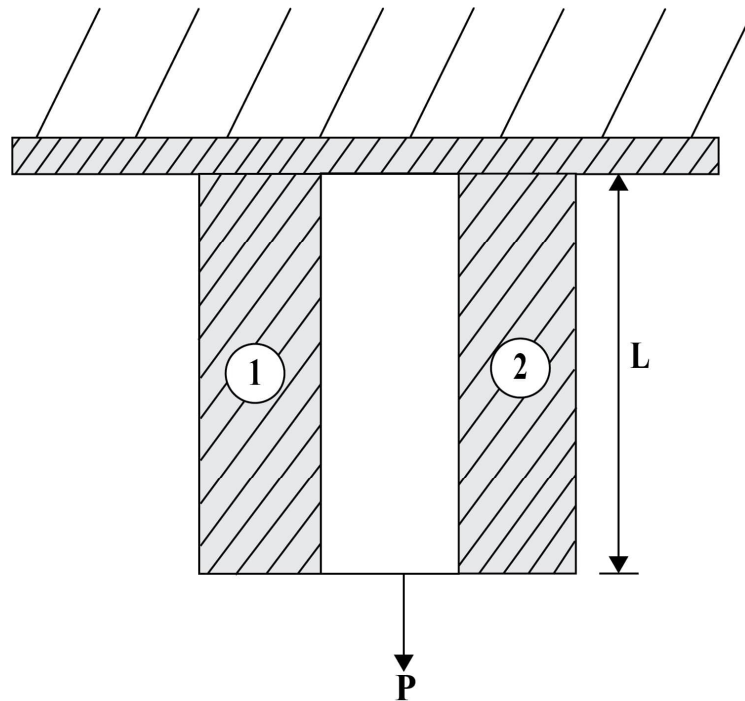


Fig. 2.25: Composite bar composed of two materials

The extension or compression is uniform in each bar, ensuring that the strain per unit length is consistent. As a result, the overall external load applied to the composite bar is the sum of the individual loads that each material carries. Fig. 2.25 shows the composite bar composed of two materials.

P: Load applied to the composite bar,

L: Length of the composite bar,

A_1 and A_2 : Area of cross-sections of bars 1 and 2

E_1 and E_2 : Modulus of elasticity of bars 1 and 2,

P_1 and P_2 : Load applied by bars 1 and 2,

σ_1 and σ_2 : Stress in the bars 1 and 2, respectively

The load applied to the composite bar is equivalent to the sum of the loads supported by the two individual bars.

$$P = P_1 + P_2 \quad \text{..... (2.26)}$$

The stress in bar 1, $= \frac{\text{Load carried by bar 1}}{\text{Area of the cross-section of bar 1}}$

$$\therefore \sigma_1 = \frac{P_1}{A_1} \quad \text{or} \quad P_1 = \sigma_1 A_1 \quad \text{.....(2.27)}$$

Similarly, stress in the bar 2,

$$\sigma_2 = \frac{P_2}{A_2} \quad \text{or} \quad P_2 = \sigma_2 A_2 \quad \text{.....(2.28)}$$

By using the values of P_1 and P_2 in eqn. (2.26), we get:

$$P = \sigma_1 A_1 + \sigma_2 A_2 \quad \text{.....(2.29)}$$

Since the ends of both bars are firmly connected, each bar will undergo the same deformation. Furthermore, each bar has the same length, which implies that the strain of both bars is the same.

Now, the strain in bar 1 is calculated as follows:

$$\text{Strain in bar 1,} = \frac{\text{Stress in bar 1}}{\text{Young's modulus of bar 1}} = \frac{\sigma_1}{E_1}$$

The strain in bar 2 is

$$\text{Strain in bar 2,} = \frac{\sigma_2}{E_2}$$

$$\text{Strain in bar 1} = \text{Strain in bar 2}$$

$$\therefore \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad \text{.....(2.30)}$$

Using Eqn. (2.27) and Eqn. (2.28), we can determine the values of σ_1 and σ_2 . By using these values in Eqn. (2.29), we can calculate the loads carried by the bar.

The ratio $\frac{E_1}{E_2}$ is referred to as the modulus ratio.

Problem 2.14

The steel rod with a diameter of 50mm is positioned at the centre of a hollow copper tube with an external and internal diameter of 70mm and 60mm, respectively. This composite bar is subsequently exposed to an axial force of 50KN. Young's modulus of steel and copper rod is 2.1×10^5 N/mm² and 1.1×10^5 N/mm², respectively. Assuming both the rod and the tube have equal lengths of 20cm, determine the following:

- (i) The stresses experienced by the tube and rod.
- (ii) The load supported by each component of the composite bar.

Solution:

Diameter of steel rod (D_s) = 50mm

$\therefore$ Area of the steel rod, (A_s) = $\frac{\pi}{4} (50)^2 = 1963.49 \text{ mm}^2$

External diameter of the hollow copper tube (D_{EC}) = 70mm

Internal diameter of the hollow copper tube (D_{IC}) = 60mm

$\therefore$ Area of the copper tube, (A_c) = $\frac{\pi}{4} (70^2 - 60^2) = 1021.01 \text{ mm}^2$

Axial force (P) = 50000 N

Length of the rod (L) = 200mm

Young's modulus of steel rod (E_s) = 2.1×10^5 N/mm²

Young's modulus of copper rod (E_c) = 1.1×10^5 N/mm²

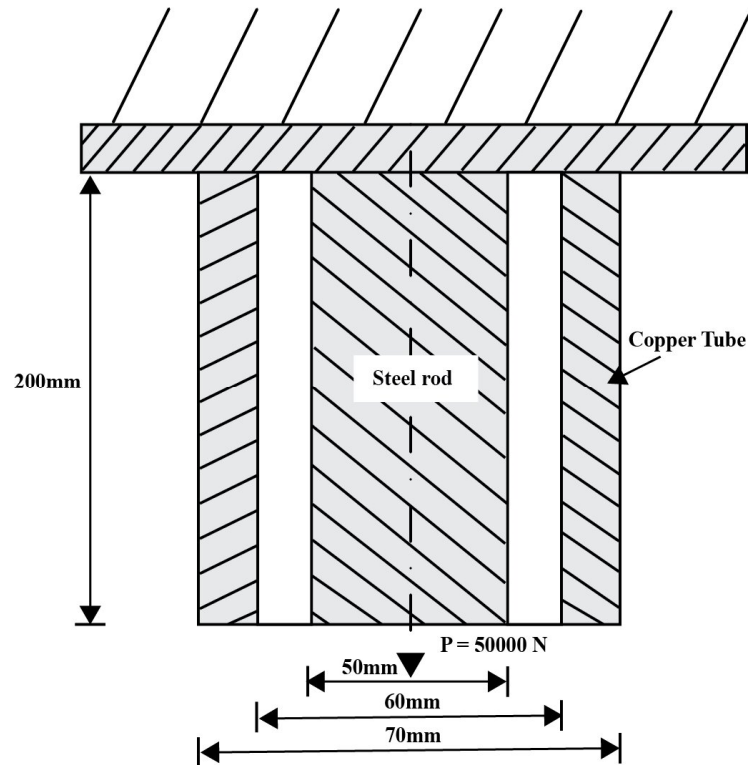


Fig. 2.26: Composite bar with steel rod inside the copper tube under axial load

(i) Stresses in the rod and tube:

Let

σ_s : Stress in the steel rod.

P_s : Load in the steel rod.

σ_c : Stress in the copper tube.

P_c : Load in the copper tube.

It is to be noted that strain in steel = Strain in copper.

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c = 1.909\sigma_c$$

$$\sigma_s = 1.909\sigma_c \quad \dots \dots (2.31)$$

$$\text{Now Stress} = \frac{\text{Load}}{\text{Area}}, \quad \therefore \text{Load} = \text{Area} \times \text{Stress}$$

Load on copper tube + Load on steel rod = Total applied load

$$(\sigma_s \times A_s) + (\sigma_c \times A_c) = P$$

$$(1.909\sigma_c \times 1963.49) + (\sigma_c \times 1021.01) = 50000$$

$$\sigma_c [(3748.30) + (1021.01)] = 50000$$

$$\sigma_c [4769.31] = 50000$$

$$\sigma_c = \frac{50000}{4769.31}$$

$$\sigma_c = 10.483 \text{ N/mm}^2$$

By using the values of σ_c in equation (2.31), we obtain the following:

$$\sigma_s = 1.909 \times 10.483$$

$$\sigma_s = 20.012 \text{ N/mm}^2$$

(ii) Loads taken by the individual bar.

$$\therefore \text{Load} = \text{Area} \times \text{Stress}$$

$\therefore$ Load in the steel rod,

$$P_s = \sigma_s \times A_s$$

$$P_s = 20.012 \times 1963.49 = 39293.36 \text{ N} = 39.293 \text{ kN}$$

$\therefore$ Load in the copper tube,

$$P_c = 50000 - 39293.36 = 10706.64 \text{ N} = 10.706 \text{ kN}$$

2.21 THERMAL STRESSES IN STRAIGHT BARS

When there is a temperature change in material and if it is allowed to expand freely, then no stresses are developed. If the material is restricted to expansion or contraction, some internal stresses are developed and are called as thermal stresses.

Consider the following condition in which a body is heated to a given temperature.

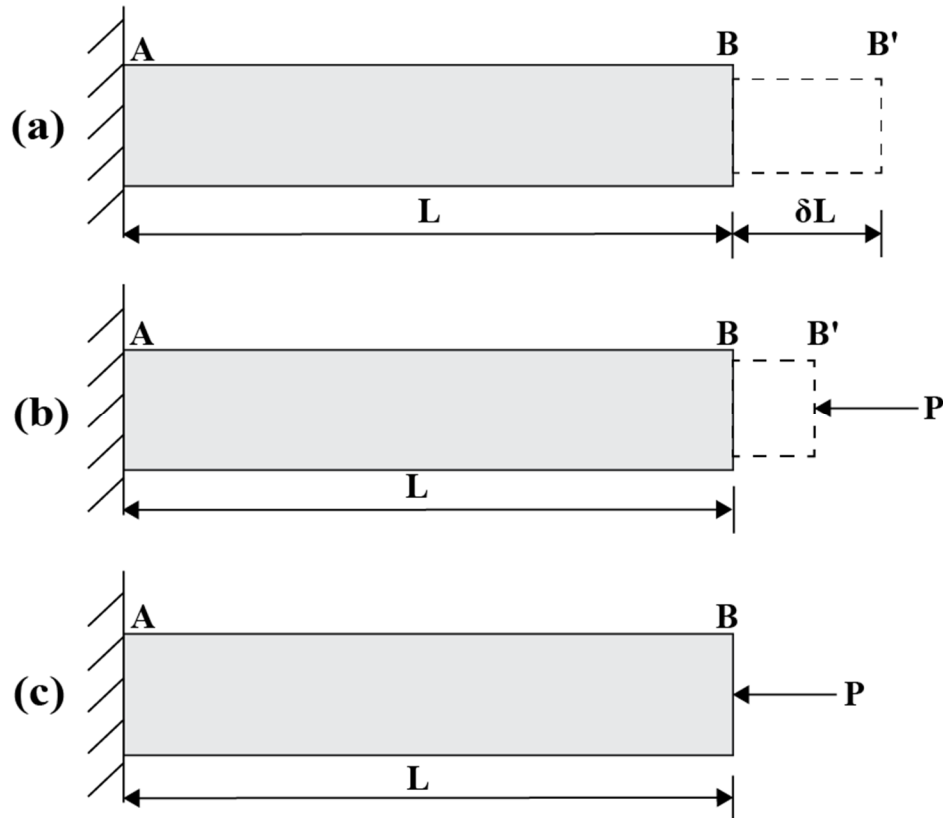


Fig. 2.27(a)-(c): Thermal stresses in straight bars

Let

L : The initial length of the body.

ΔT : Increase in Temperature

E : Modulus of elasticity

α : Linear coefficient of thermal expansion.

δL : The elongation of the rod due to the increase in the temperature.

If the rod expands freely, the extension in the rod is given by:

$$\delta L = \alpha \cdot \Delta T \cdot L \quad \dots\dots (2.32)$$

It is shown in Fig. 2.27 (a), where AB represents the initial length, and BB' represents the elongation caused by the increased temperature. Now, consider a case where an external compressive load, P is applied. It causes the rod to contract from its extended length $(L + \alpha\Delta TL)$ back to its original length, L, as shown in Fig. 2.27 (b) and (c).

When the ends of the body are firmly attached to fixed supports, thereby inhibiting any compressive stress and strain, expansion will occur within the rod. These specific stress and strain occurrences are called thermal stresses and strain.

$$\therefore \text{Thermal strain, } \epsilon = \frac{\text{Extension prevented}}{\text{Original length}}$$

$$= \frac{\delta L}{L} = \frac{\alpha \cdot \Delta T \cdot L}{L} = \alpha \Delta T \quad \dots\dots (2.33)$$

Thermal stress, $\sigma = E \times \text{Thermal strain}$

$$\sigma = \alpha \cdot \Delta T \cdot E \quad \dots\dots (2.34)$$

Thermal stress causes the thermal strain.

Problem 2.15

Determine the elongation of a 4-meter-long rod as its temperature increases from 15°C to 100°C. Calculate the stress generated in the rod if the thermal expansion is constrained. The material properties are, $\alpha = 0.000012/^{\circ}\text{C}$ and $E = 1 \times 10^5 \text{ MN/m}^2$.

Solution:

Length of the rod (L) = 4m = 4000mm

The temperature increases from $T_1 = 15^\circ\text{C}$ to $T_2 = 100^\circ\text{C}$

$\therefore$ Temperature rise, $(T) = T_2 - T_1 = 100 - 15 = 85^\circ\text{C}$

Young's modulus (E) = $1 \times 10^5 \text{ MN/m}^2 = 1 \times 10^5 \times 10^6 \text{ N/m}^2 = 1 \times 10^{11} \text{ N/m}^2$

Linear coefficient of thermal expansion (α) = $0.000012/^\circ\text{C}$

(i) Expansion of the rod due to thermal loading is given by the expression:

$$\begin{aligned}\therefore \text{Rod expansion} &= \alpha \cdot \Delta T \cdot L \\ &= 0.000012 \times 85 \times 4000 \\ \delta L &= 4.08 \text{ mm}\end{aligned}$$

(iii) When expansion is restricted, the stress induced in the rod is calculated by using the equation:

$\therefore$ Thermal stress,

$$\begin{aligned}\sigma &= \alpha \cdot \Delta T \cdot E \\ \sigma_T &= 0.000012 \times 85 \times 1 \times 10^{11} \\ \sigma_T &= 102 \times 10^6 \text{ N/m}^2 = 102 \text{ N/mm}^2\end{aligned}$$

Problem 2.16

Find the stress and the load when the temperature of a 7-meter-long steel rod with a 50 mm diameter drops from 100°C to 40°C under two conditions:

(i) When the ends do not deform and are fixed.

(ii) When the ends deform by 1.5mm.

$\alpha = 12 \times 10^{-6}/^\circ\text{C}$ and $E = 2 \times 10^5 \text{ MN/m}^2$

Solution:

Diameter of the rod (d) = 5cm = 50mm

$$\therefore \text{Area of the rod, } A = \frac{\pi}{4} \times (50)^2 = 625\pi \text{ mm}^2$$

Length of the rod (L) = 7m = 7000mm

$$T_1 = 100^\circ\text{C}$$

$$T_2 = 40^\circ\text{C}$$

$$\therefore \text{Change in temperature, } \Delta T = T_2 - T_1 = 100 - 40 = 60^\circ\text{C}$$

$$\text{Young's Modulus } E = 2 \times 10^5 \text{ MN/m}^2 \text{ (or) } E = 2 \times 10^5 \times 10^6 \text{ N/m}^2 \text{ (or) } E = 2 \times 10^{11} \text{ N/m}^2$$

$$\alpha = 12 \times 10^{-6}/^\circ\text{C}$$

(i) When the ends do not deform and stay fixed

Stress is calculated using the following equation

$$\text{Thermal Stress } \sigma = \alpha \times \Delta T \times E$$

$$= 60 \times 2 \times 10^{11} \times 12 \times 10^{-6} = 144 \times 10^6 \text{ N/m}^2 = 144 \text{ N/mm}^2$$

Force in the rod = stress $\times$ area

$$= 144 \times 625\pi = 282743.33 \text{ N}$$

(ii) When the ends deform by 1.5 mm

$$\therefore \delta = 1.5 \text{ mm}$$

The stress under conditions where the yield ends is determined using the following equation:

$$\begin{aligned} \text{Stress} &= \frac{(\alpha \cdot \Delta T \cdot L - \delta)}{L} \times E \\ \text{Stress} &= \frac{(12 \times 10^{-6} \times 60 \times 7000 - 1.5)}{7000} \times (2 \times 10^5) \text{ N/mm}^2 \\ &= \frac{(5.04 - 1.5)}{7000} \times (2 \times 10^5) \end{aligned}$$

$$= 101.14 \text{ N/mm}^2$$

$$\text{Force in the rod} = \text{Stress} \times \text{Area}$$

$$= 101.14 \times 625\pi = 198587.9 \text{ N.}$$

2.22 THERMAL STRESS IN A CIRCULAR TAPERED BAR

Consider a circular bar with a tapered section fixed at both ends and exposed to a rise in temperature, as shown in Fig. 2.28.

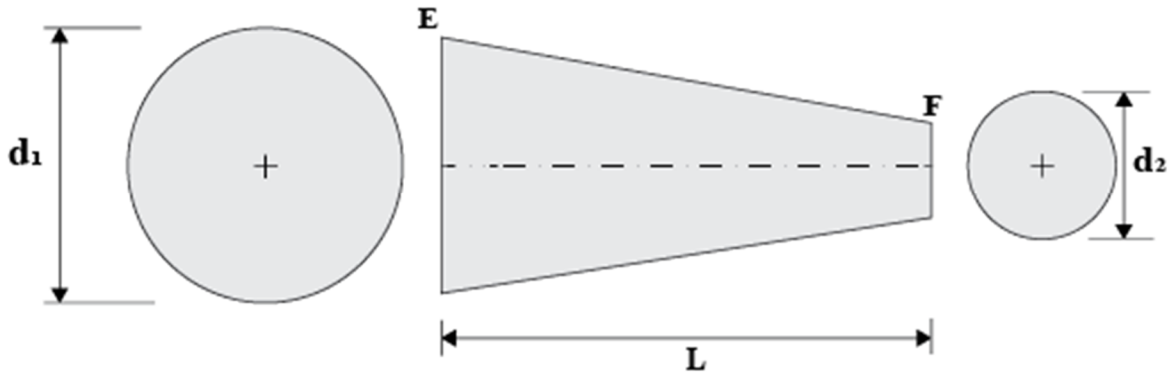


Fig. 2.28: Thermal stress in a circular tapered section bar

'L' represents the bar length.

' d_1 ' and ' d_2 ' are the larger and smaller diameters of the bar.

' ΔT ' is the increase in temperature.

' α ' is the linear coefficient of thermal expansion.

We know that,

The bar EF will usually expand when the temperature rises. However, if it is fixed at both ends, it will generate compressive stress.

$$\delta L = L.\alpha. \Delta T \quad \dots \dots (2.35)$$

Let 'P' represent the load required to regain the deformed bar to its initial length.

$$\delta L = \frac{4PL}{\pi E d_1 d_2} \quad \dots \dots (2.36)$$

Equating Eqn (2.35) and (2.36),

$$L \cdot \alpha \cdot \Delta T = \frac{4PL}{\pi E d_1 d_2} \quad \text{or} \quad P = \frac{\pi E d_1 d_2 \alpha \Delta T}{4}$$

$$\therefore \text{Max stress, } \sigma_{\max} = \frac{P}{\frac{\pi}{4} \times d_2^2} = \frac{\pi E d_1 d_2 \alpha \Delta T}{4 \times \frac{\pi}{4} \times d_2^2} = \frac{\alpha \cdot \Delta t \cdot E \cdot d_1}{d_2} \quad \dots\dots (2.37)$$

If we consider the uniform bar of diameter

$$d = d_1 = d_2$$

The stress,

$$\sigma = \alpha \cdot \Delta T \cdot E \quad \dots\dots (2.38)$$

This equation is the same as the one used for simple bars.

2.23 THERMAL STRESSES IN BARS OF DIFFERENT SECTION

Consider a bar ABC fixed at A and C ends and exposed to the rise in temperature shown in Fig. 2.29.

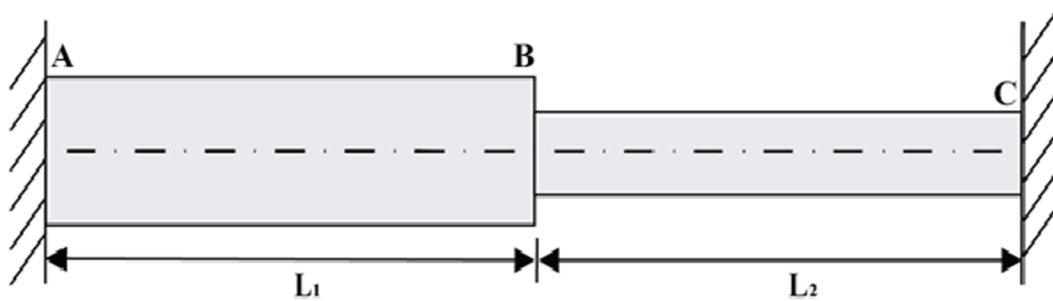


Fig. 2.29: Thermal stresses in different section bars

Let

' L_1 ' is the length of section AB.

' σ_1 ' is the stress in section AB.

' A_1 ' is the area of cross-section section AB.

' L_2 ,' ' σ_2 ,' and ' A_2 ' correspond to values for section BC.

' α ' stands for the linear coefficient of expansion.

' T ' represents the temperature rise.

We know that,

When the temperature rises, the ABC bar will begin to expand. However, since it is fixed at the A and C ends, it will generate compressive stress within the body.

Furthermore, as the thermal load is evenly distributed across both sections,

$$\sigma_1 A_1 = \sigma_2 A_2$$

Also, if we consider the total bar deformation

$$\delta L = \delta L_1 + \delta L_2 = \frac{\sigma_1 L_1}{E} + \frac{\sigma_2 L_2}{E}$$

In cases where the modulus of elasticity is different for different sections, the total deformation is

$$\delta L = \left(\frac{\sigma_1 L_1}{E_1} + \frac{\sigma_2 L_2}{E_2} \right) \quad \dots \dots (2.39)$$

2.24 THERMAL STRESSES IN COMPOSITE BAR

When the temperature of the bar composed of two or more different materials, experiences a change in temperature, it leads to either contraction or expansion of the bar. Because of the difference in the coefficient of thermal expansion, these materials experience varying degrees of expansion or contraction, leading to distinct reactions.

Consider a composite bar comprising steel and aluminum components, as shown in Fig. 2.30(a).

If the individual components of the bar (steel and aluminum) are allowed to expand freely, no internal stresses would develop. However, if both components are firmly fixed, the composite bar will undergo uniform expansion by an identical amount. It is important to note that aluminum expands higher than steel due to its higher linear expansion coefficient compared to steel.

Therefore, the expansion of aluminum will be higher than steel. Nevertheless, due to the expansion of both components, the composite bar will experience a level of expansion that falls between the expansion of aluminum (which is high) and the expansion of steel (which is less), as demonstrated in Fig. 2.30(b). As a result, the aluminum would experience compressive force, but the steel might experience tensile force, as shown in Fig. 2.30(c).

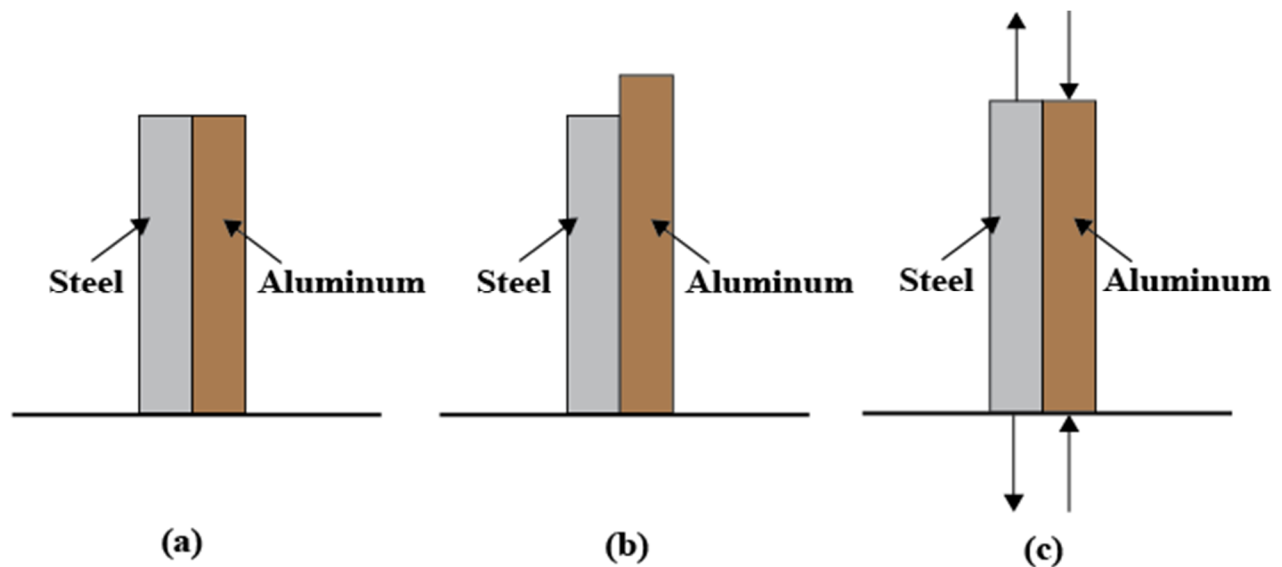


Fig. 2.30(a)-(c): Thermal stresses for steel and aluminum

Let

' σ_1 ' and ' ϵ_1 ' represent the stress and strain in aluminum.

' α_1 ' denotes the linear coefficient of thermal expansion of aluminum.

' A_1 ' is the cross-sectional area of the aluminum bar.

' σ_2 ,' ' α_2 ,' ' ϵ_2 ,' and ' A_2 ' correspond to the equivalent values for steel.

' ϵ ' represents the actual strain experienced by the composite bar per unit length.

Since the tensile load on a steel equals the compressive load on a aluminum,

$$\sigma_1 A_1 = \sigma_2 A_2$$

Now, the strain in aluminum is

$$\epsilon_1 = \alpha_1 \cdot t + \epsilon \quad \dots\dots (2.40)$$

Similarly, the strain in steel is

$$\epsilon_2 = \alpha_2 \cdot t - \epsilon \quad \dots\dots (2.41)$$

When we sum up (Eqn 2.40) and (Eqn 2.41), we obtain:

$$\epsilon_1 + \epsilon_2 = t (\alpha_1 + \alpha_2) \quad \dots\dots (2.42)$$

In the above equation, the α_1 value is taken as the larger of the two values, α_1 or α_2 .

Problem 2.17

Determine the stresses in both the copper and steel bars of the assembly. This assembly consists of a centrally passing steel rod with a 30mm diameter, a copper tube with external diameter, and internal diameter of 60mm and 50mm, respectively. The temperature in the composite system is increased by 60°C. The values Young's modulus (E) for copper and steel are 100GN/m² and 200GN/m², respectively, and the linear coefficient of thermal expansion (α) for copper = 18×10⁻⁶ per °C and steel = 12×10⁻⁶ per °C.

Solution:

Diameter of steel bar (D_s) = 30mm

$$\therefore \text{Area of steel rod, } (A_s) = \frac{\pi}{4} \times 30^2 = 225\pi \text{ mm}^2$$

External diameter of copper tube = 60mm

Internal diameter of copper tube = 50mm

$$\text{Area of Copper tube, } (A_s) = \frac{\pi}{4} \times (60^2 - 50^2) = 275\pi \text{ mm}^2$$

Rise in temperature, $T = 60^\circ\text{C}$

E for steel, $(E_s) = 200\text{GN/m}^2$

$$E_s = 200 \times 10^3 \text{ N/mm}^2$$

E for copper, $(E_c) = 100\text{GN/m}^2$

$$E_c = 100 \times 10^3 \text{ N/mm}^2$$

Coefficient of linear expansion of steel $\alpha_s = 12 \times 10^{-6} \text{ per } ^\circ\text{C}$

Coefficient of linear expansion of copper $\alpha_c = 18 \times 10^{-6} \text{ per } ^\circ\text{C}$

Due to the higher coefficient of linear expansion (α) for copper compared to steel, the copper tube will naturally undergo more expansion as the temperature increases. Both ends of the tube and the rods are firmly attached to rigid plates, with the nuts securely tightened onto the extended sections of the rod. As a result, neither of the two components can undergo unrestricted expansion.

Both the rod and the tube will expand equally. The copper tube will expand more freely than the common expansion, while the steel rod will experience less expansion in its unrestricted state than the standard expansion. The variation in expansion results in the copper tube undergoing compressive stress and the steel rod undergoing tensile stress.

Let σ_s and σ_c represent the tensile and compressive stress in steel and copper rods.

To maintain equilibrium within the system,

$$\text{Compressive load on copper} = \text{Tensile load on steel}$$

$$\sigma_s \cdot A_s = \sigma_c \cdot A_c$$

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$\sigma_s = \frac{275}{225} \times \sigma_c$$

$$\sigma_s = 1.222\sigma_c \quad \dots\dots (2.43)$$

Since the steel rod and the copper tube will experience an identical amount of expansion,

$$\text{Actual steel expansion} = \text{Actual copper expansion} \quad \dots\dots (2.44)$$

But

Actual steel expansion = (Tensile stress causes steel to expand + Expansion of steel)

$$= \sigma_s \cdot T \cdot L + \frac{\sigma_s}{E_s} \cdot L$$

Substituting these values into Eqn (2.44), we obtain:

$$\sigma_s \cdot T \cdot L + \frac{\sigma_s}{E_s} \cdot L = \sigma_c \cdot T \cdot L - \frac{\sigma_c}{E_c} \cdot L$$

$$\sigma_s \cdot T + \frac{\sigma_s}{E_s} = \sigma_c \cdot T - \frac{\sigma_c}{E_c}$$

$$(12 \times 10^{-6} \times 60) + \frac{1.222\sigma_c}{200 \times 10^3} = (18 \times 10^{-6} \times 60) - \frac{\sigma_c}{100 \times 10^3}$$

$$\frac{1.222\sigma_c}{200 \times 10^3} + \frac{\sigma_c}{100 \times 10^3} = (18 \times 10^{-6} \times 60) - (12 \times 10^{-6} \times 60)$$

$$(0.611\sigma_c \times 10^{-5}) + (10^{-5} \sigma_c) = 6 \times 10^{-6} \times 60$$

$$1.611 \times 10^{-5} \sigma_c = 36 \times 10^{-5}$$

$$\sigma_c = \frac{36}{1.611} = 22.346 \text{ N/mm}^2$$

$$\sigma_c = 22.346 \text{ N/mm}^2$$

Substituting this value in Eqn (2.43), we have:

$$\sigma_s = 1.222\sigma_c$$

$$\sigma_s = 1.222 \times 22.346$$

$$\sigma_s = 27.306 \text{ N/mm}^2$$

2.25 LONGITUDINAL STRAIN

When exposed to a compressive or tensile force, axial deformation occurs in a structural member. This strain is quantified as the ratio of the change in length (δL) to the original length (L) of the material in the direction of the force applied, and it can also be expressed as the deformation per unit length along the direction of the force applied.

Let

Change in length (δL) of the member due to the force (P) and the original length be L .

Then, longitudinal strain is $= \frac{\delta L}{L}$ (2.45)

2.26 LATERAL STRAIN

The deformation that occurs perpendicular to the direction of the applied load is known as lateral deformation. Consider a rectangular bar with dimensions breadth (b), length (L), and depth (d) subjected to an axial tensile load (P), as shown in Fig. 2.31. As a result of this load, the bar length increases while both the depth and breadth decrease.

Let

δL represents the increase in length.

δb signifies the reduction in breadth.

δd denotes the reduction in depth.

Then lateral strain $= \frac{\delta b}{b}$ and longitudinal strain $= \frac{\delta L}{L}$ (2.46)

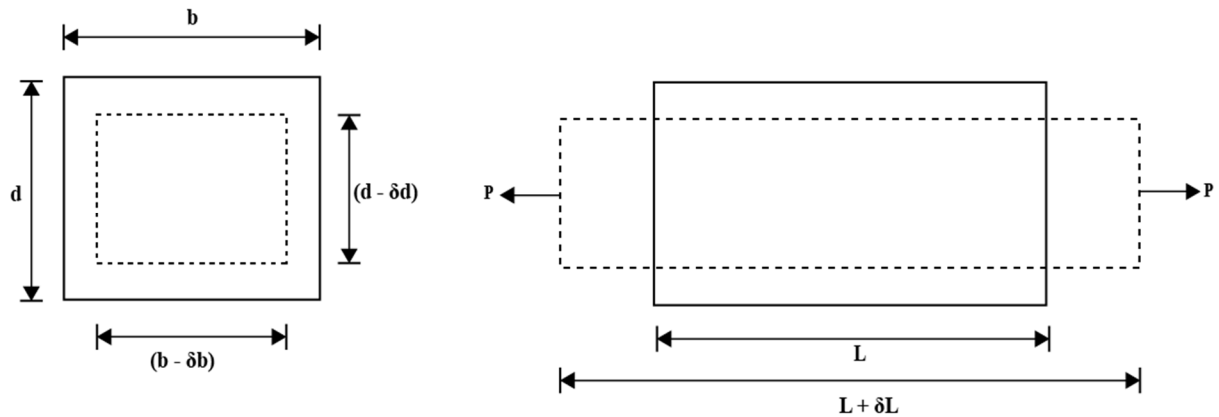


Fig. 2.31: Figure showing the lateral strain

2.27 SHEAR MODULUS

The shear modulus G , represents the relationship between shear stress and shear strain within the elastic limit.

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\delta} \quad \dots\dots (2.47)$$

2.28 POISSON'S RATIO

Poisson's ratio (μ) is the constant value of the material, representing the ratio of lateral strain to longitudinal strain when stressed within its elastic limit. Mathematically, it's defined as:

$$\text{Poisson's ratio, } \mu = \frac{-\text{Lateral strain}}{\text{Longitudinal strain}} \quad \dots\dots (2.48)$$

This relationship can also be expressed as:

$$\text{Lateral strain} = -\mu \times \text{Longitudinal strain}$$

Since lateral strain has the opposite sign of longitudinal strain, the value of Poisson's ratio typically falls within the range of 0.25 to 0.33 for most materials. However, for rubber, its value is between 0.45 to 0.50.

Problem 2.18

Calculate the change in the breadth, thickness, and length of a steel bar measuring 5m in length, 40mm in width, and 25mm in thickness when exposed to an axial pull of 40kN along its length. Poisson's ratio = 0.3 and Young's modulus of the material = 2×10^5 N/mm².

Solution:

$$L = 5\text{m} = 5000\text{mm}$$

$$b = 40\text{mm}$$

$$t = 25\text{mm}$$

$$\therefore \text{Area of cross-section, } A = b \times t = 40 \times 25 = 1000 \text{ mm}^2$$

$$P = 40\text{kN} = 40000 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.3$$

The strain in the loading direction (the longitudinal strain).

$$\begin{aligned} &= \frac{\text{Stress}}{E} && \left(\therefore \text{Stress} = \frac{\text{Load}}{\text{Area}} \right) \\ &= \frac{\text{Load}}{\text{Area} \times E} \\ &= \frac{P}{\text{Area} \times E} = \frac{40000}{1000 \times 2 \times 10^5} = 0.0002 \end{aligned}$$

$$\text{But longitudinal strain} = \frac{\delta L}{L}$$

$$\therefore \frac{\delta L}{L} = 0.0002$$

$$\therefore \delta L = 0.0002 \times L$$

$$\delta L = 0.0002 \times 5000 = 1 \text{ mm}$$

Using Poisson's ratio

$$\text{Poisson's ratio} = \frac{-\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$0.3 = \frac{-\text{Lateral strain}}{0.0002}$$

$$\therefore \text{Lateral strain} = -0.0002 \times 0.3 = -0.00006$$

We know that,

$$\text{Lateral strain} = \frac{\delta b}{b}$$

$$\delta b = b \times \text{Lateral strain}$$

$$\delta b = 40 \times (-0.00006) = -0.0024 \text{ mm}$$

2.29 BIAXIAL STRESS

Consider a two-dimensional structure, shown in Fig. 2.32 subjected to normal stresses.

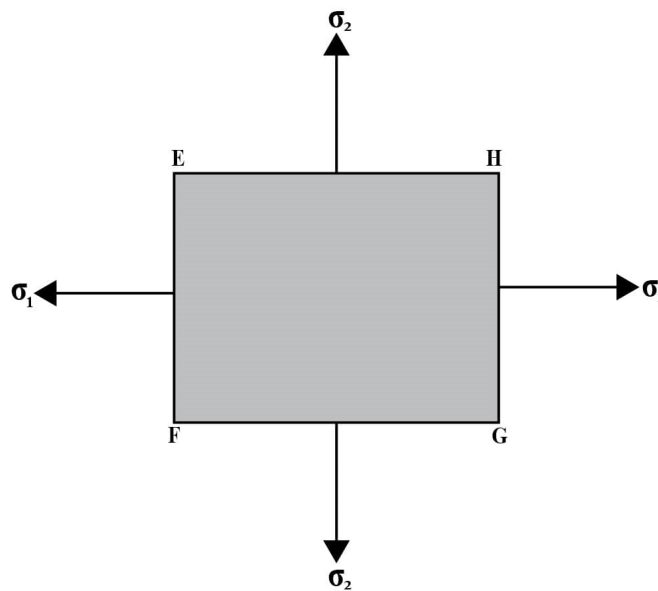


Fig. 2.32: Figure showing the biaxial stress

Let σ_1 , and σ_2 represent the normal stresses in the X and Y-directions, respectively.

When stress σ_1 is applied, the strain in the X-direction is equal to $\frac{\sigma_1}{E}$, while the strain in the Y-direction is represented as $-\mu \times \frac{\sigma_1}{E}$, where μ is Poisson's ratio.

Now, due to the load in Y- direction

The strain in the Y-direction is calculated as $\frac{\sigma_2}{E}$, while the strain in an Y-direction is provided by $-\mu \times \frac{\sigma_2}{E}$, where μ represents Poisson's ratio.

Let

ϵ_1 , ϵ_2 be the total strain in the x and y direction.

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \times \frac{\sigma_2}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \times \frac{\sigma_1}{E}$$

Thus

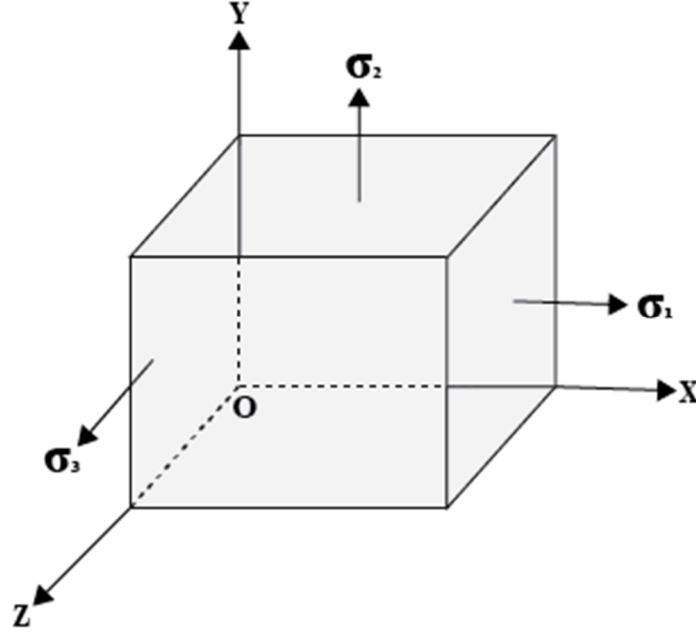
$$\therefore \epsilon_1 = \frac{\sigma_1}{E} - \mu \times \frac{\sigma_2}{E} \quad \dots\dots (2.49)$$

$$\therefore \epsilon_2 = \frac{\sigma_2}{E} - \mu \times \frac{\sigma_1}{E} \quad \dots\dots (2.50)$$

These two equations establish the relationship between stress and strain in a two-dimensional stress system.

2.30 TRIAXIAL STRESS

Fig. 2.33 shows a three-dimensional body exposed to three mutually perpendicular normal stresses: σ_1 , σ_2 , and σ_3 , acting along the X, Y and Z directions, respectively.



Fig, 2.33: Figure showing the triaxial stress

Consider the strains induced by each of these stresses separately:

- Stress σ_1 makes strain in the X, Y, and Z-directions.

The strain in the X-direction is $\frac{\sigma_1}{E}$, While the strain in the Y and Z directions is $\left(-\mu \times \frac{\sigma_1}{E}\right)$ (2.51)

- Stress σ_2 results in strain in the Y-direction, which is $\frac{\sigma_2}{E}$, and in the X and Z directions, it is $\left(-\mu \times \frac{\sigma_2}{E}\right)$ (2.52)

- Stress σ_3 induces strain in the Z-direction, by $\frac{\sigma_3}{E}$, as well as in the X and Y directions each of $\left(-\mu \times \frac{\sigma_3}{E}\right)$ (2.53)
- These relationships describe how the stresses σ_1 , σ_2 , and σ_3 produce strains in the respective directions, taking into account Young's modulus (E) and Poisson's ratio (μ).

The total strain in the direction of "X" resulting from stresses σ_1 , σ_2 , and σ_3 will be

$$\epsilon_x = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$\epsilon_y = \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E}$$

$$\epsilon_z = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

ϵ_x , ϵ_y , and ϵ_z are the total strains in the X, Y and Z-directions.

$$\epsilon_x = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} \quad \dots \dots (2.54)$$

$$\epsilon_y = \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} \quad \dots \dots (2.55)$$

$$\epsilon_z = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \quad \dots \dots (2.56)$$

These three equations establish the stress and strain relationship for a three-dimensional system subjected to normal stresses.

2.31 VOLUMETRIC STRAIN

Volumetric strain is important when a structural element is subjected to various stresses, causing deformation in multiple directions and altering its volume. This

strain is quantified as the ratio of the change in volume to the original volume and is mathematically expressed as:

Thus,
$$\epsilon_v = \frac{\delta V}{V} \quad \dots\dots (2.57)$$

In the equation, where e_v denotes the volumetric strain, δV represents the volume change, and V signifies the initial volume. The volumetric strain results from the combined effects of strains occurring in three mutually perpendicular directions, which is given by:

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z \quad \dots\dots (2.58)$$

For example, consider a bar with breadth b , length L , and depth d , as shown in Fig. 2.34. The volume V is the product of L , b , and d .

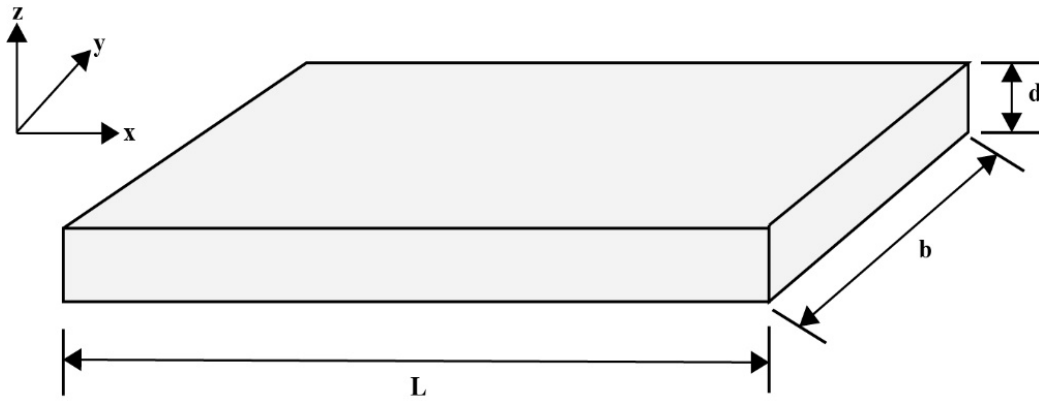


Fig. 2.34: Figure showing the volumetric strain

Now,

$$V = L \times b \times d \quad \dots\dots (2.59)$$

As the volume is determined by the dimensions L, b, and d, we can express it as a function of these variables.

$$\delta V = \delta L \, b \, d + L \, \delta b \, d + L \, b \, \delta d \quad \dots \dots (2.60)$$

$\therefore$ Volumetric strain,

$$\begin{aligned} \epsilon_v &= \frac{\delta V}{V} \\ \epsilon_v &= \frac{(\delta L \, b \, d) + (L \, \delta b \, d) + (L \, b \, \delta d)}{(L \times b \times d)} \\ \epsilon_v &= \frac{\delta L}{L} + \frac{\delta d}{d} + \frac{\delta b}{b} \quad \dots \dots (2.61) \\ \epsilon_v &= \epsilon_x + \epsilon_y + \epsilon_z \end{aligned}$$

But $\frac{\delta L}{L}$ = Longitudinal strain and $\frac{\delta d}{d}$ and $\frac{\delta b}{b}$ are lateral and transverse strains.

Upon these values into the Eqn. (2.61), we obtain the following result if the section is cubical:

$$\epsilon_v = \text{Longitudinal strain} + (2 \times \text{Lateral strain}) \quad \dots \dots (2.62)$$

Based on the equation for Poisson's ratio,

$$\therefore \quad \text{Lateral strain} = (-\mu \times \text{Longitudinal strain}) \quad \dots \dots (2.63)$$

By replacing the lateral strain value in Eqn. (2.63), we obtain:

$$\begin{aligned} \epsilon_v &= \text{Longitudinal strain} (1 - 2\mu) \\ \epsilon_v &= \frac{\delta L}{L} (1 - 2\mu) \quad \dots \dots (2.64) \end{aligned}$$

Problem 2.19

Find the change in volume when a steel bar measuring 400mm in length, 60mm in width, and 50mm in thickness is exposed to a tensile force of 350kN along its length. Given the material properties, Poisson's ratio (μ) = 0.25 and Young's modulus of elasticity (E) = 2×10^5 N/mm². Calculate the change in volume.

Solution:

Length of the bar (L) = 400mm

Width of the bar (b) = 60mm

Thickness of the bar (t) = 50 mm

Tensile force (P) = 350kN = 350×10^3 N

Poisson's ratio (μ) = 0.25

Original volume, $V = L \times b \times t$

$$V = 400 \times 60 \times 50 = 1200000 \text{ mm}^3$$

The longitudinal strain is provided by

$$\frac{dL}{L} = \frac{\text{Stress in the direction}}{E}$$

However, stress in the direction of the applied load can be expressed as.

$$= \frac{P}{A} = \frac{P}{b \times t} = \frac{350 \times 10^3}{60 \times 50} = 116.66 \text{ N/mm}^2$$

$$\frac{dL}{L} = \frac{116.66}{2 \times 10^5} = 0.00058$$

Now, volumetric strain is provided by equation

$$\epsilon_v = \frac{dL}{L} (1 - 2\mu)$$

$$\epsilon_v = 0.00058 (1 - 2 \times 0.25)$$

$$\epsilon_v = 0.00029$$

Let δV = Change in volume.

Then $\frac{\delta V}{V}$ is volumetric strain.

$$\frac{\delta V}{V} = 0.00029$$

$$\delta V = 0.00029 \times V$$

$$\delta V = 0.00029 \times 1200000$$

$$\delta V = 348 \text{ mm}^3$$

2.32 BULK MODULUS

When a solid body is subjected to an equivalent stress (p) operating in three perpendicular directions, it undergoes consistent changes in all three dimensions without alteration in its shape. The ratio between the volume change and the initial volume is the volumetric strain (ϵ_v). Bulk modulus (K) is defined as

$$K = \frac{p}{\epsilon_v} \quad \dots \dots (2.65)$$

p represents the uniform pressure applied in the three mutually perpendicular directions.

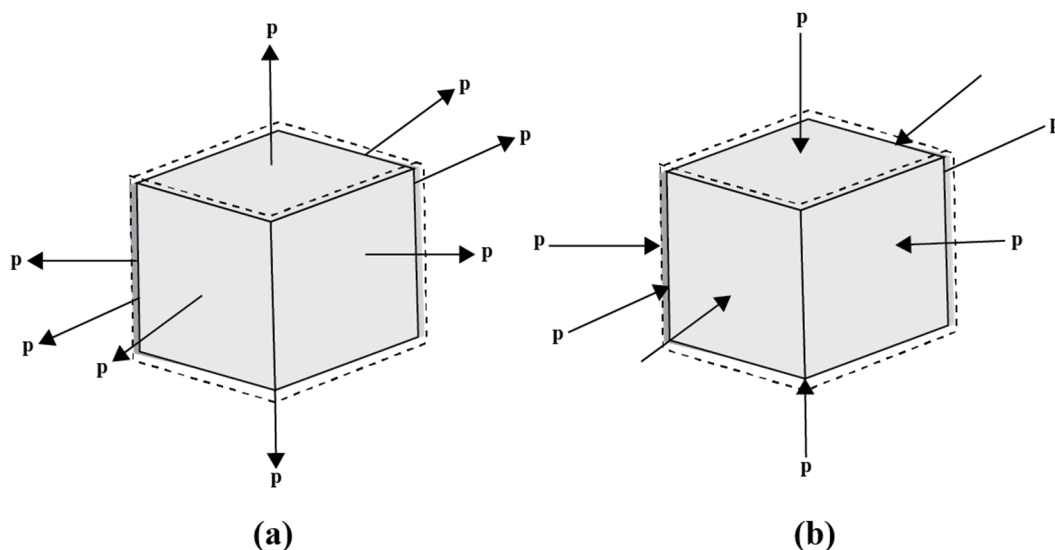


Fig. 2.35(a)-(b): Figure showing the loading condition for bulk modulus

$$\text{Volumetric strain } \epsilon_v = \frac{\delta V}{V}$$

δV = Change in volume

V = Original volume

Fig. 2.35(a)-(b) shows a solid body experiencing uniform tension and compressive force 'p' in the three mutually perpendicular directions. Because this type of stress mimics hydrostatic pressure, it is called hydrostatic pressure.

2.33 RELATIONSHIP BETWEEN SHEAR MODULUS (G) AND YOUNG'S MODULUS (E)

When a square block of unit thickness is subjected to a shear stress of magnitude on the section of AB, CD, AD and CB, then the principal stress along the diagonal is τ .

Total tensile strain along the diagonal $BD = \frac{\tau}{E} + \mu \cdot \left(\frac{\tau}{E}\right)$

$$BD = \frac{\tau}{E} (1 + \mu) \quad \dots \dots (2.66)$$

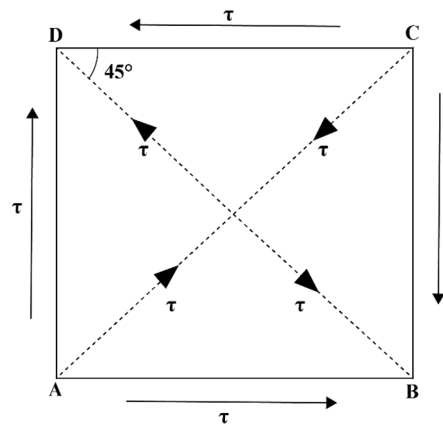


Fig. 2.36: Figure showing the shear stress over a surface

Due to the shear stresses acting on the section. The section ABCD will be deformed to position ABC₁D₁ as shown in Fig. 2.37.

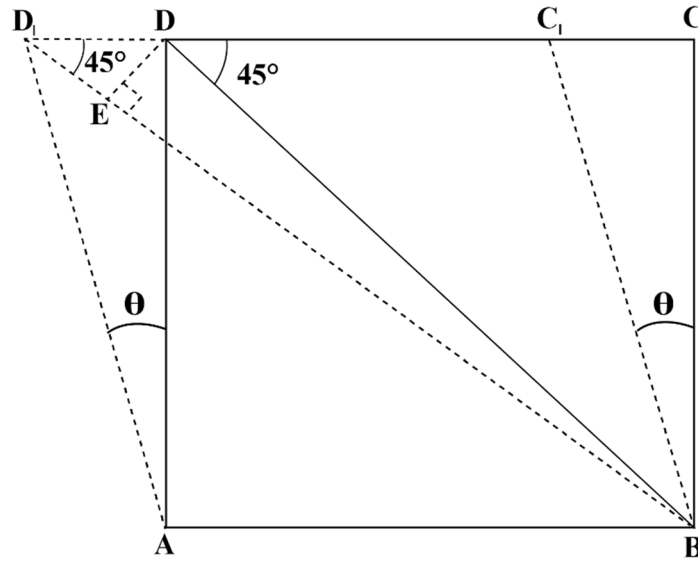


Fig. 2.37: Figure showing the square block is deformed to position ABC₁D₁ due to shear stress intensity

$$\text{Shear strain} = \theta = \frac{DD_1}{AD}$$

Now, an increase in the length of the diagonal $BD = BD_1 - BD$

$$\begin{aligned} \therefore \text{Tensile strain along } BD &= \frac{BD_1 - BD}{BD} \\ &= \frac{BE + D_1E - BE}{BD} \\ &= \frac{D_1E}{BD} \end{aligned}$$

$\therefore$ Now in $\triangle DD_1E$,

$$D_1E = DD_1 \times \cos 45^\circ$$

$$D_1E = DD_1 \times \frac{1}{\sqrt{2}}$$

$\therefore$ In $\triangle ABD$,

$$\begin{aligned} BD &= \sqrt{AB^2 + AD^2} \\ &= \sqrt{AD^2 + AD^2} \end{aligned} \quad (\because AB = AD)$$

$$BD = \sqrt{2} AD$$

$$\text{Tensile strain along BD} = \frac{DD_1}{BD}$$

$$BD = \frac{DD_1 \times \frac{1}{\sqrt{2}}}{\sqrt{2} AD}$$

$$BD = \frac{1}{2} \frac{DD_1}{AD} \quad \left(\theta = \frac{DD_1}{AD} \right)$$

$$BD = \frac{1}{2} \theta \quad \dots \dots (2.70)$$

Equating the tensile strain along BD due to the forces along BD (Tension) and along AC (Compression), we get

$$\frac{\tau}{E} (1 + \mu) = \frac{1}{2} \theta \quad \left(\because \theta = \frac{\tau}{G} \right)$$

$$\frac{\tau}{E} (1 + \mu) = \frac{1}{2} \left(\frac{\tau}{G} \right)$$

$$E = 2G (1 + \mu) \quad \dots \dots (2.67)$$

Where

μ - Poisson's ratio, G - Shear Modulus, and E - Young's Modulus.

2.34 RELATIONSHIP BETWEEN BULK MODULUS (K) AND YOUNG'S MODULUS (E)

When a body is exposed to a tri-axial stress system,

We know that its volumetric strain is given by

$$\epsilon_v = \frac{\delta V}{V}$$

$$\epsilon_v = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} (1 - 2\mu)$$

Now

$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\therefore \epsilon_v = \frac{(3\sigma)}{E} (1 - 2\mu)$$

But
$$K = \frac{\sigma}{\epsilon_v} = \frac{\sigma}{\frac{(3\sigma)}{E} (1-2\mu)}$$

$$\therefore K = \frac{E}{3(1-2\mu)} \quad \dots \dots (2.68)$$

2.35 RELATIONSHIP BETWEEN SHEAR MODULUS (G), YOUNG'S MODULUS (E), AND BULK MODULUS (K)

We know that

$$E = 2G (1 + \mu) \quad \dots \dots (2.69)$$

And
$$E = 3K (1 - 2\mu) \quad \dots \dots (2.70)$$

From Eqn. (2.70)

$$(1 + \mu) = \frac{E}{2G}$$

$$\therefore \mu = \frac{E}{2G} - 1$$

Equating this value in Eqn. (2.71)

$$E = 3K \left[1 - 2 \left(\frac{E}{2G} - 1 \right) \right] = 3K \left[1 - \frac{E}{G} + 2 \right]$$

$$E = 3K \left[3 - \frac{E}{G} \right] = 3K \left[\frac{3G - E}{G} \right]$$

$$EG = 3K (3G - E) = 9KG - 3KE$$

$$\therefore EG + 3KE = 9KG$$

$$\therefore E (G + 3K) = 9KG$$

$$\therefore E = \frac{9KG}{3K+G} \quad \dots \dots (2.71)$$

Problem 2.20

Calculate the Poisson's ratio and Bulk modulus of a material, given that its Shear modulus is $4.8 \times 10^4 \text{ N/mm}^2$ and Young's modulus is $1.2 \times 10^5 \text{ N/mm}^2$.

Solution:

$$E = 1.2 \times 10^5 \text{ N/mm}^2$$

$$G = 4.8 \times 10^4 \text{ N/mm}^2$$

Let

μ = Poisson's ratio

Using the (Eqn 2.71),

$$E = 2G (1+\mu)$$

$$1.2 \times 10^5 = 2 \times (4.8 \times 10^4) (1+\mu)$$

$$(1+\mu) = \frac{1.2 \times 10^5}{2 \times 4.8 \times 10^4} = 1.25$$

$$(1+\mu) = 1.25$$

$$\mu = 1.25 - 1$$

$$\mu = 0.25$$

Using the formula for bulk modulus,

$$\text{We get } K = \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^5}{3(1-2 \times 0.25)}$$

$$K = 8 \times 10^4 \text{ N/mm}^2$$

UNIT SUMMARY

- The chapter on simple stress and strain delves into the fundamental concepts underlying the behavior of materials under load, providing a comprehensive understanding of their applications in engineering, manufacturing, and everyday life. Beginning with an exploration of stress, students learn to calculate stress by dividing the force applied to a material by its cross-sectional area, gaining insights into the internal forces and deformation experienced by materials.
- The unit further explores stress-strain curves, providing students with a graphical representation of material's mechanical properties, including its elasticity, yield strength, ultimate strength, and ductility. Students learn to analyze stress-strain curves to assess the suitability of materials for specific applications, such as determining the structural integrity of bridges, buildings, and mechanical components.
- Moreover, students gain an understanding Young's modulus, shear modulus, and bulk modulus, which quantify a material's resistance to deformation under different types of loading.
- By the end of the unit, students should have acquired a solid foundation in simple stress and strain analysis, equipping them with the knowledge and skills necessary to excel in fields such as mechanical engineering, materials science, and structural design.

EXERCISE

A. Multiple Choice Questions

- 2.1 The ratio of stress and strain is known as
(a) Modulus of elasticity (c) Both a and b
(b) Young's modulus (d) None of the above
- 2.2 Two parallel, equal and opposite forces acting tangentially to the surface of the body is called as
(a) Complementary stress (c) Shear stress
(b) Compressive stress (d) Tensile stress
- 2.3 The actual failure stress in a true stress-strain diagram is the ratio of
(a) Load at failure and original cross-sectional area (c) Maximum load and original cross-sectional area
(b) Failure load and reduced cross-sectional area (d) Yield load and original cross-sectional area
- 2.4 The ratio between ultimate stress and working stress is called
(a) Factor of safety (c) Breaking stress
(b) Proof stress (d) Poisson's ratio
- 2.5 According to Hooke's law, stress is proportional to
(a) Elasticity (c) Strain
(b) Longitudinal stress (d) Tensile stress
- 2.6 If a part is constrained to move and heated, it will develop
(a) Principal stresses (c) Compressive stress
(b) Tensile stress (d) Shear stress
- 2.7 If a piece of material neither expands nor contracts in volume when subjected to stresses, then the poisson's ratio must be
(a) 0 (c) 0.33
(b) 0.25 (d) 0.5
- 2.8 Engineering strain of a mild steel sample is recorded as 0.100%. The true strain is
(a) 0.010% (c) 0.099%
(b) 0.055% (d) 0.101%

2.9 For an isotropic, homogenous and elastic material obeying Hooke's law, number of independent elastic constants

- | | |
|--------|-------|
| (a) 21 | (c) 3 |
| (b) 9 | (d) 2 |

2.10 Modulus of rigidity is the ratio of

- | | |
|--------------------------------------|-----------------------------------|
| (a) Lateral strain and linear strain | (c) Shear stress and shear strain |
| (b) Linear stress and lateral strain | (d) Shear strain and shear stress |

2.11 The relation between modulus of elasticity (E), modulus of rigidity (G) and bulk modulus (K) is given as

- | | |
|--------------------|--------------------|
| (a) $K+G / (3K+G)$ | (c) $3KG / (9K+G)$ |
| (b) $3KG / (3K+G)$ | (d) $9KG / (3K+G)$ |

2.12 If the poisson's ratio of an elastic material is 0.4, the ratio of modulus of rigidity to young's modulus is

- | | |
|----------|----------|
| (a) 0 | (c) 0.36 |
| (b) 0.25 | (d) 0.5 |

2.13 When a rectangular bar is uniaxially loaded, the volumetric strain (e_v) is given as

- | | |
|------------------------------|-------------------------------|
| (a) $\sigma_x / E (1 - \mu)$ | (c) $\sigma_x / E (1 - 2\mu)$ |
| (b) $\sigma_x / E (1 + \mu)$ | (d) $\sigma_x / E (1 + 2\mu)$ |

2.14 The work done in producing strain on a material per unit volume is called

- | | |
|----------------|----------------|
| (a) resilience | (c) elasticity |
| (b) ductility | (d) plasticity |

2.15 The property by virtue of which a metal can be beaten into plates is called

- | | |
|----------------|------------------|
| (a) ductility | (c) malleability |
| (b) resilience | (d) plasticity |

2.16 The proof resilience is given by

- | | |
|---------------------|-------------------|
| (a) $2\sigma^3/E$ | (c) $\sigma^2/2E$ |
| (b) $2E / \sigma^2$ | (d) $E/2\sigma^2$ |

2.17 The thermal stress is given by

- | | |
|-------------------|----------------------|
| (a) $E \alpha T$ | (c) $E \alpha / T$ |
| (b) ET / α | (d) $1 / E \alpha T$ |

Answer to Multiple-Choice Questions

2.1 (c), 2.2 (c), 2.3 (b), 2.4 (a), 2.5 (c), 2.6 (c), 2.7 (d), 2.8 (c), 2.9 (d),
2.10 (d), 2.11 (d), 2.12 (c), 2.13 (c), 2.14 (a), 2.15 (c), 2.16 (c), 2.17 (a)

B. Subjective Questions

- 2.1 What is the procedure of finding thermal stresses in a composite bar?
- 2.2 What is principle of superposition? Explain its uses.
- 2.3 Define composite bar. How will you find the stresses and load carried by each member of a composite bar?
- 2.4 What do you mean by a bar of uniform strength?
- 2.5 Define modulus ratio, thermal stresses, thermal strains and poisson's ratio.
- 2.6 Distinguish between (i) Stress and Strain, (ii) Force and Stress, and (iii) Tensile stress and Compressive stress.
- 2.7 Define the terms (i) Elasticity, (ii) Elastic limit, (iii) Young's modulus and (iv) Modulus of rigidity.
- 2.8 Explain the different types of stresses and strains.

C. Numerical Problems

1. Determine the following for a rod measuring 3000mm in length and having a diameter of 40mm when exposed to a 40kN axial tensile force: (i) the stress. (ii) the strain and (iii) the extension of the rod.

(Ans: Stress=31.830N/mm², Strain=0.000159, extension=0.477mm)

2. The Young's modulus of the material is 2×10^5 N/mm². Calculate the following for a bar comprising three sections with lengths of 400mm, 300mm, and 250mm, and diameters of 30mm, 50mm, and 60mm, respectively, when exposed to a 50000 N

axial load: (i) The stresses in individual portions and (ii) The overall bar expansion.

(Ans: Stress in section 1, 2 and 3 = 70.735 N/mm^2 , 25.444 N/mm^2 and 17.683 N/mm^2 ; overall extension = 0.2579 mm)

3. Determine the overall elongation of brass bar with an area of cross-sectional of 1000 mm^2 is exposed to axial forces, as shown in Fig. 2.34 and $E = 1.05 \times 10^5 \text{ N/mm}^2$.

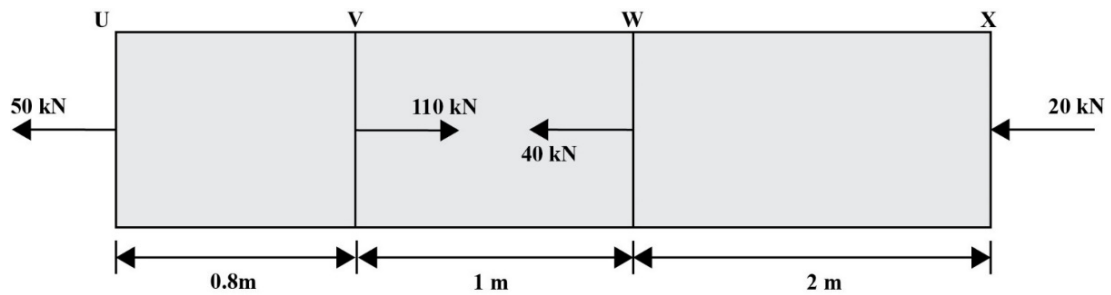


Fig. 2.36: Brass bar subjected to axial forces

(Ans: Overall elongation 0.57995 mm)

4. Determine the elongation of a tapered rod, which uniformly narrows from a 70 mm diameter to a 50 mm diameter over a 600 mm length, under an axial load of 7000N. Given that the Young's modulus (E) is $2 \times 10^5 \text{ N/mm}^2$.

(Ans: 0.00763 mm)

5. Determine the elongation of a steel rectangular bar, 5000 mm in length and 15mm in thickness, under an axial tensile load of 60kN. The width of the bar ranges from 80mm at one end to 35mm at the other. Given that Young's Modulus of material (E) is $2 \times 10^5 \text{ N/mm}^2$.

(Ans: 1.8370 mm)

6. Determine the following for a composite bar consisting of a central steel rod with a 30mm diameter enclosed within a hollow copper tube with an external diameter of 60mm and an internal diameter of 40mm. The composite bar is subjected to an

axial force of 60000N, and each component has a length of 30cm. (i) the stress experienced by the rod and the tube, and (ii) the load carried by each member.

(Ans: $\sigma_c = 20.54 \text{ N/mm}^2$, $\sigma_s = 39.22 \text{ N/mm}^2$, $P_s = 22725.01 \text{ N}$, $P_c = 32274.98 \text{ N}$)

7. Determine the elongation of a 5-meter-long rod as it is subjected to a temperature increase from 20°C to 100°C . Calculate the resulting stress in the rod if this expansion is restricted. Consider the material properties: Young's modulus (E) is $1 \times 10^{11} \text{ N/m}^2$, and the coefficient of linear expansion (α) is 0.000012°C .

(Ans: 4.8mm, 96 N/mm^2)

8. Determine the change in the length, thickness, and width of a steel bar measuring 6 meters in length, 50mm in width, and 35mm in thickness when it undergoes a 40kN axial tension along its length. Poisson's ratio = 0.32 and $E = 2 \times 10^5 \text{ N/mm}^2$.

(Ans: 0.6857mm, 0.001827mm, 0.001278mm)

9. Calculate the following for a 25mm diameter bar subjected to a 60kN tensile load with a measured extension of 0.15mm in a gauge length of 300mm and a diameter change of 0.00375mm: (i) Young's modulus, Bulk modulus, and Poisson's ratio.

(Ans: $G = 94023.84 \text{ N/mm}^2$, $\mu = 0.3$, $K = 203718.325 \text{ N/mm}^2$)

KNOW MORE

- *Engineering Design:* Simple stress and strain calculations are fundamental in engineering design, particularly in determining the strength and durability of structures and materials. Engineers use stress and strain analysis to ensure that components like bridges, buildings, and mechanical systems can withstand expected loads and environmental conditions.
- *Material Testing:* In material science and manufacturing, stress and strain testing are essential for evaluating the mechanical properties of materials. Engineers conduct tests to determine the stress-strain curves of materials, which provide valuable insights into their strength, stiffness, and ductility. This information helps in selecting appropriate materials for specific applications.
- *Aerospace and Automotive Industries:* Stress and strain analysis are critical in the aerospace and automotive industries for designing and testing aircraft components, spacecraft, and vehicles. Engineers use stress and strain calculations to optimize the design of structural elements, such as wings, fuselages, and chassis, ensuring safety and performance.
- *Construction:* In construction, stress and strain considerations are vital for ensuring the stability and integrity of buildings, bridges, and other infrastructure. Engineers analyze the stress and strain distribution in various building materials, such as concrete, steel, and wood, to design structures that can withstand loads from gravity, wind, and seismic activity.
- *Product Design:* Stress and strain considerations are also relevant in product design across various industries. Designers analyze the stress and strain on consumer products, electronics, and packaging to ensure durability, reliability, and safety for end-users.

REFERENCE AND SUGGESTED READINGS

1. J.L. Meriam and L.G. Kraige, Engineering mechanics: Dynamic, Wiley Publisher, New York
2. D.S. Bedi, “Engineering Mechanics”; Khanna publications, New Delhi.
3. Khurmi, R.S., Strength of Materials, S Chand and Co. Ltd. New Delhi.
4. Bansal RK, “A text book of Engineering Mechanics”; Laxmi publications, New Delhi
5. Bedi D.S., Strength of Materials, Khanna Publishing House, Delhi, Ed. 2018
6. Punmia B C, Strength of Materials, Laxmi Publications (p) Ltd. New Delhi.
7. Rattan S.S., Strength of Materials, McGraw Hill Education; New Delhi.
8. Subramaniam R, Strength of Materials, Oxford University Press.

Dynamic QR Code for Further Reading

Basic Concepts of simple stress and strain



Superposition Principle



Application of stress and strain



Concept of true stress strain



Concept of temperature stress strain



Stress strain relation



Torsion



Relation between Young's Modulus and Bulk Modulus



3

SHEAR FORCE AND BENDING MOMENT

UNIT SPECIFICS

Upon completion of this chapter, the reader will have acquired the following abilities:

- *Understand the structural behaviour of beams.*
- *Analyze the influence of external loads, including axial force (A.F), and shear force (SF), and bending moment (BM), on different sections of beams using appropriate sign conventions and relationships between load intensity of SF, and BM.*
- *Draw the A.F, SF and BM diagrams for statically determinate beams and frames.*
- *Draw SF and BM diagrams for determinate beams using graphical methods, and comprehend the singularity function approach for determining SF and BM in beams.*

This unit aims to familiarize readers with the fundamental principles of shear force and bending moment analysis, which are essential for understanding the behaviour of structures under various loads. Readers will gain proficiency in calculating and interpreting shear force and bending moment diagrams, enabling them to assess the internal forces within structural elements accurately. Through clear explanations and illustrative examples, this unit provides a comprehensive overview of the

distribution and effects of shear and bending forces, equipping readers with the necessary tools to analyze and design efficient and stable structures. Whether for students or professionals in civil and mechanical engineering, this unit serves as a foundational guide for mastering the principles of structural mechanics and design.

Apart from this at the end of the unit, a succinct recapitulation of the overall broad concepts is provided in the form of a unit summary. Besides, a large number of multiple-choice questions as well as descriptive-type questions with Bloom's taxonomy action verbs are included. A list of references and suggested readings is given in the unit so that one can go through them for practice. It is important to note that to get more information on various topics of interest, some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge. Video resources along with QR codes are given for getting more information on various topics of interest which can be surfed or scanned through mobile phones for viewing.

RATIONALE

The concepts of shear force and bending moment find widespread application in structural engineering and mechanics, where they play a fundamental role in analyzing and designing various structures such as beams, bridges, buildings, and mechanical components. Shear force refers to the transverse force acting perpendicular to the longitudinal axis of a structural element while the bending moment represents the internal moment that induces bending or flexural deformation in the element. Understanding these concepts is crucial for engineers to ensure structural integrity, assess load-bearing capacity, and predict potential failure modes of a given design. By calculating shear force and bending moment along the length of a structure, engineers can determine critical points of stress and

deformation, aiding in the optimization of material usage, structural stability, and overall safety. Moreover, these principles extend beyond traditional engineering disciplines, finding applications in fields such as biomechanics, materials science, and even in metaphorical contexts like decision-making processes, as we explored earlier. Overall, shear force and bending moment concepts form the backbone of structural analysis, providing engineers with essential tools for designing resilient and efficient structures across diverse domains.

PRE-REQUISITES

Before reading this unit, the students are advised to revisit the following:

Engineering Mechanics: Sem I

Physics: Mechanical properties of solids (Class XI)

UNIT OUTCOMES

After completing this unit, the student will be able to:

U3-O1: Identify and classify different types of supports used in structural engineering, including roller, hinged, and fixed supports.

U3-O2: Understand the behaviour of various types of beams, such as simply supported, cantilever, overhanging, fixed, and continuous beams, under different loading conditions.

U3-O3: Recognize the types of loads applied to beams, such as uniformly distributed, uniformly varying, and concentrated or point loads.

U3-O4: Analyze and construct shear force and bending moment diagrams for beams subjected to different loading configurations, including cantilever, simply supported, and overhanging beams.

U3-O5: Apply the principles learned to analyze and design structural members to withstand various loading conditions, enhancing proficiency in structural analysis and engineering design.

Unit-3 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES (1-Weak Correction; 2- Medium Correction; 3- Strong Correction)				
	CO-1	CO-2	CO-3	CO-4	CO-5
U3-O1	3	1	2	3	1
U3-O2	3	1	2	3	1
U3-O3	3	1	2	3	1
U3-O4	3	1	2	3	1
U3-O5	3	1	2	3	1

3.1 INTRODUCTION

The term "**beam**" refers to a structural member which is one-dimensional, the length is much higher than the cross-sectional dimension, and takes transverse loads. The analysis of beams involves determining shear force (SF) and bending moment (BM) at different sections of beams and for different end conditions. This chapter focuses on calculating SF and BM in different beams. The next chapter will discuss the details of beam deflection.

Beams can undergo bending or deflection due to the applied loads. Point loads are typically considered concentrated forces, even though they may be distributed over a small area in practice. Distributed loads, on the other hand, are spread over a specific length of the beam. The loading rate can be either uniform or variable at different points along the beam.

3.2 TYPES OF SUPPORTS

Supports are elements used to restrain or hold a beam in place, enabling it to resist various loads and maintain stability. Supports are essential for distributing the forces acting on a beam and ensuring it remains structurally sound. There are different types of supports commonly used in beam construction, which include:

- Roller support or Triangular support
- Hinged support
- Fixed or Built-in support

These are some of the primary types of supports used in beam construction. The selection of a specific support type depends on various factors, including the structural requirements, load conditions, and the desired behavior of the beam under different loads.

3.2.1 ROLLER SUPPORT

Roller support, permits rotation of the beam and only supports force normal to its surface as in the case of a beam resting on a sliding surface or a flat surface like a masonry wall, allows beam rotation and sustains a force only normal to its surface. The reaction along the rolling surface is zero due to the potential movement allowed on the supporting surface. Thus, the roller support reacts solely in a direction perpendicular to the surface, enabling rotational movement. The typical representation of different supports is shown in Fig. 3.1.

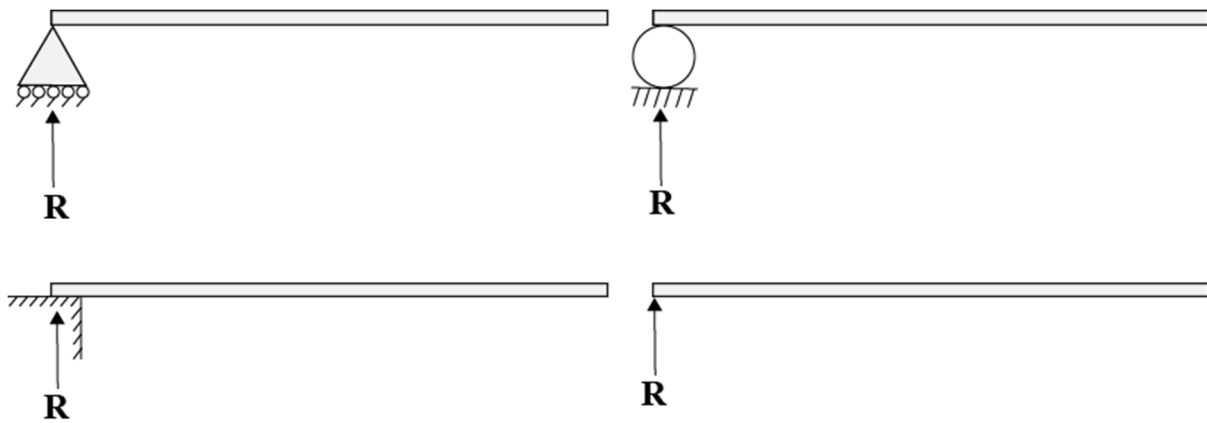


Fig. 3.1: Types of roller support

3.2.2 HINGED SUPPORT

A hinged support, also known as a pinned support, restricts the translational displacement of the beam while allowing free rotation. Unlike roller supports, hinged support can sustain reactions in any direction, including vertical and horizontal components. As the hinged support permits rotation, beams with hinged joints experience zero bending moments at the hinge. In some instances, beams are equipped with hinged joints, eliminating the reaction typically in hinged supports. The conventional methods of representing hinged supports and hinged joints are shown in Fig. 3.2.



Fig. 3.2: Hinged Support and Hinged Joint

3.2.3 FIXED OR BUILD-IN SUPPORT

Fixed support, also referred to as built-in support, rigidly holds a beam in place without permitting any movement or rotation (as shown in Fig. 3.3). It applies a restraining moment and reaction forces to the beam.

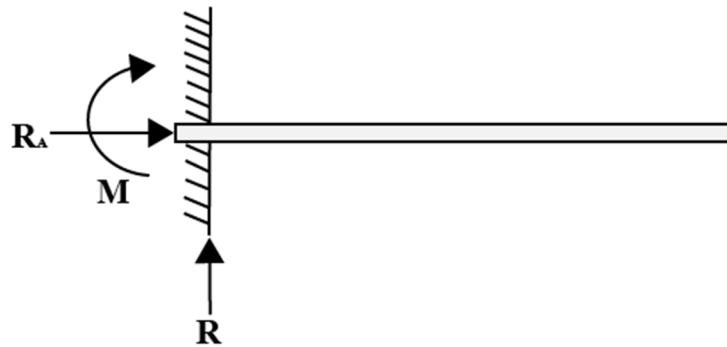


Fig. 3.3: Figure showing the fixed support

3.3 BEAMS AND TYPES

In general, beams are typically one-dimensional members with the length much higher than the cross-sectional dimensions. These beams experience loads primarily in a transverse direction. The main effect of these loads is to induce bending in the beam, as indicated by the dotted line in Fig. 3.4. Although this bending causes small changes in load geometry, they are usually neglected. In this case, the beams being discussed are described as planar, indicating that their longitudinal axis and the lines along which the loads and reactive forces act are confined to a single plane.

These beams have straight longitudinal axes, typically horizontal, although inclined beams are not unusual.

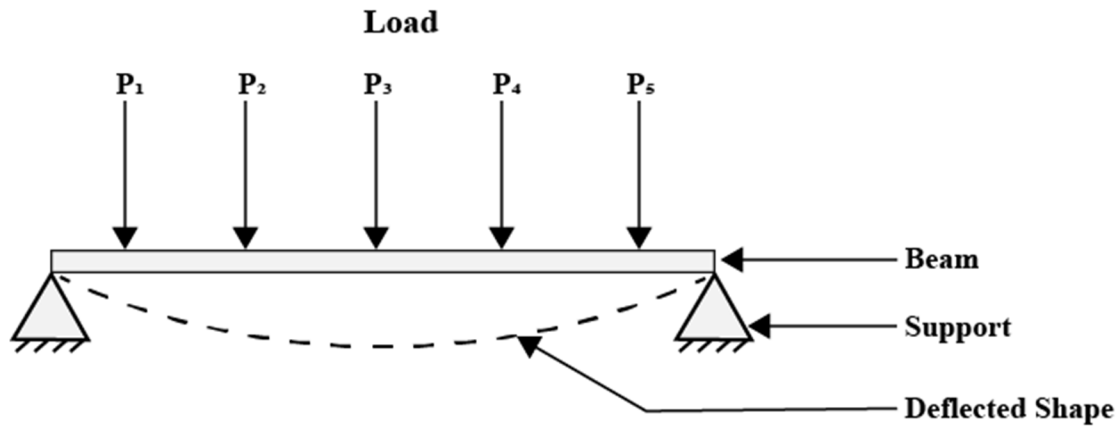


Fig. 3.4: Bending of beam due to applied loads

Before drawing the SF and BM diagrams, it is necessary to understand the beam types, support conditions, and the different types of loads that exert forces on the beams. The most frequent types of beams are as follows:

- Simply supported beam
- Cantilever beam
- Over-hanging beam
- Continuous beam
- Fixed-beam
- Fixed-Propped beam

3.3.1 SIMPLY SUPPORTED BEAM

When a beam is supported by triangular support at both ends or when one end is supported by a roller and the other end is hinged (as shown in Fig. 3.5), it is referred to as a simply supported beam.

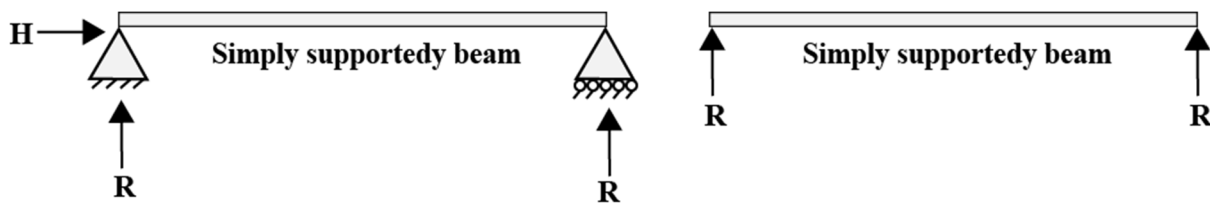


Fig. 3.5: Figure showing the simply supported beam

3.3.2 CANTILEVER BEAM

Fig. 3.6 shows a type of beam known as cantilever beam. The beam is firmly fixed or supported at one end while free or unsupported at the other end.

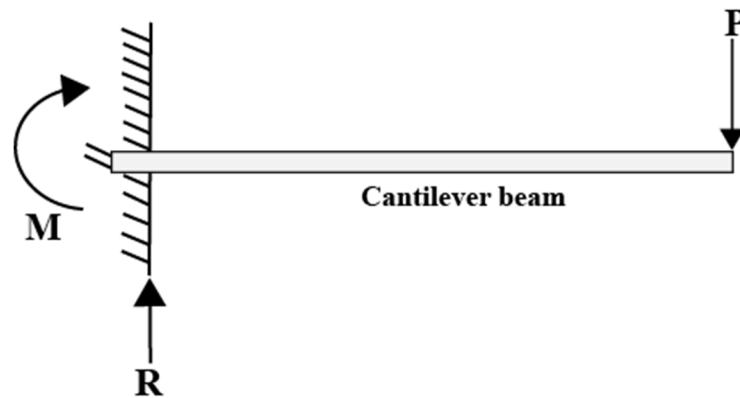


Fig. 3.6: Figure showing the cantilever beam

3.3.3 OVERHANGING BEAM

A beam that has its length extended beyond its supports is referred to as an overhanging beam (Fig. 3.7).

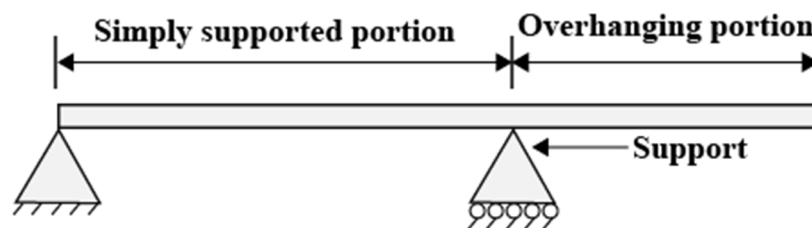


Fig. 3.7: Figure showing the overhanging beam

3.3.4 FIXED-BEAM

Fig. 3.8 shows a fixed or built-in beam. This beam type is distinguished by having both ends fixed or supported, providing a rigid support at both ends.

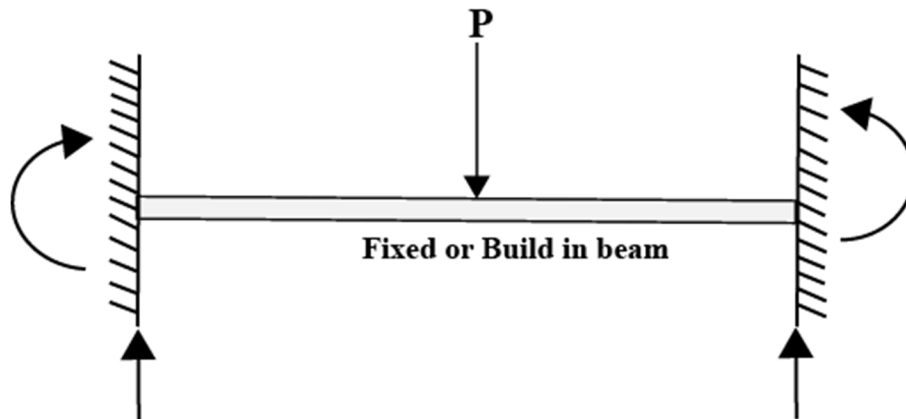


Fig. 3.8: Figure showing the fixed-beam

3.3.5 CONTINUOUS BEAM

Fig. 3.9 shows a beam that has more than two supports known as a continuous beam.

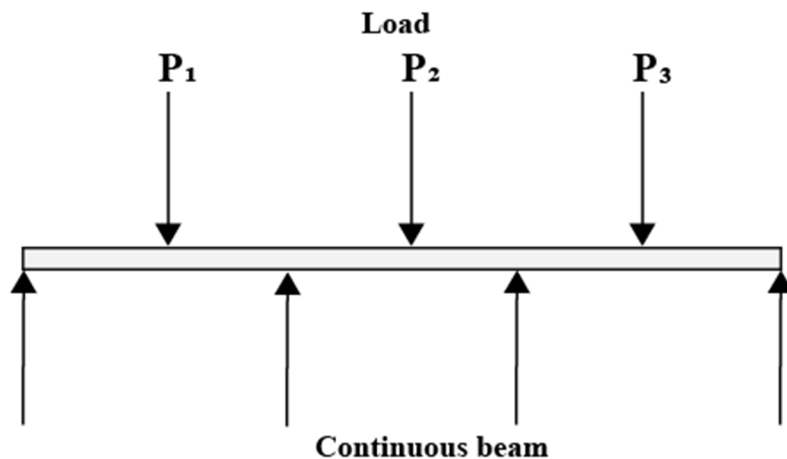


Fig. 3.9: Figure showing the continuous beam

3.4 LOADS AND THEIR TYPES

Loads are forces that are applied to a structure or body that can induce stress, and strain. Beams, which are mostly horizontal, experience various types of vertical loads. The primary loads in the beam are:

- Uniformly distributed load
- Uniformly varying load
- Concentrated or point load

Apart from these loads, there can be constant moments which can also cause bending in the beam.

3.4.1 UNIFORMLY DISTRIBUTED LOAD (UDL)

The beam is subjected to a UDL when the load intensity remains constant over the entire length of the beam. It indicates that the same amount of force per unit length is applied over the beam. In Fig. 3.10, the UDL is represented by the symbol "w" and is measured in Newton per meter (N/m).

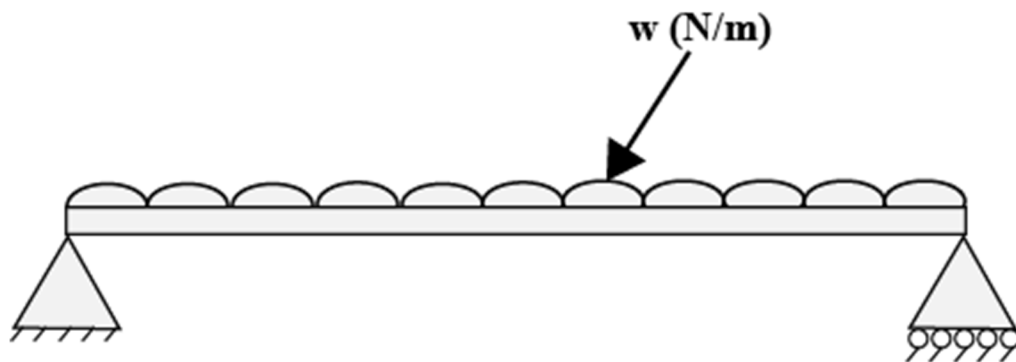


Fig. 3.10: Figure showing the uniformly distributed load

3.4.2 UNIFORMLY VARYING LOAD (UVL)

The UVL is distributed along the length of the beam, in which the load varies from minimum to maximum. This type of load is commonly represented by a triangular

shape, as depicted in Fig. 3.11. In this case, the beam load varies from zero and increases linearly to the other end, resulting in a triangular load distribution.

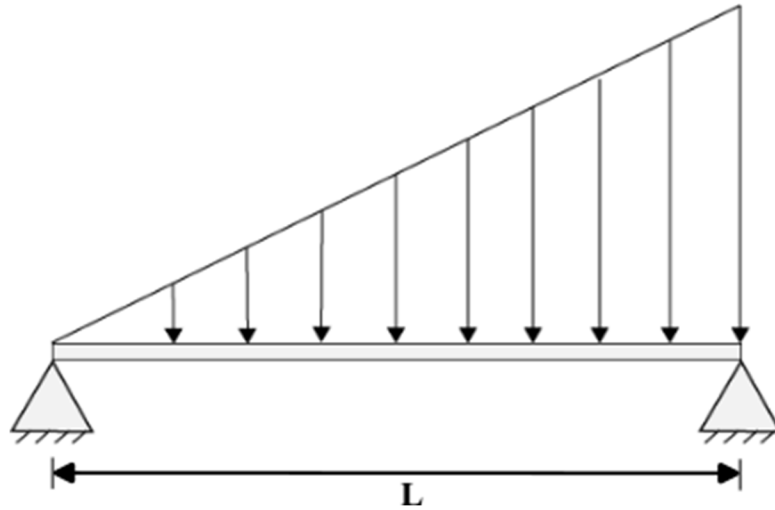


Fig. 3.11: Figure shows the Uniformly Varying Load

3.4.3 CONCENTRATED LOAD (CL)

Generally, a CL is acting at a single point, Fig. 3.12, the symbol "P" represents the point load applied at a specific location along the beam.

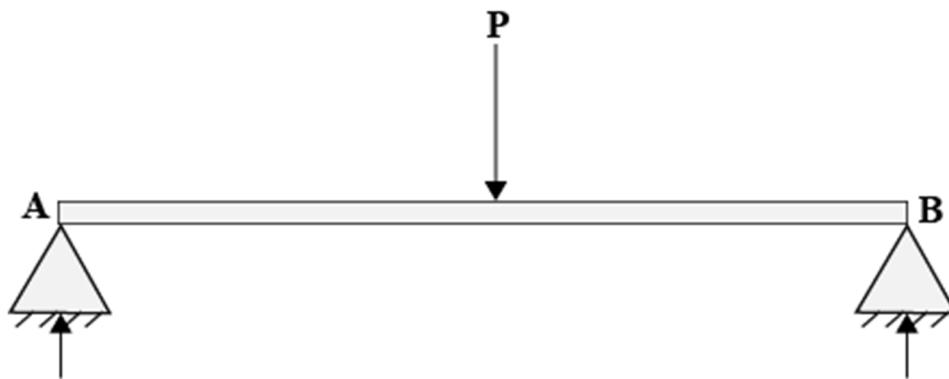


Fig. 3.12: Figure showing the concentrated load

3.5 SHEAR FORCE (SF)

Shear force at a section in a beam is defined as the algebraic sum of forces acting either to the left or right of the section.

Assume the beam shown in Fig. 3.13 is supported at the ends and is subjected to point loads. R_a and R_b are the reactions at the supports. Consider the beam section, the sum of total forces on the left side of the portion results in an upward force F (assuming an upward direction). For the beam to maintain equilibrium, the combination of the forces on the right side of the portion must result in a downward force F . This force F is known as the S.F.

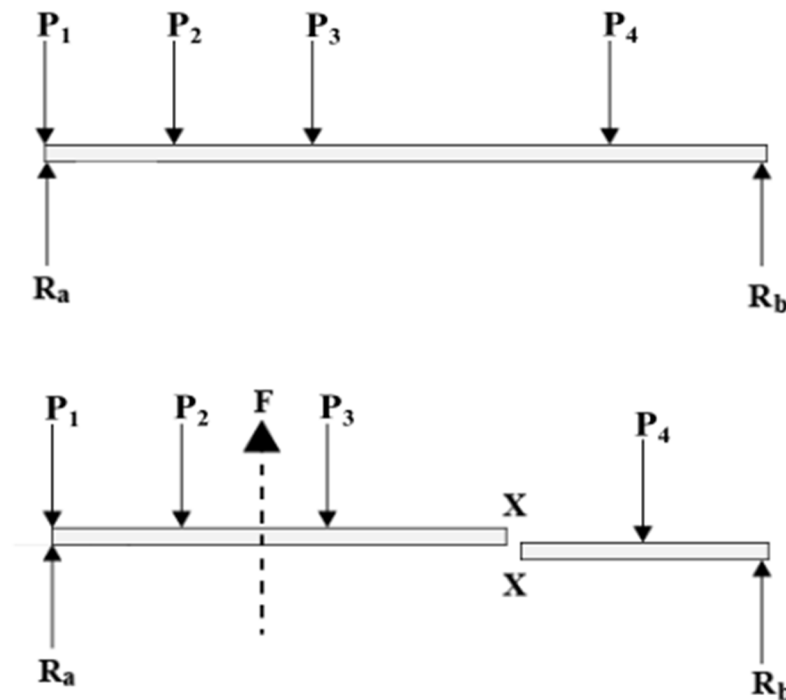


Fig. 3.13: Beam supported at two locations and subjected to four loads with reactions R_a and R_b

The sign convention for shear force is that defines it is positive when the total force to the left side is upward or when the total force to the right of a section is downward.

3.6 BENDING MOMENT

The bending moment at a section of the beam is defined as the algebraic sum of the moments of all the forces acting either to the left or to the right of the section. For

the beam to remain in equilibrium, if the BM of the forces to the left side of section X-X is clockwise (Fig. 3.14), the BM of the forces to the right of X-X must be anti-clockwise.

The sign convention for BM defines it as positive if the moment on the left section is in the clockwise direction or if the moment on the right section is in the anti-clockwise direction. It is commonly known as **sagging**, and it causes an upward concavity in the beam. Conversely, a BM that causes upward convexity is considered negative and is referred to as **hogging**.

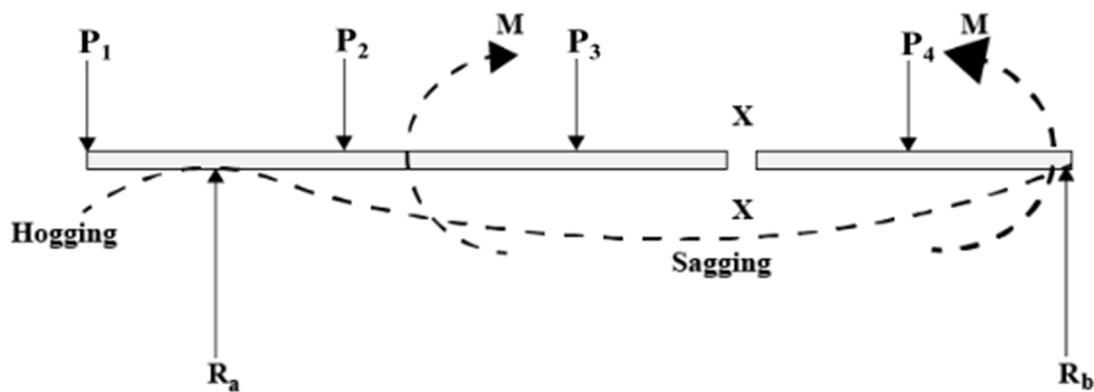


Fig. 3.14: Principle of equilibrium

3.7 RELATIONSHIP BETWEEN LOAD, S.F, AND B.M

- The relationships between loads, S.F and B.M, in a beam are essential in analyzing beam behavior. The following relations are significant in this regard:
- When a point load is applied at a specific beam section, the shear force changes suddenly.
- When the shear force is constant over the length of the beam the bending moment varies linearly over the length.
- When the shear force varies linearly over the length of the beam the bending moment will have parabolic variation.

- In the case of uniformly varying load, the shear force varies parabolically and bending moment variation is cubic.
- These relationships help us to understand the variations of S.F and B.M over the length of the beam for different loading conditions.

3.8 S.F AND B.M DIAGRAMS FOR CANTILEVERS BEAM

Consider a cantilever beam that supports either a UDL or a concentrated load.

3.8.1 CONCENTRATED LOAD APPLIED AT ITS FREE END

Assume a cantilever beam of 'L' length subjected to a 'P' concentrated load applied at its free end, as depicted in Fig. 3.15(a).

(i) S.F Diagram

Let F_x be the SF at x , and

M_x be the BM at x .

When analyzing a specific section of the beam located at ' x ' distance from the free end, the force acting to the right of that section is a constant downward force, denoted as P. This force is constant over the whole length of the beam, independent of x . Consequently, the SF is positive, as indicated in Eqn. (3.1). This positive SF results in an SF diagram that forms a horizontal straight line, as shown in Fig. 3.15(b).

$$F_x = +P \quad \text{.....(3.1)}$$

(ii) B.M Diagram

Applying a moment about the section,

$$M_x = -P \times x \quad \dots\dots(3.2)$$

In Fig. 3.15(c), the BM diagram exhibits a negative slope indicating that the BM on the right of the section is in the positive direction. This negative BM can also be interpreted as **hogging**, where the beam is convex downwards and experiencing compressive stresses at the bottom layer.

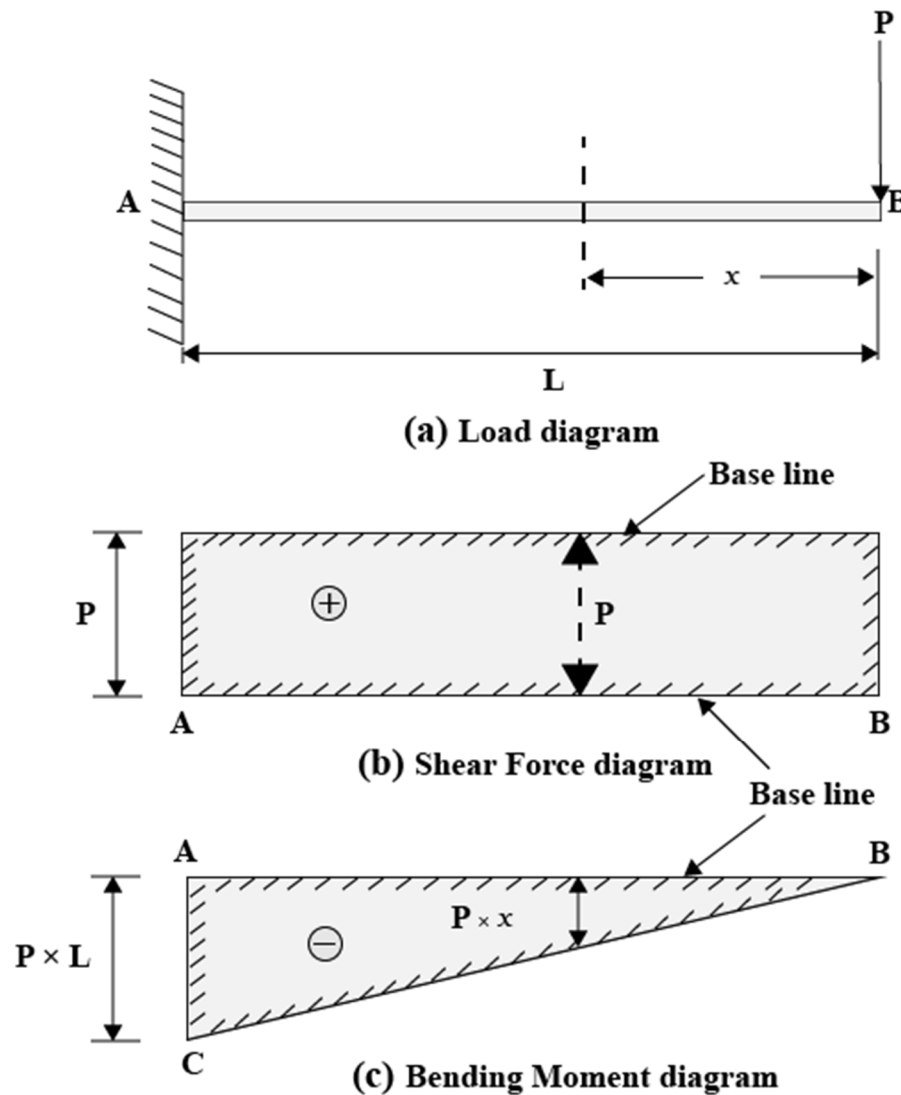


Fig. 3.15(a)-(c): SF and BM diagram of concentrated load applied at its free end

At point B, $x = 0$, therefore, $BM = 0$

At point A, $x = L$, therefore, $BM = P \times L$

Maximum $BM = P \times L$ at the fixed end.

SF and BM diagrams for multiple concentrated point loads that act on a cantilever share similar characteristics and trends.

In the SF diagram, each concentrated load will introduce a step change in the magnitude of the SF. Positive shear forces will arise from upward forces from the left, while negative shear forces will result from downward forces from right. The shear force diagram will show the cumulative effect of these loads over the length of the cantilever.

Each load will contribute a moment that causes either sagging (bending in the same direction as the load) or hogging (bending in the opposite direction). The bending moment diagram will show the cumulative effect of these moments throughout the length of the cantilever. Both diagrams will exhibit variations in magnitude and direction at the positions where the concentrated loads are applied. The specific shapes and slopes of the diagrams will depend on the magnitudes, and directions of the concentrated loads.

SOLVED PROBLEMS

Problem 3.1

The point loads are carried by a 4 m long cantilever beam, as depicted in Figure 3.16. Draw the S.F and B.M diagrams.

Solution:

(i) S.F diagram

- SF at point B is +1000N. SF remains constant between B and D. Due to the point load at point D, the SF becomes $(1000 + 600) = 1600$ N. From points D

and C, the SF remains 1600 N. At point C again, the SF becomes $(1600 + 400) = 2000$ N.

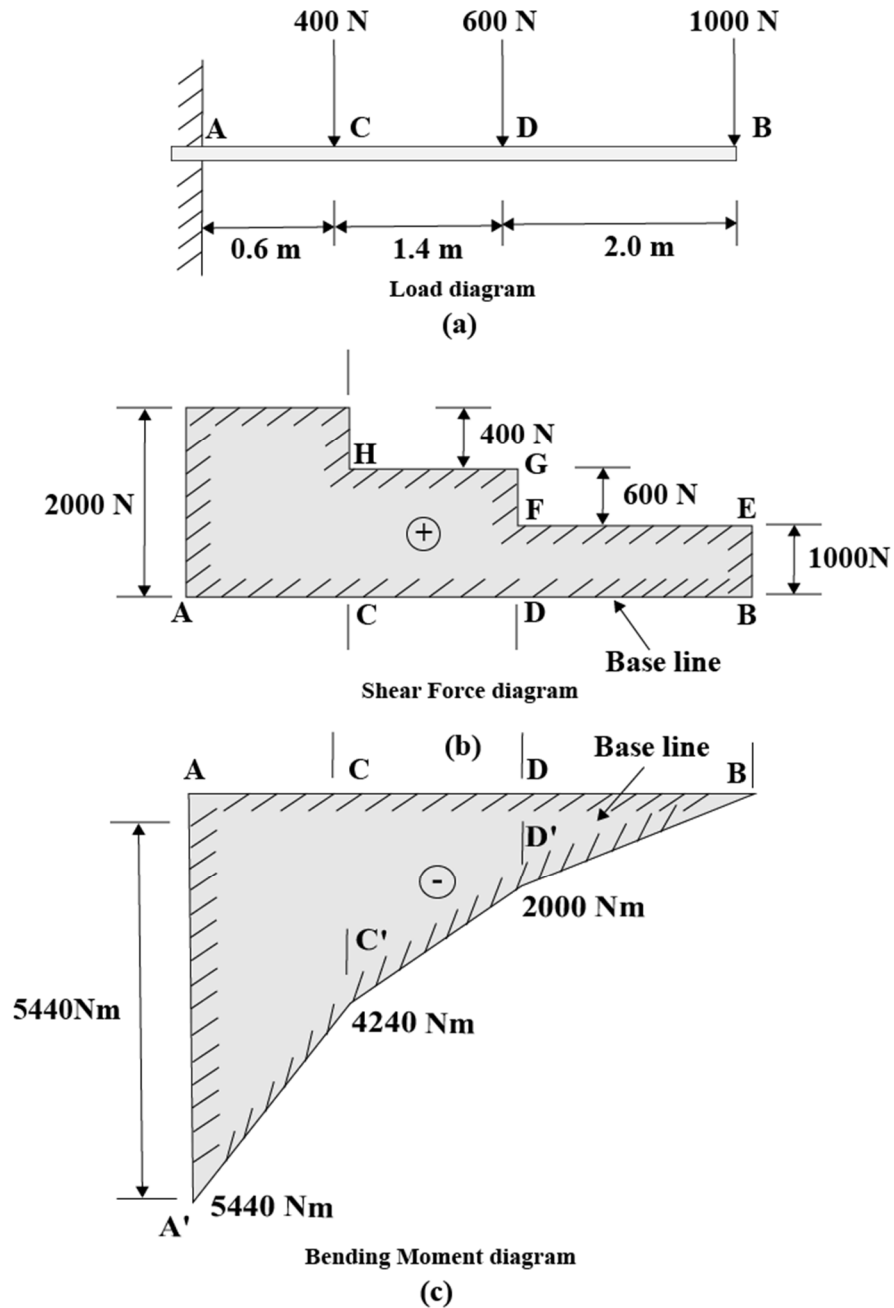


Fig. 3.16(a)-(c): S.F and B.M diagram of the cantilever beam

- SF between points A and C remains constant and is 2000 N. Therefore, the SF variation across the length of the beam, is given by

- S.F at point B, $F_B = + 1000 \text{ N}$
- S.F at point D, $F_D = + 1600 \text{ N}$
- S.F at point C, $F_C = + 2000 \text{ N}$
- S.F at point A, $F_A = + 2000 \text{ N}$

Fig. 3.16(b) shows the S.F diagram, created as follows.

(ii) B.M diagram:

B.M at point B is zero: $M_B = 0$

- BM at a point between D and B at a distance ' x ' from B, is given by

$$M_D = - 1000 \times x$$

At the point D, $x = 2.0\text{m}$. Therefore, the BM at D point,

$$M_D = - 1000 \times 2.0 = - 2000 \text{ Nm.}$$

$$M_D = -2000 \text{ Nm}$$

- BM at a section between C and D at x distance at B point is given by
(At point D, $x = 2.0 \text{ m}$; at point C, $x = 2.0 + 1.4 = 3.4$. Therefore, the value of x differs from 2.0 m to 3.4 m)

$$M_C = - 1000x - 600 (x - 2.0) \quad \text{.....(i)}$$

- B.M at point C is obtained by replacing $x = 3.4 \text{ m}$ in Eqn. (i),

$$M_C = - (1000 \times 3.4) - 600 (3.4 - 2.0)$$

$$M_C = - 3400 - 600 (1.4) = - 3400 - 840$$

$$M_C = - 4240 \text{ Nm}$$

- The BM at a section between C and A at x distance at point B is given by
(At point C, $x = 3.4 \text{ m}$; at point A, $x = 3.4 + 0.6 = 4$. Therefore here, the value of x differs from 3.4 m to 4 m)

$$M_A = - 1000x - 600 (x - 2.0) - 400 (x - 3.4) \quad \text{.....(ii)}$$

- BM at point A is obtained by replacing $x = 2.4$ in Eqn. (ii),

$$M_A = - (1000 \times 4.0) - 600 (4.0 - 2.0) - 400 (4.0 - 3.4)$$

$$M_A = - 4000 - 1200 - 240$$

$$M_A = - 5440 \text{ Nm}$$

Therefore, B.M will be represented as follows at different points:

$$M_B = 0$$

$$M_D = - 2000 \text{ Nm}$$

$$M_C = - 4240 \text{ Nm}$$

$$M_A = - 5440 \text{ Nm}$$

Fig. 3.16(c) shows the B.M diagram.

3.8.2 UNIFORMLY DISTRIBUTED LOAD

Assume a cantilever beam of length 'L' that is subjected to a UDL, denoted as 'P' per unit length, over the length of the cantilever. This load distribution is depicted in Fig. 3.17(a).

(i) S.F Diagram

Let F_x be the SF at x , and

M_x be the BM at x .

When analyzing a specific section of the beam located at a distance ' x ' from the free end, the force acting on the right side of that section is w_x , where x represents the distance, and w represents a downward force per unit length. This force varies linearly over the length. Consequently, the SF at the section is positive, as indicated in Eqn. (3.3), resulting in a shear force diagram that varies linearly, as shown in Fig. 3.17(b).

$$F_x = + (w \times x) \quad \dots\dots\dots (3.3)$$

At point B, $x = 0$, therefore, $F_x = 0$

At point A, $x = L$, therefore, $F_x = w \times L$

(ii) B.M Diagram

According to Art. 3.4.2, the UDL acting over a length can be transferred into point load which will act the CG of the beam.

When calculating the BM about section x . The following expression is used:

$M_x = -$ (Total load over the section) $\times$ (CG of the section from the free end)

$$\begin{aligned} M_x &= -(w \cdot x) \cdot \frac{x}{2} \\ &= -w \cdot \frac{x^2}{2} \end{aligned} \quad \text{..... (3.4)}$$

The BM diagram exhibits a negative sign, indicating a hogging effect, due to the clockwise direction of the moment on the right-side section of the section. This BM diagram is graphically represented by a parabolic curve that progressively rises as the value of x increases, as illustrated in Fig. 3.17(c).

Eqn. (3.4) indicates that the BM at a particular section is directly proportional to the square of its distance from that section at the free end. This relation is satisfied by the parabolic law, where the BM increases quadratically with the distance from the free end.

At point B, $x = 0$, therefore, $M_x = 0$

At point A, $x = L$, therefore $M_x = -w \times \frac{L^2}{2}$

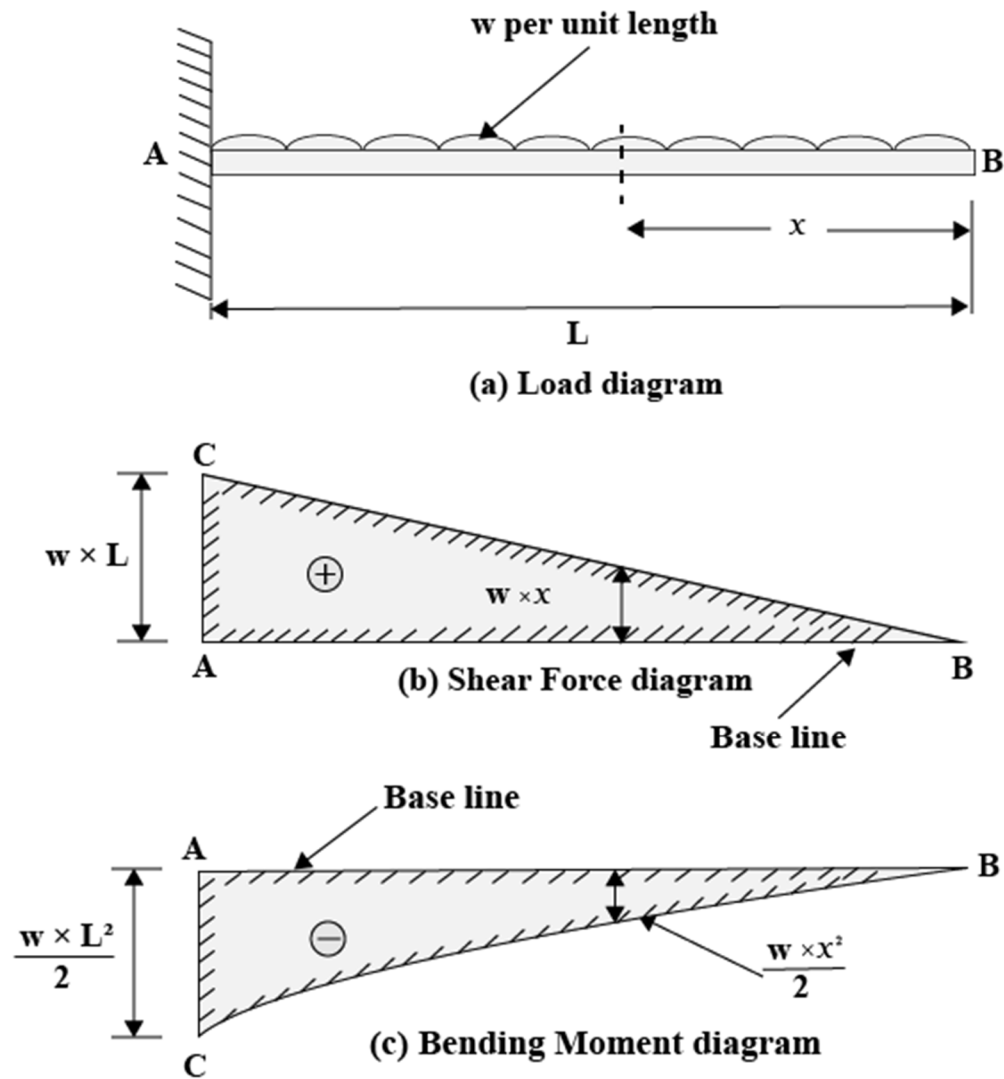


Fig. 3.17(a)-(c): SF and BM diagram of the uniformly distributed load

The load intensity is constant and negative, resulting in a downward force. Consequently, the shear force diagram has a consistent negative slope, forming a straight line slanting downward to the right. The SF remains positive along the beam, reaching its maximum at A and decreasing to zero at B. In alignment with this, the bending moment diagram exhibits a maximum slope at A, zero at B, and a continuously positive slope throughout the beam.

Problem 3.2:

The UDL of 6 kN/m is carried out by a 5 m-long cantilever beam and a 5 kN load at the free end, as shown in Figure 3.18. Draw SF and BM diagram for the cantilever beam.

Solution:

Length of the beam = 5 m; UDL = 6kN/m; and Point load at free end = 5kN

(i) S.F diagram

S.F at point B = 5 kN

Consider the section at x distance from the free B end. SF at a section is given by,

$$F_x = 5.0 + w \cdot x$$

(Positive sign is because of the download force on the right-side section)

$$F_x = 5.0 + (6) \cdot x \quad (\because w = 6 \text{ kN/m})$$

The above equation shows that SF obeys a straight-line law.

At point B, $x = 0$, therefore $F_B = 5.0$ kN

At point A, $x = 5$ m, therefore, $F_A = 5.0 + (6) \cdot (5) = 35$ kN

Fig. 3.18(b) shows the SF diagram in which $F_B = 5$ kN and $F_A = 35$ kN.

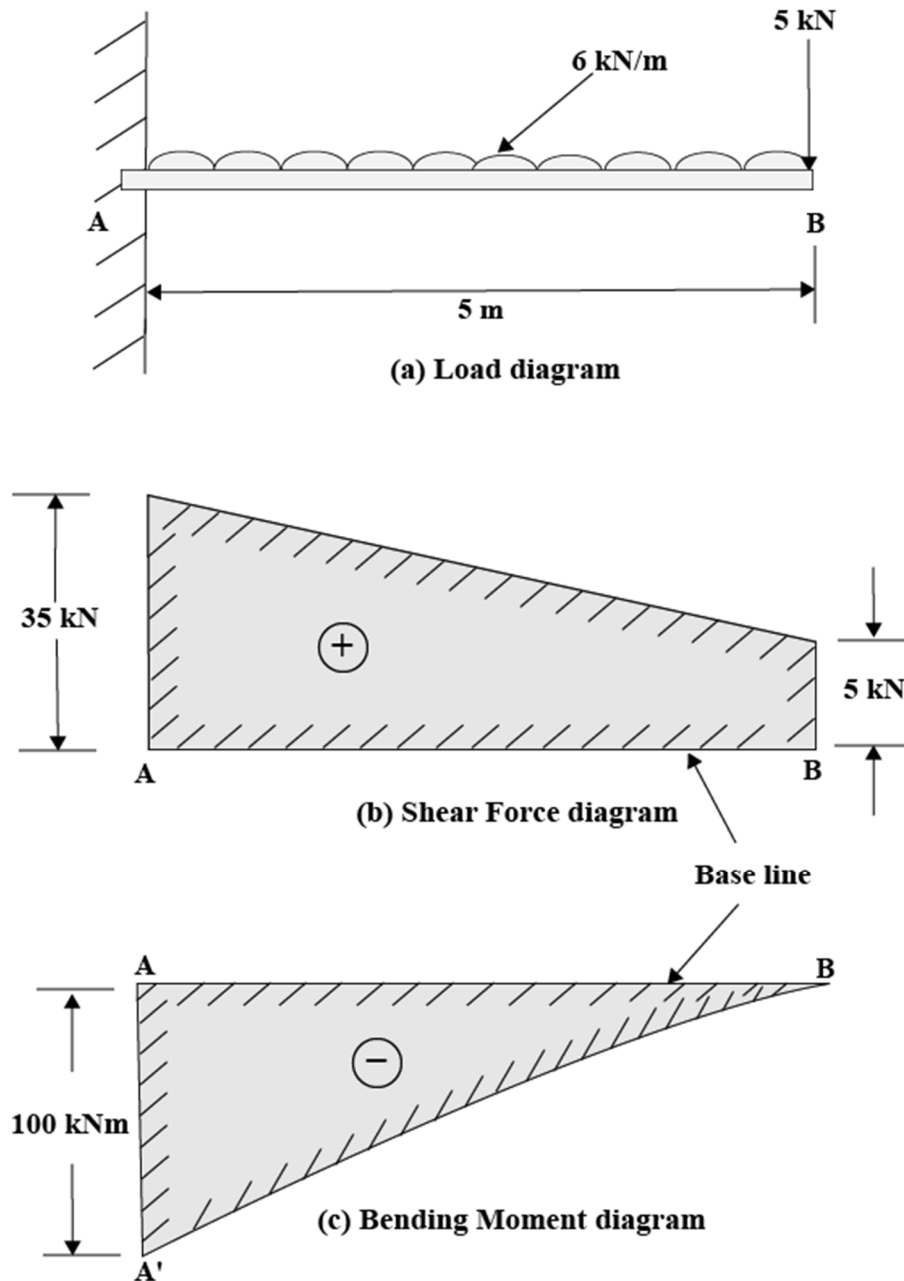


Fig 3.18(a)-(c): SF and BM diagram of the cantilever beam.

(ii) B.M diagram

BM at an x distance from the free B end is given by:

$$M_x = -\left(5x + wx \cdot \frac{x}{2}\right)$$

$$M_x = - \left(5x + \frac{6x^2}{2} \right) \quad (\because w = 6 \text{ kN/m})$$

$$M_x = -(5x + 3x^2) \quad \dots\dots (i)$$

(Negative sign is because of the right-side section, BM of load at x is clockwise)

Eqn (i) shows that the BM changes by the parabolic law. In Eqn (i), we obtain,

$$\text{At point B, } x = 0, \text{ therefore, } M_B = - [5 \times 0 + 3(0^2)] = 0$$

$$\text{At point A, } x = 5, \text{ therefore, } M_A = - [5 \times 5 + 3(5^2)] = 100 \text{ kNm}$$

Fig. 3.18(c) shows a BM diagram in which $AA' = 100 \text{ kNm}$ and the points A' and B join in a parabolic curve.

Problem 3.3:

The UDL of 3 kN/m is carried by a 6 m long cantilever beam that runs the whole length and a 5 kN point load at a 3 m distance from the free end. Draw the SF and BM diagram of the beam.

Solution:

Length of the beam = 6 m ; UDL = 3 kN/m ; and Point load = 5 kN ; It is acting at 3 m from the free end.

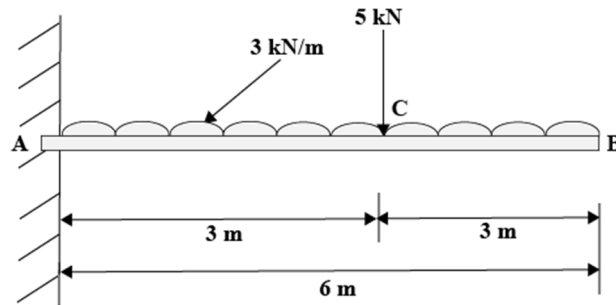
(i) S.F diagram

- Consider a point between B and C at a distance x from the free end. SF at the section is represented as,

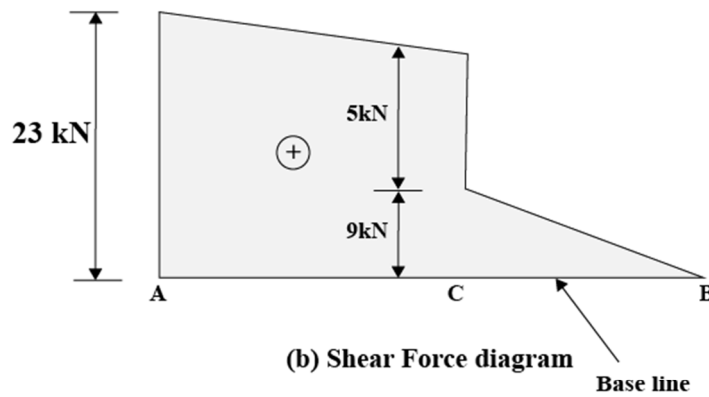
$$F_x = + w \times x$$

(Positive sign is because of the downward force on the right-side section)

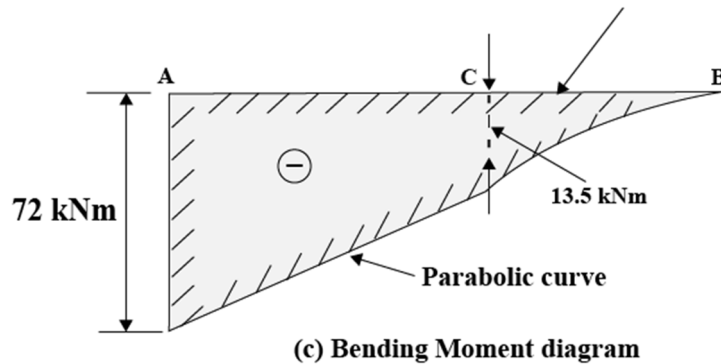
$$F_x = +3 \times x \quad (\because w = 3\text{kN/m}) \dots\dots (i)$$



(a) Load diagram



(b) Shear Force diagram



(c) Bending Moment diagram

Fig 3.19(a)-(c): SF and BM diagram of cantilever beam carrying the uniformly distributed load

Eqn (i), x changes from 0 to 3. Eqn (i) shows that SF varies linearly between B and C.

At point B, $x = 0$, therefore, $F_B = 3 \times 0 = 0$

At point C, $x = 3$, therefore, $F_C = 3 \times 3 = 9 \text{ kN}$

- Assume a point between Z and Y at distance x from the free end. S.F at the portion is represented as,

$$F_x = + (w \times x) + 5 \text{ kN}$$

(Positive sign is because of the download force on the right-side section)

$$F_x = + (3 \times x) + 5 \quad (\because w = 3\text{kN/m}) \quad \dots\dots\dots(\text{ii})$$

Eqn (ii), x changes from 3 to 6. Eqn (ii) shows that SF varies linearly between A and C.

At point C, $x = 3$, $F_C = 3 (3) + 5 = 14 \text{ kN}$

At point A, $x = 6$, $F_A = 3 (6) + 5 = 23 \text{ kN}$

Fig. 3.19(b) shows an SF diagram at A, $F_A = 23 \text{ kN}$, $F_B = 14 \text{ kN}$, and $F_C = 9 \text{ kN}$.

(ii) B.M diagram

- Assume a point between Z and Y at an x distance from the free end. B.M at the portion is represented as,

$$M_x = - (w \times x) \cdot \frac{x}{2}$$

$$M_x = - (3 \times x) \cdot \frac{x}{2} \quad (\because w = 3\text{kN/m})$$

$$M_x = - \frac{3x^2}{2} \quad \dots\dots\dots(\text{iii})$$

(Negative sign is because of the right-side section, BM of load at x is clockwise)

Eqn. (iii), x varies from 0 to 3. Eqn (iii) shows that BM varies parabolic between B and C.

At point B, $x = 0$, therefore, $M_B = -\frac{3(0)^2}{2} = 0$

At point C, $x = 0$, therefore, $M_C = -\frac{3(3)^2}{2} = -\frac{27}{2} = -13.5 \text{ kN/m}$.

- Consider a section between A and C at distance x from the free end B is given as

$$M_x = - (w \times x) \cdot \frac{x}{2} - 6(x - 3) \quad (\because w = 3\text{kN/m})$$

$$M_x = - (3 \times x) \cdot \frac{x}{2} - 6(x - 3) \quad \dots\dots\dots(\text{iv})$$

In Eqn. (iv), x varies from 3 to 6. Eqn. (iv) shows that BM varies a parabolic between A and C.

At point C, $x = 3$, therefore, $M_C = - (3 \times 3) \cdot \frac{3}{2} - 6(3 - 3)$

$$M_C = - \left(\frac{27}{2} \right) - 6(0)$$

$$M_C = - \frac{27}{2} = -13.5 \text{ kN}$$

$$M_C = - 13.5 \text{ kNm}$$

At point A, $x = 6$, therefore, $M_A = - (3 \times 6) \cdot \frac{6}{2} - 6(6 - 3)$

$$M_A = - \left(\frac{108}{2} \right) - 6(3)$$

$$M_A = -54 - 18 = -72$$

$$M_A = -72 \text{ kNm}$$

Fig. 3.19(c) shows a BM diagram in which $M_C = 13.5 \text{ kNm}$, and $M_A = 72 \text{ kNm}$. The shape is parabolic.

3.8.3 UNIFORMLY VARYING LOAD

Consider a cantilever beam of length 'L' exposed to a load distribution that progressively varying between zero at the free end and 'w' force per unit length at the fixed end. To calculate the loading rate in section D, we can examine the change in load over a specific length. At point B, the loading rate is zero. At point A, the loading rate is w per unit length. Therefore, the loading rate at any point is $\left(\frac{w}{L} \cdot x\right)$ per unit length.

(i) S.F Diagram

Let F_x = SF at D, and

M_x = BM at D.

SF at x distance from the free end of the cantilever is determined by the equation,

F_x is the shear force at a distance x from the free end.

B = Triangle area (BCD)

$$= \frac{DB \times CD}{2} = \frac{x \times \left(\frac{w \times x}{L}\right)}{2} = \frac{w \times x^2}{2L} \quad \dots \dots (3.5)$$

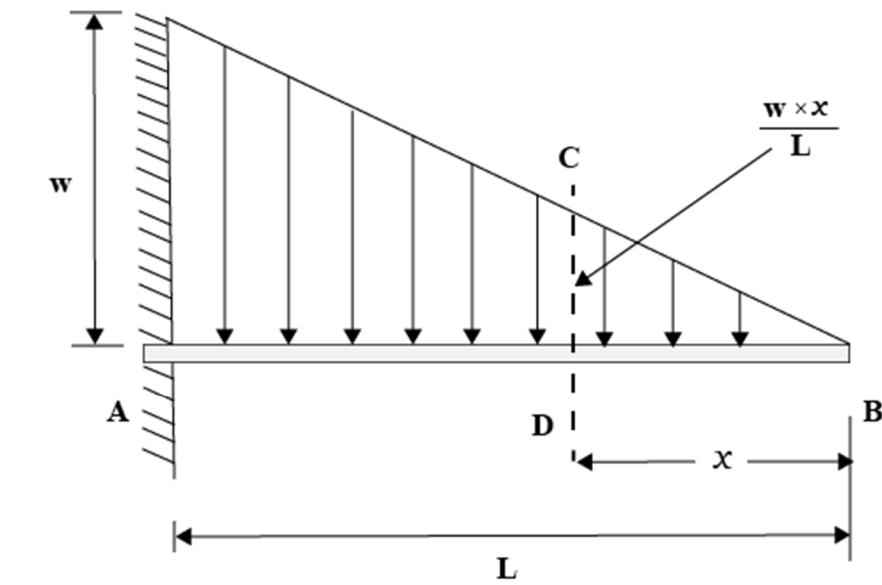
Where $\left(DB = x, DC = \frac{w \cdot x}{L}\right)$

Eqn. 3.5 shows that the shear force varies parabolically.

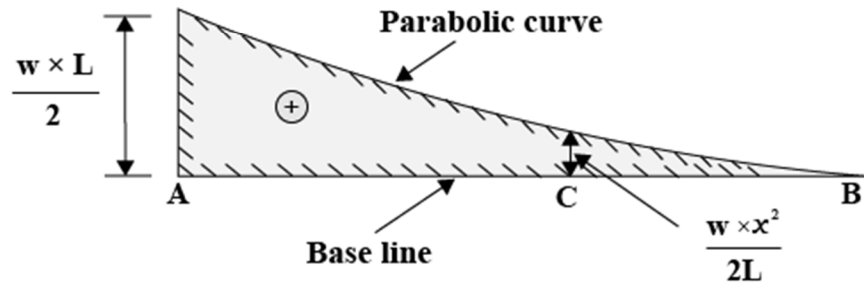
At point B, $x = 0$, therefore

$$F_B = \frac{w \times 0^2}{2L} = 0$$

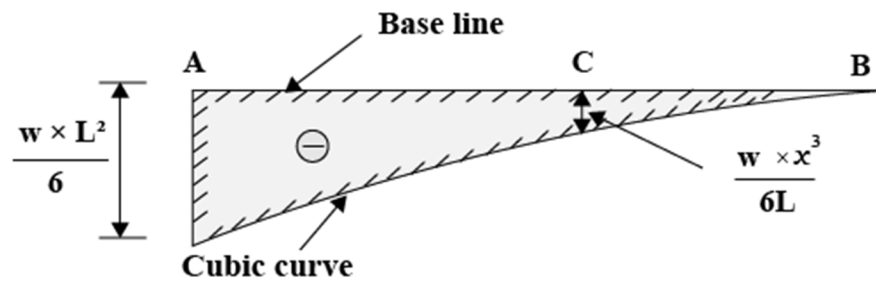
$$F_B = 0$$



(a) Load diagram



(b) Shear Force diagram



(c) Bending Moment diagram

Fig. 3.20(a)-(c): SF and BM diagram of the uniformly varying load

At point A, $x = L$, therefore

$$F_A = \frac{w \times L^2}{2L} = \frac{w \times L}{2}$$

(ii) BM diagram

BM at point D at ' x ' distance from the free end B, is given by:

$$\begin{aligned} M_x &= - (\text{Distance of the load from CD}) \times (\text{Overall load for a length } x) \\ &= - (\text{Distance of C.G of the triangle from CD}) \times (\text{Area of triangle BCD}) \\ &= - \left(\frac{x}{3}\right) \times \left(\frac{w \times x^2}{2L}\right) = - \frac{w \times x^3}{6L} \quad \dots \dots \dots (3.6) \end{aligned}$$

Eqn. (3.6) shows that the BM varies cubically.

At point B, $x = 0$, therefore

$$\begin{aligned} M_B &= - \frac{w \times 0}{6L} = 0 \\ M_B &= 0 \end{aligned}$$

At point A, $x = L$, therefore

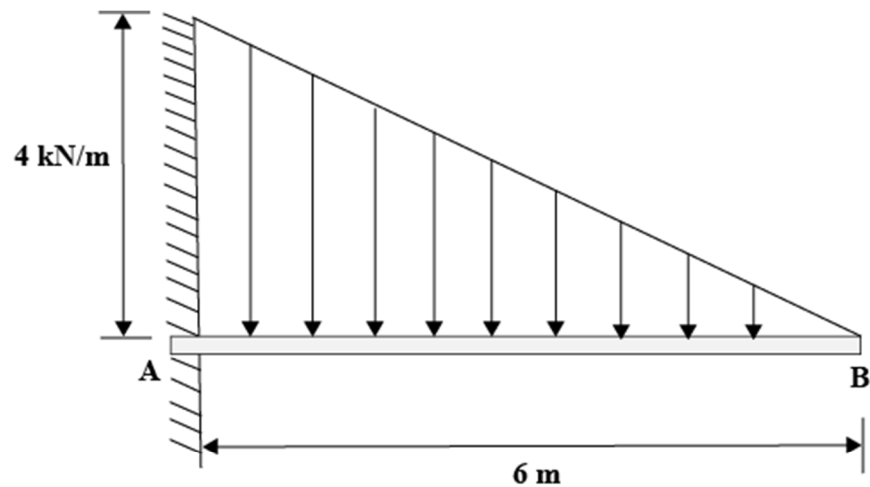
$$M_A = - \frac{w \times L^3}{6L} = - \frac{wL^2}{6}$$

Problem 3.4

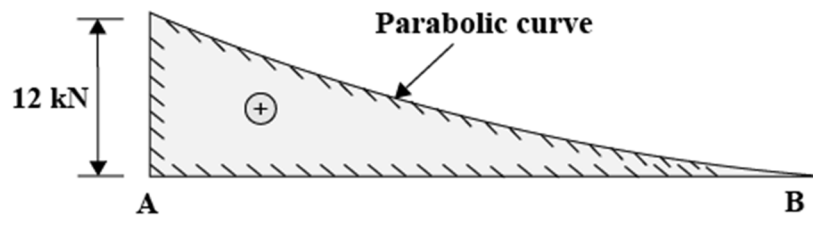
A 6 m long cantilever beam that carries a load that gradually increases from zero at the free end to 4 kN/m at the fixed end. Draw the SF and BM of the beam.

Solution:

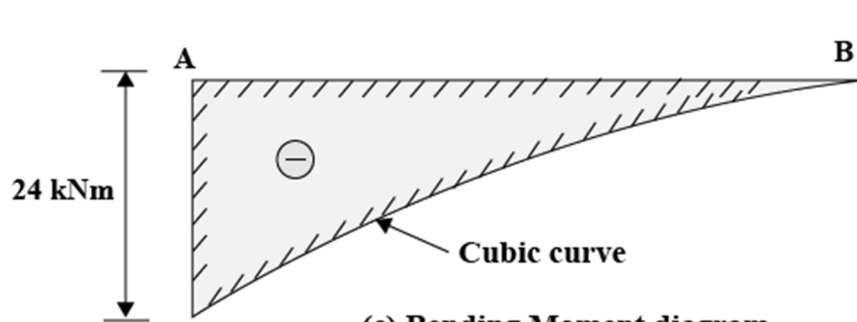
Length of the beam = 6 m; Load at fixed end = 4 kN/m



(a) Load diagram



(b) Shear Force diagram



(c) Bending Moment diagram

Fig. 3.21(a)-(c): SF and BM diagram of cantilever beam carrying gradually increase in load

(i) SF diagram

SF is zero at point B. The SF at point A will equal the area of the load diagram.

$$\therefore \text{Thus, SF at point A} = \frac{4 \times 6}{2} = \frac{24}{2} = 12 \text{ kN}$$

The SF between A and B varies parabolically.

(ii) BM diagram

$$\text{BM is zero at point B. The BM at A} = -\frac{w \cdot L^2}{6} = -\frac{4 \times 6^2}{6} = -24 \text{ kNm}$$

$\therefore$ The bending moment varies cubically between A and B.

3.9 S.F AND B.M DIAGRAM FOR SIMPLY SUPPORTED BEAM

A simply supported beam can support both UDL and concentrated point loads.

3.9.1 SIMPLY SUPPORTED BEAM WITH POINT LOAD AT ITS MID-POINT

Consider a simply supported beam of length 'L', where point load 'P' is applied at its mid-point C, as shown in Fig. 3.22(a).

The reaction at the supports will be balanced and equal in magnitude. In other words, each support will experience half of the load P.

$$\text{Therefore, } R_A = R_B = \frac{P}{2}.$$

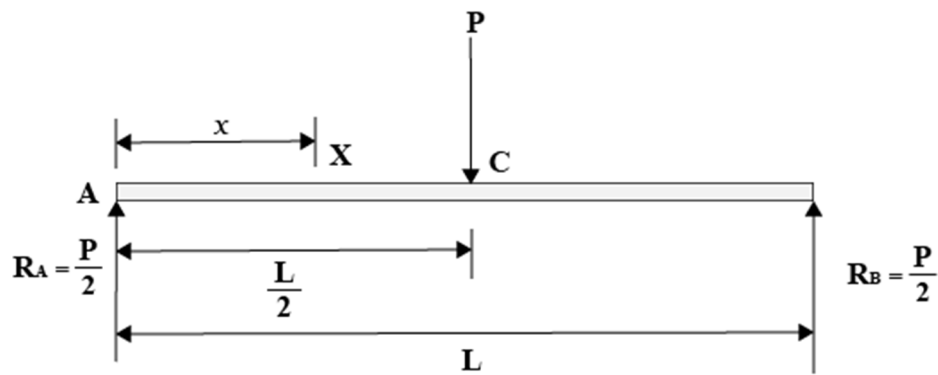
The BM and SF at a distance x from the end A.

(i) S.F Diagram

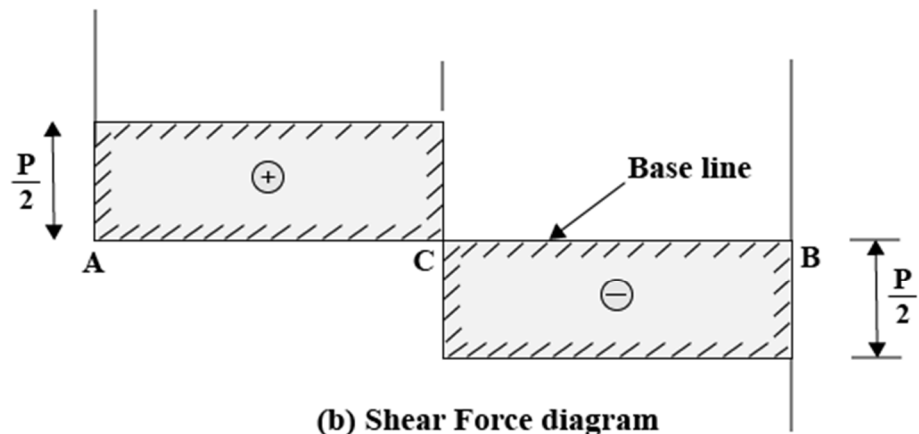
Let

F_x = SF at distance x , and

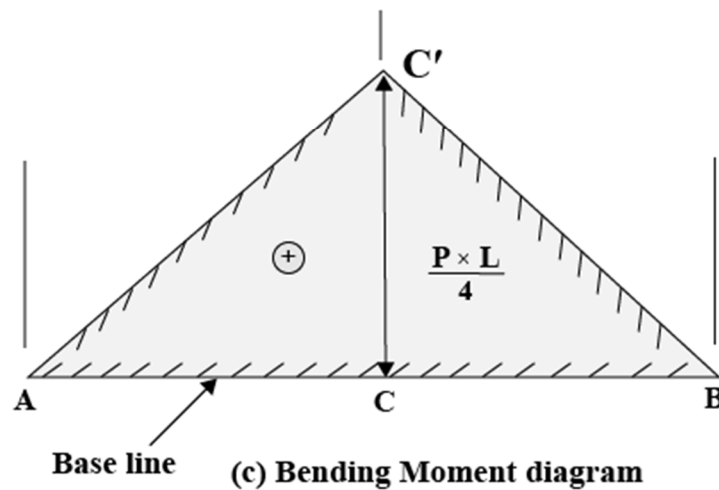
M_x = BM at distance x .



(a) Load diagram



(b) Shear Force diagram



(c) Bending Moment diagram

Fig. 3.22(a)-(c): SF and BM diagram of the simply supported beam with point load at its mid-point

When considering the left side of section X there is the sectional force $P/2$ and no other force act on it.

$$F_x = + \frac{P}{2} \quad \dots \dots (3.7)$$

Therefore, the SF between points A and C is considered positive and equal to $\frac{P}{2}$.

Now consider a point on the right side between points B and C at a ' x ' distance from A.

$$F_x = \left(\frac{P}{2} - P \right) = - \frac{P}{2} \quad \dots \dots (3.8)$$

Therefore, the SF between points C and B on the beam will remain constant and is equal to $-\frac{P}{2}$.

In section C, the S.F shifts from $+\frac{P}{2}$ to $-\frac{P}{2}$

Fig. 3.22(b) shows the S.F diagram.

(ii) B.M diagram

The BM at any section between points A and C at a distance of x from the end A is given by the equation:

$$M_x = R_A \times x \text{ or } M_x = + \frac{P}{2} \times x \quad \dots \dots (3.9)$$

BM for the left section will be positive as it is acting clockwise direction. BM for the right section will be acting in the anti-clockwise direction.

At point A, $x = 0$, therefore, $M_A = \frac{P}{2} \times 0 = 0$

$$M_A = 0$$

At point C, $x = \frac{L}{2}$, therefore, $M_c = \frac{P}{2} \times \frac{L}{2} = \frac{P \cdot L}{4}$

$$M_c = \frac{P \cdot L}{4}$$

From Eqn. (3.9), it is clear that the BM varies linearly between points A and C. BM is zero at point A and is $\frac{P \cdot L}{4}$ at point C.

BM at the section between points B and C from a distance of x from the end A is given by the equation:

$$\begin{aligned} M_x &= R_A x - P \times \left(x - \frac{L}{2}\right) \\ &= \left(\frac{P}{2} \times x - Px\right) + \left(P \times \frac{L}{2}\right) \\ &= \frac{PL}{2} - \frac{Px}{2} \end{aligned} \quad \dots \dots (3.10)$$

At point C, $x = \frac{L}{2}$ hence $M_C = \frac{PL}{2} - \left(\frac{P}{2} \times \frac{L}{2}\right) = \frac{PL}{4}$

At point B, $x = L$ hence $M_B = \frac{PL}{2} - \left(\frac{P}{2} \times L\right) = 0$

Therefore, BM at C is $\frac{PL}{4}$, decreasing to zero at B. The BM diagram as shown in Fig. 3.22(c).

3.9.2 SIMPLY SUPPORTED BEAM WITH UNIFORMLY DISTRIBUTED LOAD

In Fig. 3.23(a), a beam AB of length 'L' is shown as being simply supported at the ends of A and B. The beam carries a UDL of 'w' per unit length over the entire length. The support reactions will be equal, with a magnitude equal to the half of the total load.

(i) S.F diagram

Let

R_A = Reaction at A, and

R_B = Reaction at B

$$\therefore R_A = R_B = \frac{w \times L}{2}$$

Consider a section at a distance x from the left end A. The SF at the section (i.e., F_x) is given by the equation:

$$\begin{aligned} F_x &= +R_A - (w \cdot x) \\ F_x &= +\left(\frac{w \cdot L}{2}\right) - (w \cdot x) \end{aligned} \quad \dots \dots (3.11)$$

It is obvious from Eqn. (3.11) that the SF varies linearly. The SF values at different points are:

$$\text{At point A, } x = 0, \text{ therefore, } F_A = +\left(\frac{w \times L}{2}\right) - \left(\frac{w \times 0}{2}\right) = +\frac{w \cdot L}{2}$$

$$\text{At point B, } x = L, \text{ therefore, } F_B = +\left(\frac{w \times L}{2}\right) - (w \times L) = -\frac{w \cdot L}{2}$$

$$\text{At point C, } x = \frac{L}{2}, \text{ therefore, } F_C = +\left(\frac{w \times L}{2}\right) - \left(\frac{w \times L}{2}\right) = 0$$

Fig. 3.23(b) shows the SF diagram.

(ii) B.M diagram

Consider a section at a distance x from the left end A. The BM at the section (i.e., M_x) is given by the equation:

$$M_x = +(R_A \times x) - \left(w \times x \times \frac{x}{2} \right)$$

$$M_x = \left(\frac{w \times L}{2} \times x \right) - \left(\frac{w \times x^2}{2} \right) \quad \left(\because R_A = \frac{w \times L}{2} \right) \quad \dots \dots (3.12)$$

From Eqn. (3.12), the BM varies parabolically.

The BM values at different points are:

At point A, $x = 0$, therefore, $M_A = \left(\frac{w \times L}{2} \times 0 \right) - \left(\frac{w \times 0^2}{2} \right) = 0$

At point B, $x = L$, therefore, $M_B = \left(\frac{w \times L}{2} \times L \right) - \left(\frac{w}{2} \times L^2 \right) = 0$

At point C, $x = \frac{L}{2}$, therefore,

$$M_c = \left(\frac{w \times L}{2} \times \frac{L}{2} \right) - \left(\left(\frac{w}{2} \right) \times \left(\frac{L}{2} \right)^2 \right) = \left(\frac{w \times L^2}{4} \right) - \left(\frac{w \times L^2}{8} \right)$$

$$M_c = \frac{(w \times L^2)}{8}$$

Therefore, the BM varies parabolically from zero at point A to $+\frac{w \cdot L^2}{8}$ at the centre of the beam. The BM drops symmetrically from this maximum value to zero at point B as shown in Fig. 3.23(c).

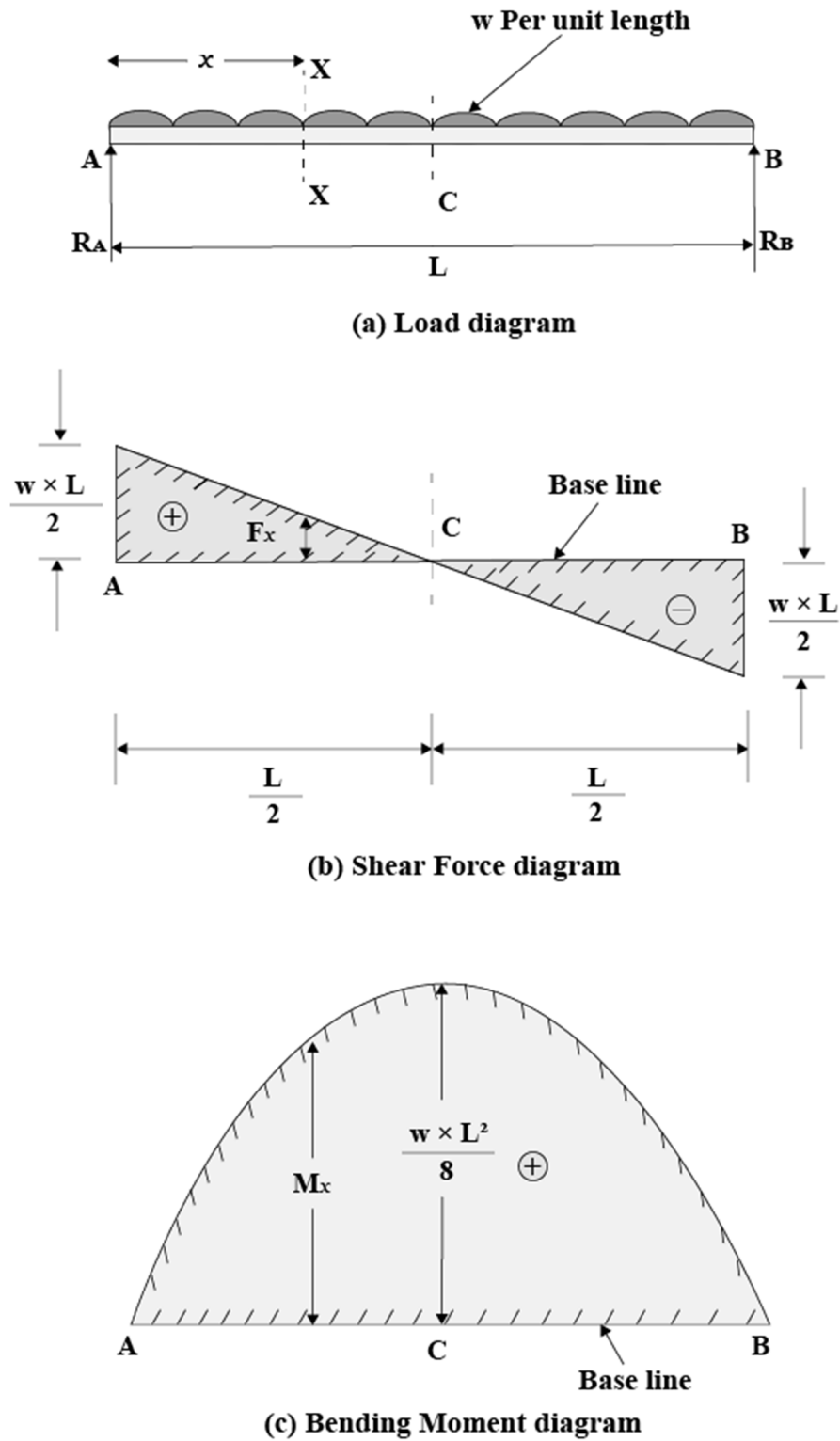


Fig. 3.23(a)-(c): SF and BM diagram of the simply supported beam with uniformly distributed load

Problem 3.5

Draw SF and BM diagram of a 15 m long simply supported beam with a UDL of 12 kN/m over a length of 10 m from the left end.

Solution

Initially, determine the reactions R_A and R_B .

Taking moments of the forces about X, we obtain

$$R_B \times 15 = 12 \times 10 \times \frac{10}{2}$$

$$R_B \times 15 = 600$$

$$R_B = \frac{600}{15}$$

$$R_A = \text{Total load} - R_B$$

$$R_A = (12 \times 10) - 40$$

$$R_A = 120 - 40$$

$$R_A = 80 \text{ kN}$$

$$R_B = 40 \text{ kN}$$

(i) SF diagram

Consider a section X-X at a distance x between A and C. SF at the X-X section is given as

$$F_x = + R_A - 12x$$

$$F_x = + 80 - 12x \quad \dots\dots\dots(i)$$

Eqn (i) shows the SF varies linearly between A and C.

At point A, $x = 0$, therefore, $F_A = 80 - 12(0) = 80 \text{ kN}$

At point C, $x = 10$, therefore, $F_C = 80 - 12(10) = 80 - 120 = - 40 \text{ kN}$

SF at point A is +80 kN, and at point C is -40 kN. Also, SF between A and C varies linearly. It implies that SF is zero somewhere between A and C. Let SF be zero at x m from A. Then, in Eqn (i), we replace the SF value (i.e., F_x) equal to zero, and we obtain

$$F_x = 80 - 12x$$

$$0 = 80 - 12x$$

$$12x = 80$$

$$x = \frac{80}{12}$$

$$x = 6.66 \text{ m}$$

Therefore, SF is zero at a 6.66 m distance from A. Fig. 3.24(b) shows the SF diagram.

In the SF diagram, point D is at a distance of 6.66 m from A.

(ii) BM diagram

BM at the section between A and B at a distance x from A is given by,

$$M_x = (R_x \times x) - \left(12 \times x \times \frac{x}{2}\right) = 80x - 6x^2 \quad \dots\dots\dots (ii)$$

The BM values at different points are:

At point A, $x = 0$, therefore, $M_A = 80 (0) - 6 (0)^2 = 0$

At point C, $x = 10$ m, therefore, $M_C = 80 (10) - 6 (10)^2 = 800 - 600 = 200 \text{ kNm}$

At point D, $x = 6.66$ m, therefore, $M_D = 80 (6.66) - 6 (6.66)^2 = 532.8 - 266.13$

$$M_D = 266.7 \text{ kNm}$$

BM between C and B varies linearly. Fig. 3.24(c) shows the BM diagram.

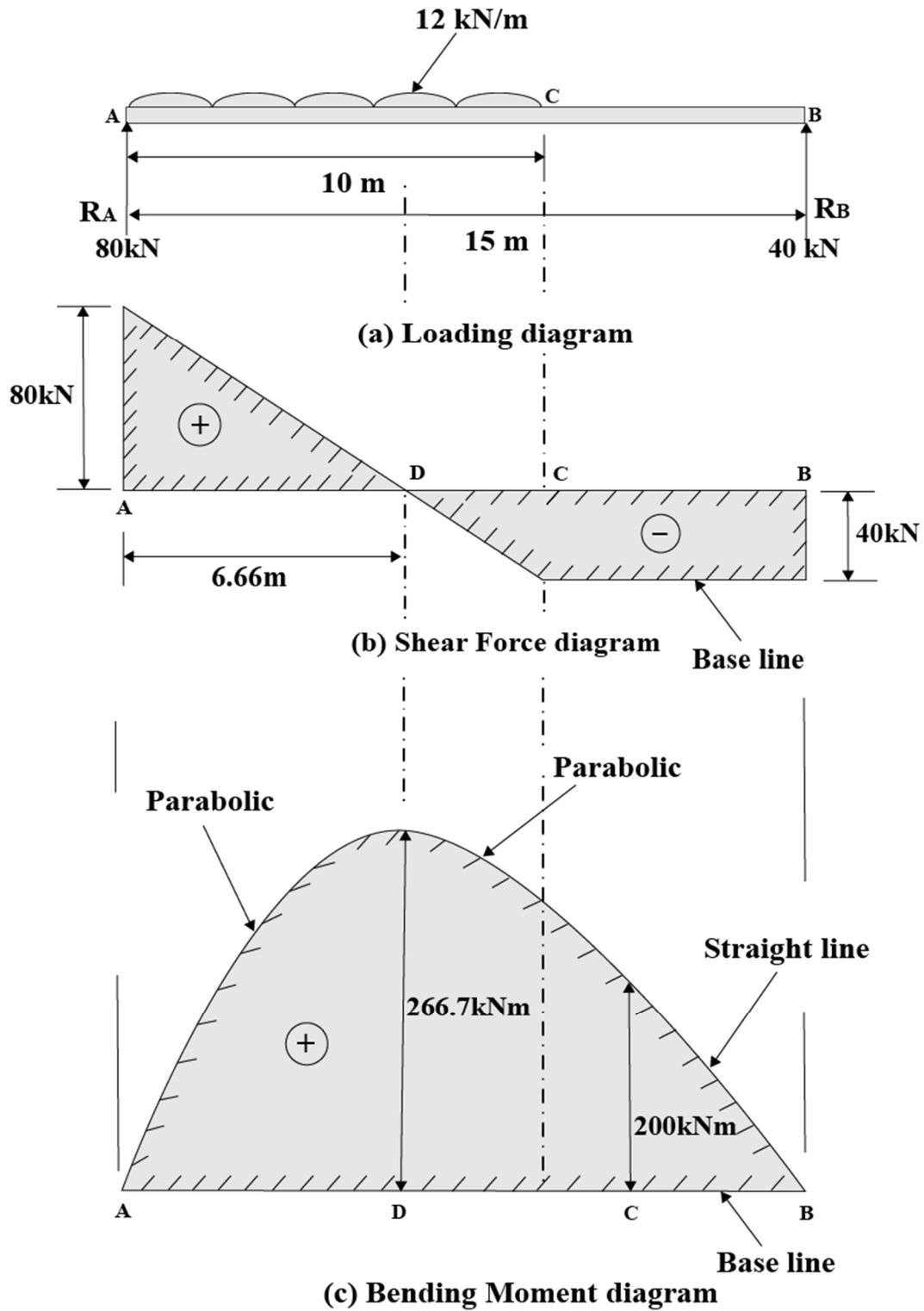


Fig. 3.24(a)-(c): SF and BM diagram of the simply supported beam carrying the uniformly distributed load

Problem 3.6

Draw SF and BM diagram for a simply supported beam of 12 m long and carrying a UDL of 14 kN/m at a 2 m distance from the end A as shown in Fig. 3.25

Solution:

Initially, determine the reactions R_A and R_B .

Taking moments of the force about A, we obtain

$$R_B \times 12 = 14 \times 6 \times \left(2 + \frac{6}{2}\right)$$

$$R_B \times 12 = 14 \times 6 \times (5)$$

$$R_B \times 12 = 420$$

$$R_B = \frac{420}{12}$$

$$R_B = 35 \text{ kN}$$

$$R_A = \text{Overall load on beam} - R_B$$

$$R_A = (14 \times 6) - 35$$

$$R_A = 49 \text{ kN}$$

(i) SF diagram

SF at point A = + 49 kN. SF remains constant between A and C and equals + 49 kN. SF at point B = - 35 kN. SF remains constant between B and D. SF at distance x from B is given as,

$$F_X = + 49 - 14 (x - 2) \quad \dots\dots\dots(i)$$

At point C, $x = 2$, therefore, $F_C = 49 - 14 (2 - 2) = 49 - 14 (0) = + 49 \text{ kN}$

At point D, $x = 8$, therefore, $F_D = 49 - 14 (8 - 2) = 49 - 14 (6) = - 35 \text{ kN}$

SF at point C is + 49 kN, and at point D is -35 kN. SF between C and D varies linearly. It implies that the SF is zero somewhere between A and B. Let be zero at (x) m from X. Then, in Eqn (i), we substitute the SF value (i.e., zero) and get,

$$F_X = 49 - 14(x - 2)$$

$$0 = 49 - 14(x - 2)$$

$$0 = 49 - 14x + 28$$

$$14x = 77$$

$$x = \frac{77}{14}$$

$$x = 5.5 \text{ m}$$

Therefore, SF is zero at a 5.5 m distance from A.

Thus, the SF diagram is shown in Fig. 3.25(b).

(ii) BM diagram

BM at point A is zero and

BM at point B is also zero

$$\text{BM at point C} = R_X \times 2 = 49 \times 2 = 98 \text{ kN}$$

BM at the section between C and D at a distance x from A is represented as, M_X

$$= (R_X \times x) - 14(x - 2) \times \frac{(x-2)}{2} = (49x) - 14 \frac{(x-2)^2}{2} \quad \dots\dots(ii)$$

$$\text{At point C, } x = 2, \text{ therefore, } M_C = (49 \times 2) - 14 \frac{(2-2)^2}{2} = 98 - 7(0) = 98 \text{ kNm}$$

$$\text{At point D, } x = 8, \text{ therefore, } M_D = (49 \times 8) - 14 \frac{(8-2)^2}{2} = 392 - 7(6)^2 = 392 - 252$$

$$M_D = 140 \text{ kNm}$$

$$\text{At point E, } x = 5.5, \text{ therefore, } M_E = (49 \times 5.5) - 14 \frac{(5.5-2)^2}{2} = 269.5 - 7(3.5)^2$$

$$M_E = 269.5 - 85.75 = 183.75 \text{ kNm}$$

The BM from varies linearly to 98 kNm at point C. Between point C and D, the BM varies by parabolically, and BM is maximum at point E. From D and B, the BM will decrease from 183.75 kNm to 140 kNm and at B point BM is zero. The BM diagram shown in Fig. 3.25(c).

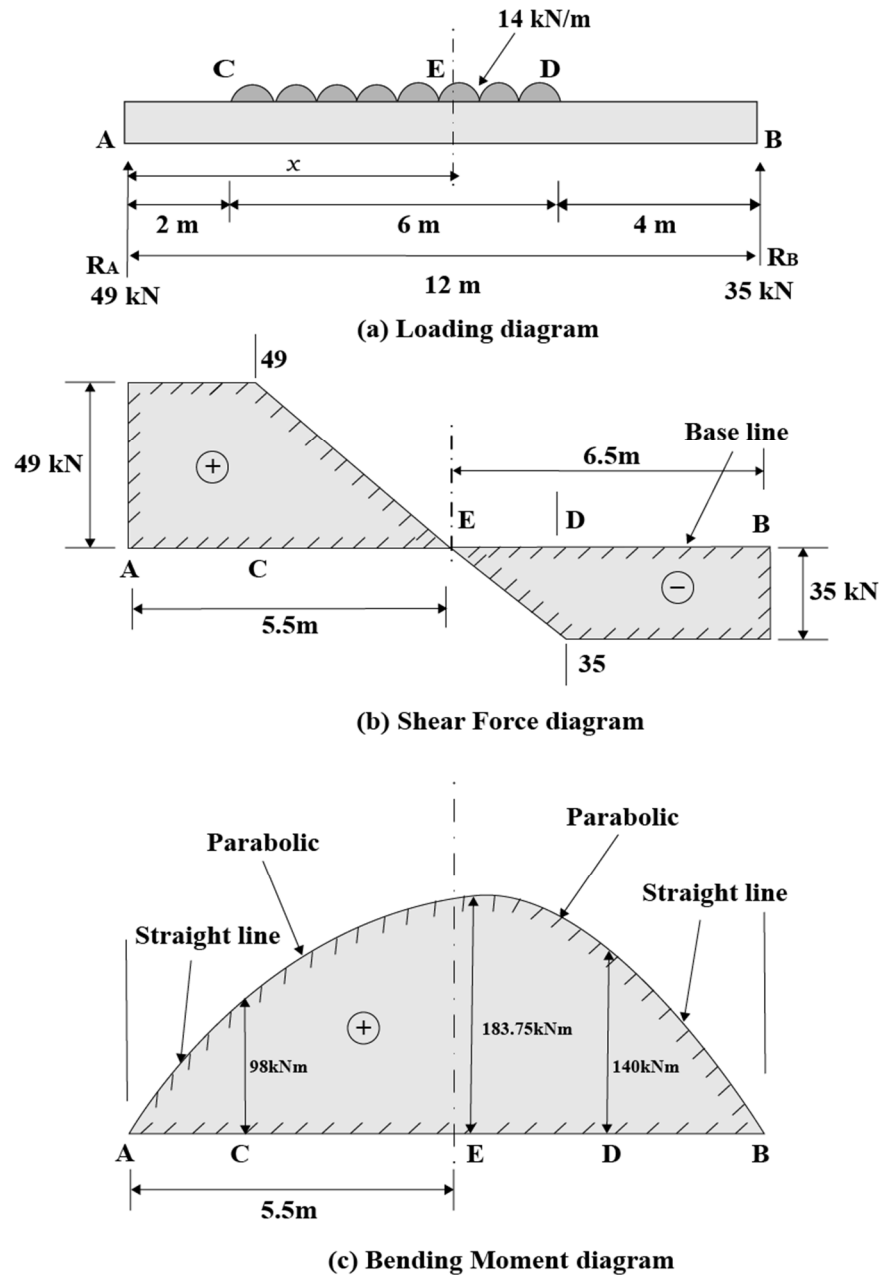


Fig. 3.25(a)-(c): SF and BM of the simply supported beam carries the uniformly distributed load

3.9.3 TRIANGULAR LOAD VARIES FROM ZERO AT BOTH ENDS TO 'w' PER UNIT LENGTH IN THE MIDDLE

In Fig. 3.26(a), a beam of L length is simply supported at the ends A and B. The UVL of the beam begins from zero at both ends and increases to 'w' load per unit length in the middle. Because the load on the beam is symmetrical, the responses at the supports will be equal, with a magnitude equal to half of the total load applied over the beam.

The Area of load diagram ABC = The total load on the beam

$$= \frac{AB \times CO}{2}$$

$$= \frac{L \times P}{2} \text{ or } \left(\frac{w \times L}{2} \right)$$

$R_A = R_B = \text{half the total load}$

$$= \frac{1}{2} \left(\frac{w \times L}{2} \right) = \left(\frac{w \times L}{4} \right)$$

(i) SF diagram

Consider a section X between points A and C at a distance x from end A.

Vertical distance XD in the load diagram = The loading rate at X

$$= \frac{x}{\left(\frac{L}{2}\right)} \times w \quad \left[\because \frac{CO}{\left(\frac{L}{2}\right)} = \frac{XD}{x} \therefore XD = \frac{x \times CO}{\left(\frac{L}{2}\right)} = \frac{2 \times x \times w}{L} \right]$$

$$= \left(\frac{2 \times w}{L} \right) \times x$$

Now, the Area of load diagram AXD = load on the length AX of the beam

$$= \frac{x \times XD}{2} = \frac{x \times \frac{2 \times w}{L} \times x}{2} \quad \left(\because XD = \frac{2 \times w}{L} \times x \right)$$

$$= \left(\frac{w}{L} \times x^2 \right)$$

This load acts at a specific distance of $\frac{x}{3}$ from A

Now SF at X is given by the equation,

$F_x = R_A - \text{load on the AX length}$

$$= \left(\frac{w \times L}{4} \right) - \left(\frac{w}{L} \times x^2 \right) \quad \left(\because R_A = \frac{w \times L}{4} \right) \quad \dots \dots (3.13)$$

Eqn. (3.13) shows that shear force varies between points A and C by parabolically.

At point A, $x = 0$, therefore, $F_A = \left(\frac{w \times L}{4} \right) - \left(\frac{w}{L} \times 0 \right) = \frac{w \times L}{4}$

At point C, $x = \frac{L}{2}$, therefore,

$$F_C = \left(\frac{w \times L}{4} \right) - \left(\left(\frac{w}{L} \right) \times \left(\frac{L}{2} \right)^2 \right) = \left(\frac{w \times L}{4} \right) - \left(\frac{w \times L}{4} \right) = 0$$

The shear force at B = $R_B = \left(\frac{w \times L}{4} \right)$

The SF is shown in Fig. 3.26(b). The SF between A to C and C to B is parabolic.

(ii) BM diagram

At points A and B, BM is zero.

The equation gives the BM at any point X,

$$\begin{aligned} M_x &= (R_A \times x) - \text{Length of the load} \left(AX \times \frac{x}{3} \right) \\ M_x &= \left[\left(\frac{w \times L}{4} \right) \times x \right] - \left[\left(\frac{w \times x^2}{L} \right) \times \left(\frac{x}{3} \right) \right] \\ M_x &= \left[\left(\frac{w \times L}{4} \right) \times x \right] - \left[\left(\frac{w \times x^3}{3L} \right) \right] \quad \dots \dots (3.14) \end{aligned}$$

Eqn (3.14) shows that the BM between points A and C varies cubically.

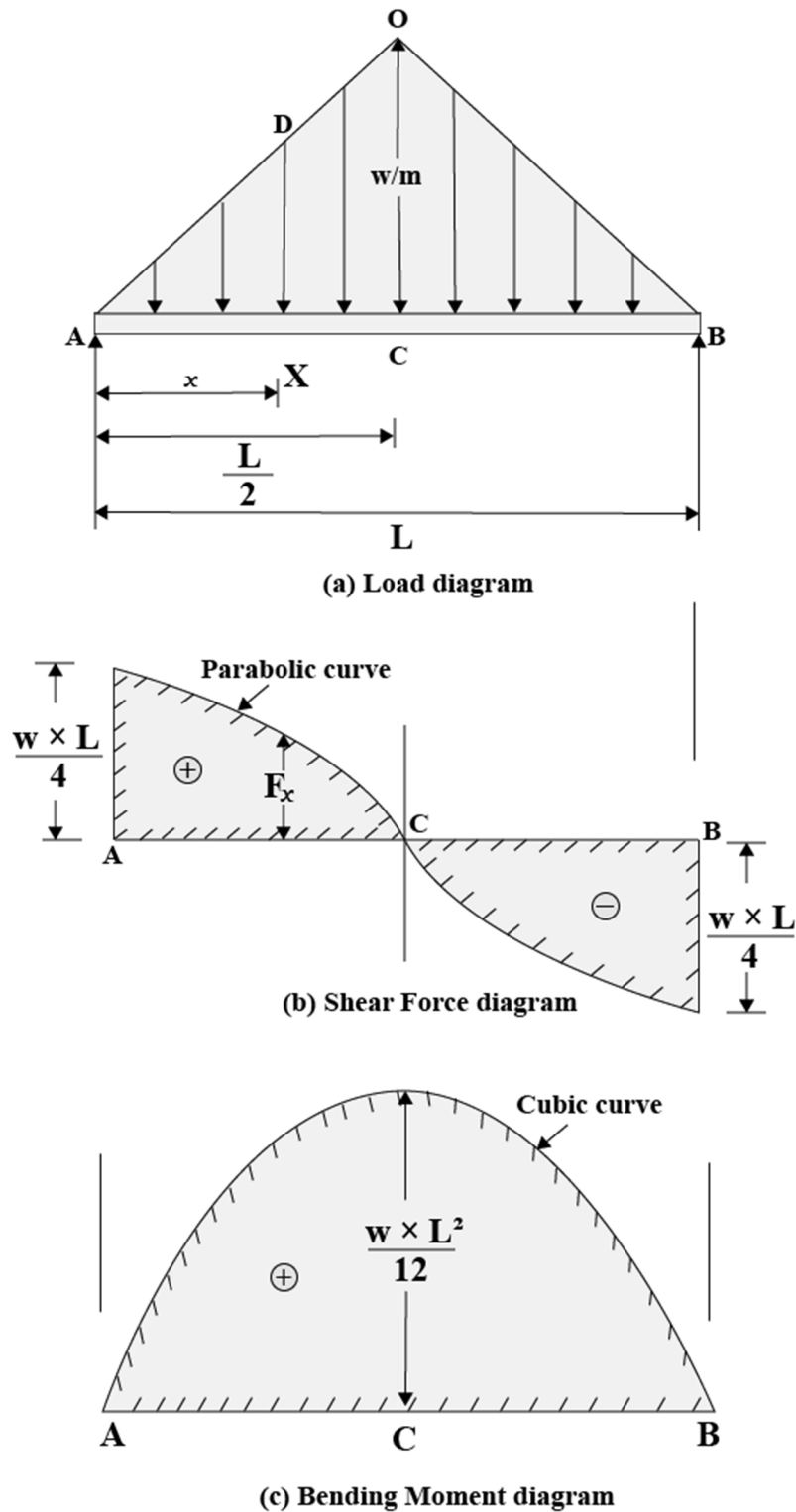


Fig. 3.26(a)-(c): SF and BM diagram of the simply supported beam carries triangular load

At point A, $x = 0$, therefore, $M_A = 0$,

At point C, $x = \frac{L}{2}$, therefore, $M_c = \left[\left(\frac{w \times L}{4} \right) \times \left(\frac{L}{2} \right) \right] - \left[\left(\frac{w}{3L} \right) \times \left(\frac{L}{2} \right)^3 \right]$

$$M_c = \left[\left(\frac{w \times L^2}{8} \right) - \left(\frac{w \times L^2}{24} \right) \right]$$

$$M_c = \left[\left(\frac{3w \times L^2}{24} \right) - \left(\frac{w \times L^2}{24} \right) \right]$$

$$M_C = \frac{w \times L^2}{12}$$

The maximal BM is at the mid-span of the beam. It corresponds to the location where the shear force is zero and undergoes a change in sign.

$\therefore$ Maximum BM is at point C, $M_c = \frac{w \times L^2}{12}$

And the total load = $\frac{w \times L}{2}$

Maximum BM = $\left(\frac{w \times L}{2} \right) \times \left(\frac{L}{6} \right) = \left(\frac{w \times L^2}{12} \right)$

The BM is shown in Fig. 3.26(c).

3.9.4 LINEARLY VARYING LOAD ON A BEAM FROM ZERO AT ONE END TO ‘w’ PER UNIT LENGTH AT OTHER END

In Fig. 3.27(a), a beam AB of length L is shown which is simply supported at points A and B. The UVL of the beam begins at zero at end A and increases to ‘w’ per unit length at end B. To determine the reactions of R_A and R_B at the supports.

Considering moment about A, we obtain

$$R_B \times L = \left(\frac{w \times L}{2} \right) \times \left(\frac{2}{3} \times L \right)$$

$$R_B = \left(\frac{w \times L}{3} \right) \text{ and } \left[\text{Total load} = \left(\frac{w \times L}{2} \right) \text{ acts } \left(\frac{2}{3} \times L \right) \text{ from A} \right]$$

$$R_A = \text{Total load on beam} - R_B$$

$$R_A = \left(\frac{w \times L}{2}\right) - \left(\frac{w \times L}{3}\right)$$

$$R_A = \left(\frac{w \times L}{6}\right)$$

(i) SF diagram

Consider any section X at a distance x from end A. The equation gives the SF at X:

$$F_x = R_A - \text{load on length AD} = \left[\left(\frac{w \times L}{6}\right) - \left[\left(\frac{w \times x}{L}\right) \times \left(\frac{x}{2}\right)\right]\right]$$

$$\left(\because \text{Load on AD} = \frac{AD \times CD}{2} = \frac{x \times w \times x}{2 \times L}\right)$$

$$F_x = \left[\left(\frac{w \times L}{6}\right) - \left(\frac{w \times x^2}{2 \times L}\right)\right] \dots \dots (3.15)$$

Eqn (3.15) shows that SF varies parabolically.

$$\text{At point A, } x = 0, \text{ therefore } F_A = \left[\left(\frac{w \times L}{6}\right) - \left(\frac{w}{2L} \times 0\right)\right] = \left(\frac{w \times L}{6}\right)$$

$$\text{At point B, } x = L, \text{ therefore, } F_B = \left[\left(\frac{w \times L}{6}\right) - \left(\frac{w \times L^2}{2 \times L}\right)\right] = \left[\left(\frac{w \times L}{6}\right) - \left(\frac{w \times L}{2}\right)\right]$$

$$F_B = \left[\frac{(w \times L) - (3w \times L)}{6}\right]$$

$$F_B = -\frac{2w \times L}{6}$$

$$F_B = -\frac{w \times L}{3}$$

The SF is $+\frac{wL}{6}$ at A, decreasing to $-\frac{wL}{3}$ at B with a parabolic variation.

Between points A and B, there exists a point where the SF becomes zero. Assuming that the SF is zero at a distance x from point A, we may use Eqn (3.15) to get the SF:

$$0 = \left(\frac{w \times L}{6} \right) - \left(\frac{w \times x^2}{2 \times L} \right) \text{ or } \left(\frac{w \times x^2}{2 \times L} \right) = \left(\frac{w \times L}{6} \right)$$

$$x^2 = \left[\left(\frac{w \times L}{6} \right) \times \left(\frac{2 \times L}{w} \right) \right] = \left(\frac{L^2}{3} \right)$$

$$x = \left(\frac{L}{\sqrt{3}} \right) = 0.577L$$

Fig. 3.27(b) shows the SF diagram.

(ii) BM diagram

At points A and B, BM is zero.

The equation gives the BM at section X at a distance x from the end A.

$$M_x = (R_A \times x) - \text{load on length AD} \times \left(\frac{x}{3} \right) \left(\because \text{load on AX acts at } \frac{x}{3} \text{ from X} \right)$$

$$= \left[\left(\frac{w \times L}{6} \right) \times (x) \right] - \left[\left(\frac{w \times x^2}{2L} \right) \times \left(\frac{x}{3} \right) \right]$$

$$= \left[\left(\frac{w \times L}{6} \right) \times (x) \right] - \left(\frac{w \times x^3}{6L} \right) \quad \dots \dots (3.16)$$

Eqn (3.16) shows that BM varies between A and B cubically.

The maximum BM is reached at a point where the shear force is zero.

The point is located at a distance of $\frac{L}{\sqrt{3}}$ from point A.

Therefore, by substituting $x = \frac{L}{\sqrt{3}}$ in the Eqn (3.16).

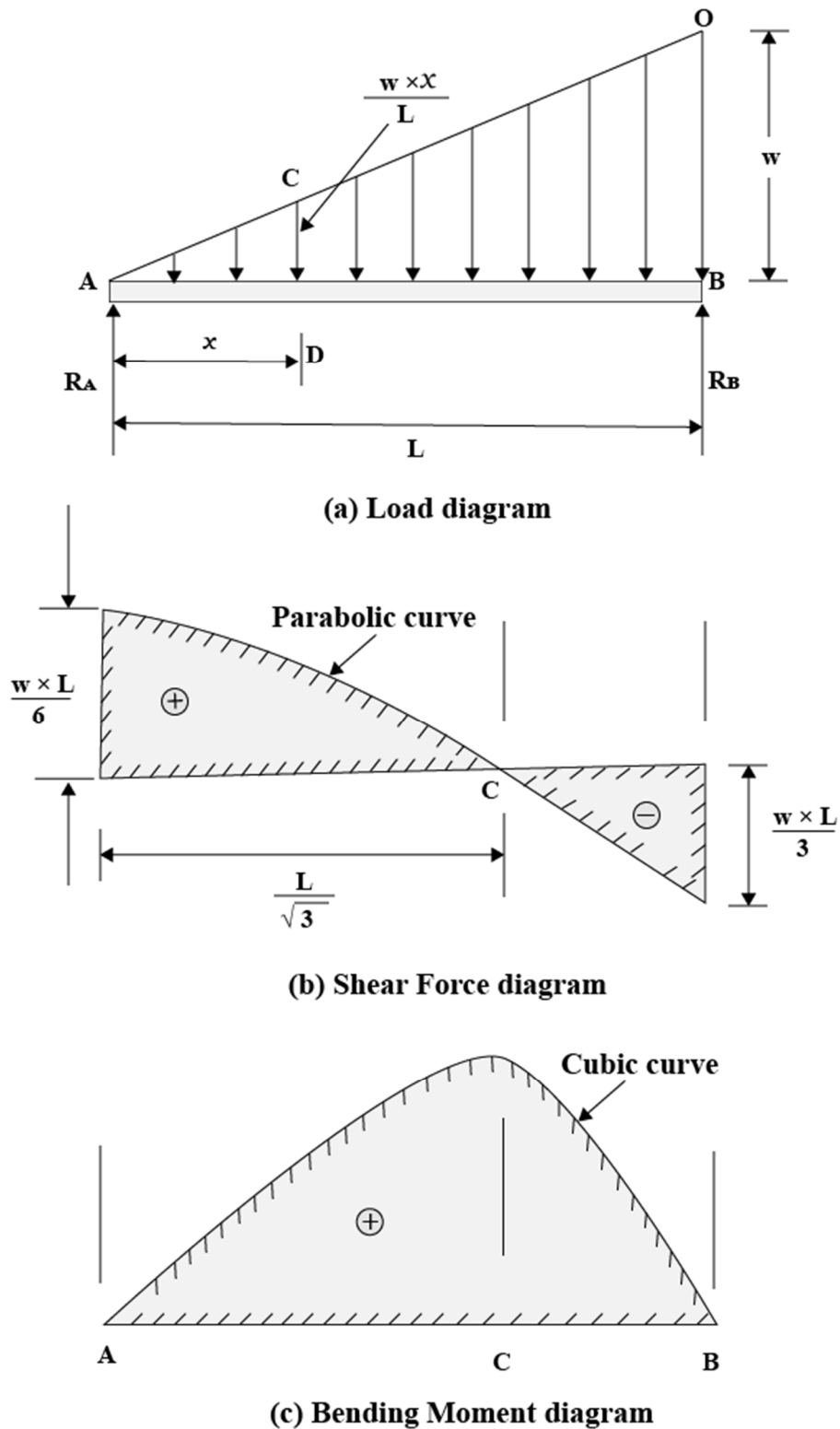


Fig. 3.27(a)-(c): SF and BM diagram of the linearly varying load on a beam

$$\begin{aligned}
\therefore \text{Maximum BM} &= \left[\left(\frac{w \times L}{6} \right) \times \left(\frac{L}{\sqrt{3}} \right) \right] - \left[\left(\frac{w}{6 \times L} \right) \times \left(\frac{L}{\sqrt{3}} \right)^3 \right] \\
&= \left[\left(\frac{w \times L^3}{6 \times \sqrt{3}} \right) - \left(\frac{w \times L^2}{18\sqrt{3}} \right) \right] \\
&= \left[\frac{(3w \times L^2) - (w \times L^2)}{9 \times \sqrt{3}} \right] \\
&= \left[\frac{w \times L^2}{9 \times \sqrt{3}} \right]
\end{aligned}$$

Fig. 3.27(c) shows the BM diagram.

Problem 3.7

A 10 m long simply supported beam carries a UDL of 1000 N/m from one end to 2000 N/m at the other. Draw SF and BM diagrams. Calculate the location and magnitude of the maximum BM also.

Solution:

The load may have a UDL of 1000 N/m throughout the whole span and a UVL ranging from zero at A to 1000 N/m at B.

Load on the beam as a result of UDL of 1000 N/m = $1000 \times 10 = 10000$ N

Load on the beam as a result of triangular loading = $\frac{1}{2} \times 1000 \times 10 = 5000$ N

Now determine the reaction R_A and R_B

(i) SF diagram

Considering the moments about point A, we obtain

$$R_B \times 10 = \left(10000 \times \frac{10}{2} \right) + \left(5000 \times \frac{2}{3} \text{ of } 10 \right)$$

$$R_B \times 10 = (50000) + (5000 \times 6.66)$$

$$R_B \times 10 = 50000 + 33300$$

$$R_B = \frac{83300}{10} = 8330 \text{ N}$$

$$R_B = 8330 \text{ N}$$

$$R_A = \text{Total load on the beam} - R_B$$

$$R_A = (10000 + 5000) - 8330$$

$$R_A = 6670 \text{ N}$$

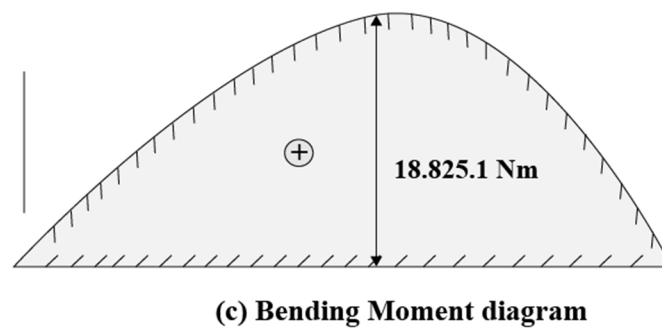
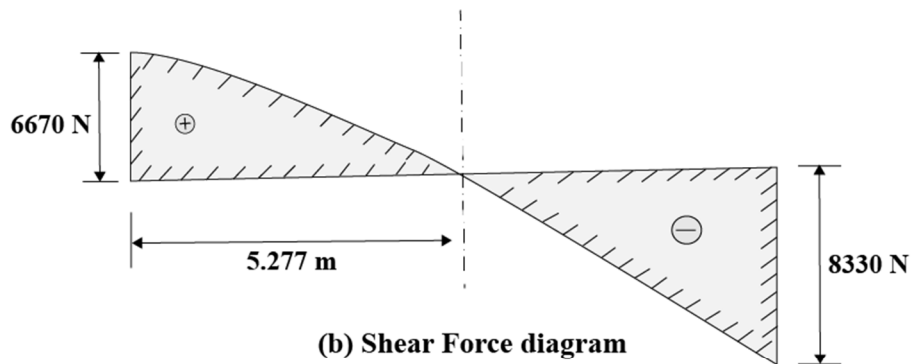
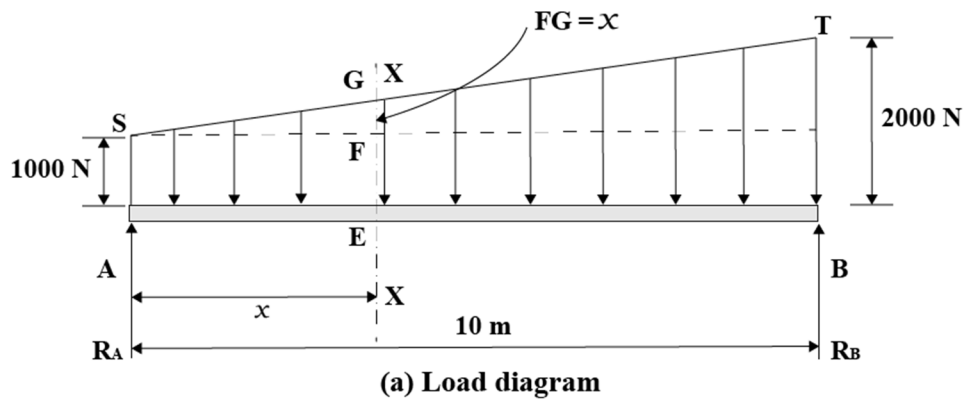


Fig. 3.28(a)-(c) SF and BM diagram of the simply supported beam carries uniformly varying load

Considering the section, X-X at a distance x from A.

Loading rate at the section X-X

$$= \text{Length EG} = \text{EF} + \text{FG}$$

$$= 1000 + \left(\frac{x}{10} \times 1000\right)$$

$$= 1000 + 100x$$

Total load on length AX

$$= \text{Area of load diagram AEFGS}$$

$$= \text{Area of } \triangle \text{FGS} + \text{Area of rectangle}$$

$$= \left(\frac{100x \times x}{2}\right) + (1000 \times x)$$

$$= 50x^2 + 1000x$$

Now, SF at section X-X is represented as

$$F_x = R_A - \text{load on length AX}$$

$$F_x = 6670 - (50x^2 + 1000x)$$

$$F_x = 6670 - 50x^2 - 1000x \quad \dots\dots\dots(i)$$

Eqn (i) shows that SF varies between A and B parabolically.

At point A, $x = 0$, therefore, $F_A = 6670 - 50(0) - 1000(0) = 6670 \text{ N}$

At point B, $x = 10$, therefore, $F_B = 6670 - 50(10)^2 - 1000(10) = -8330 \text{ N}$

Find the location of zero shear. Use Eqn (i) equal to zero, we obtain

$$0 = 6670 - 50x^2 - 1000x \quad (\because \text{Divide by } 50)$$

$$x^2 + 20x - 133.4 = 0$$

$$= \frac{-20 \pm \sqrt{20^2 - 4(1)(-133.4)}}{2(1)}$$

$$\begin{aligned}
&= \frac{-20 \pm \sqrt{400 + 533.4}}{2(1)} \\
&= \frac{-20 \pm 30.554}{2} \\
&= \frac{10.554}{2} \quad (\because \text{Neglecting -ve root}) \\
&= 5.277 \text{ m}
\end{aligned}$$

(ii) BM diagram

BM at section X-X is given by,

$$\begin{aligned}
M_x &= (R_X \times x) - \left(1000 \times x \times \frac{x}{2}\right) - \left(\frac{1}{2} \times x \times 100x \times \frac{x}{3}\right) \\
M_x &= (6670x) - 500x^2 - \frac{50x^3}{3} \quad \dots\dots(ii)
\end{aligned}$$

Eqn (ii) shows that BM between A and B varies cubically.

At point A, $x = 0$, therefore, $M_A = 0$

$$\begin{aligned}
\text{At point B, } x &= 10, \text{ therefore, } M_B = (6670 \times 10) - (500 \times 10^2) - \frac{50 \times 10^3}{3} \\
M_B &= 0
\end{aligned}$$

Maximum BM exists where SF is zero. But SF is zero at an x distance of 5.277 m from A. Therefore, maximum BM is obtained by substituting $x = 5.277$ m in Eqn (ii)

$$\text{The max BM} = (6670 \times 5.277) - (500 \times 5.277^2) - \frac{50 \times 5.277^3}{3} = 18825.1 \text{ Nm}$$

3.9.5 ECCENTRIC POINT LOAD

Fig. 3.29(a) shows a beam AB with a length of L that is simply supported at ends A and B. The beam sustains a point load P applied at point C at a distance 'a' from the end A.

(i) SF diagram

Let

Reaction at the support A = R_A , and

Reaction at the support B = R_B .

We can take BM about points A or B to calculate the reaction at the supports.

Taking BM about A,

$$R_B \times L = P \times a$$
$$R_B = \frac{P \times a}{L} \quad \dots \dots \dots (3.17)$$

$$R_A = P - R_B = P - \left(\frac{P \times a}{L} \right)$$

$$R_A = P \left(1 - \frac{a}{L} \right)$$

$$R_A = P \left(\frac{L - a}{L} \right)$$

$$R_A = \frac{P \times b}{L} \quad (\because L - a = b) \quad \dots \dots \dots (3.18)$$

Consider section X which between A to C at a distance x from the end A.

The equation gives the SF F_x in the section:

$$F_x = +R_A = + \frac{P \times b}{L} \quad \dots \dots \dots (3.19)$$

At section X, SF will be positive because the total force on the left side section is acting in the upward direction.

The SF between points A to C is constant and is equal to $\frac{Pb}{L}$

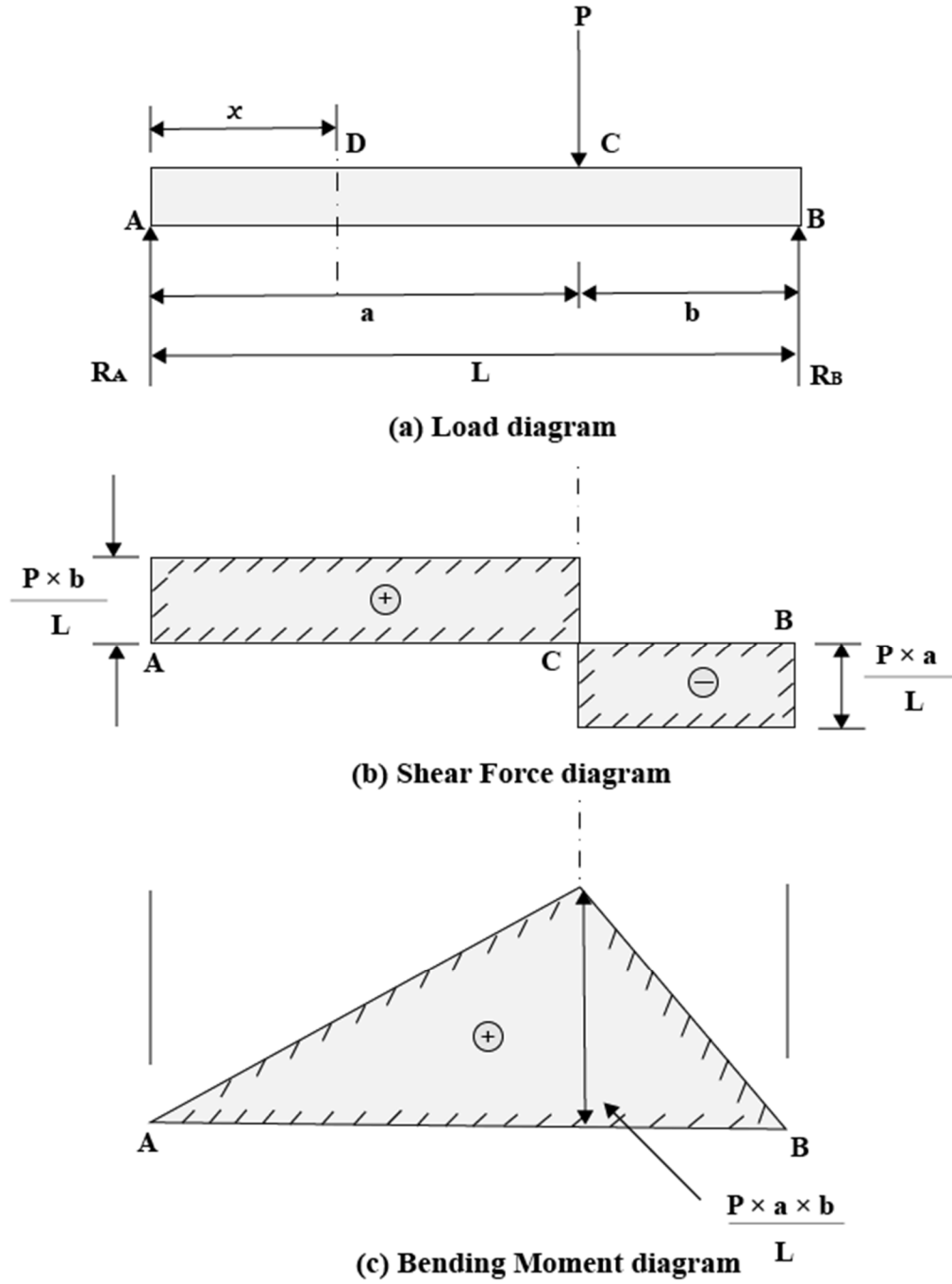


Fig. 3.29(a)-(c): SF and BM of the Eccentric Point Load

Consider a section between B and C at a distance x from the end A. The force on the left of this section will be equal to $R_A - P$.

$$R_A - P = \left(\frac{P \times b}{L} \right) - P$$

$$\begin{aligned}
&= P \times \left(\frac{b - L}{L} \right) \\
&= -P \left(\frac{L - b}{L} \right) \\
&= -\frac{P \times a}{L} \quad (\because L - b = a)
\end{aligned}$$

The SF between points C to B is $-\frac{Pa}{L}$. At point C, the SF changes from $+\frac{Pb}{L}$ to $-\frac{Pa}{L}$. Fig. 3.29(b) shows the SF diagram.

(ii) BM diagram

BM between A and C at a distance x from the point A end is given by the equation:

$$\begin{aligned}
M_x &= R_A \times x \\
M_x &= + \left(\frac{P \times b}{L} \right) \times x \quad \text{(Plus sign due to sagging)(3.20)}
\end{aligned}$$

At point A, $x = 0$, therefore, $M_A = \left(\frac{P \times b}{L} \right) \times 0 = 0$

At point C, $x = a$, therefore, $M_C = \left(\frac{P \times b}{L} \right) \times a = \frac{P \times b \times a}{L}$

Therefore, the BM varies from zero at point A to $\frac{Pab}{L}$ at point C in accordance with straight-line law.

At B point, BM is zero. As a result, BM will reduce from $\frac{Pab}{L}$ at point C to zero at point B, in accordance with the straight-line law. Fig. 3.29(c) shows BM.

SF and BM diagrams show that maximum BM is at C point when SF changes sign.

Problem 3.8

A 14 m long simply supported beam has a point load of 6 kN and 10 kN at 4 m and 10 m distances from the left end, respectively. Draw the SF and BM diagram for the beam.

Solution:

To begin, find the reactions R_A and R_B .

Considering moments of the forces about point A, we obtain

$$R_B \times 14 = (6 \times 4) + (10 \times 10)$$

$$R_B \times 14 = 24 + 100$$

$$R_B = \frac{124}{14} = 8.85$$

$$R_B = 8.85$$

$$R_A = \text{Total load on the beam} - R_B$$

$$R_A = (6 + 10) - 8.85$$

$$R_A = 16 - 8.85$$

$$R_A = 7.15$$

(i) S.F diagram

- SF at point A, $F_A = + R_A = +7.15$ kN.
- SF between the points A and C is constant and is + 7.15 kN.
- SF at point C, $F_C = +7.15 - 6 = +1.15$ kN.
- SF between the points C and D is constant and is + 1.15 kN.
- SF at point D, $F_D = +1.15 - 10 = -8.85$ kN.
- SF between the points D and B is constant and is - 8.85 kN.
- SF at point B, $F_B = - R_B = -8.85$ kN.

Fig. 3.30(b) shows the SF diagram.

(ii) B.M diagram

- BM at point A, therefore, $M_A = 0$
- BM at point C, therefore, $M_C = (R_A \times 4) = (7.15 \times 4) = 28.6 \text{ kNm}$
- BM at point D, therefore, $M_D = (R_A \times 10) - (6 \times 6) = (7.15 \times 10) - (36)$
 $M_D = 71.5 - 36 = 35.5 \text{ kNm}$
- BM at point B, therefore, $M_B = 0$

Fig. 3.30(c) shows the BM diagram.

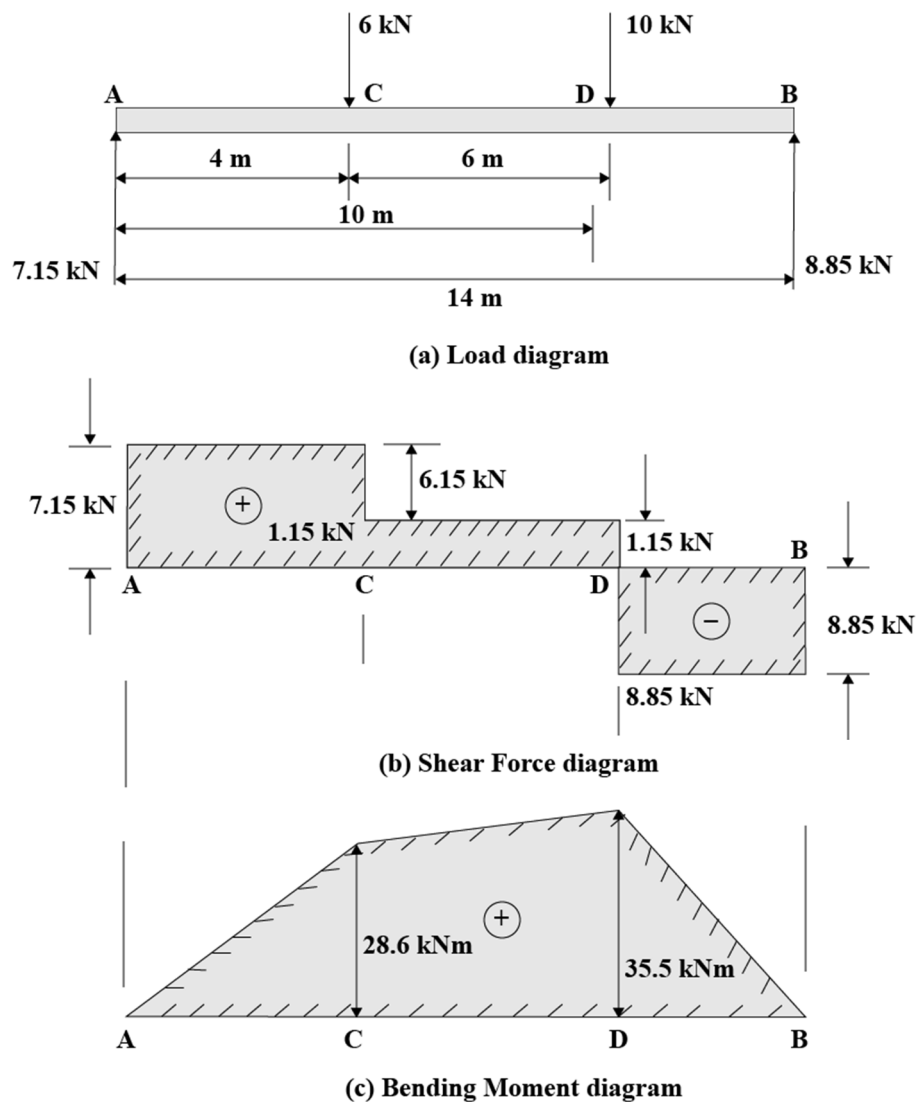


Fig. 3.30(a)-(c): SF and BM diagram for the simply supported beam

3.10 SF AND BM DIAGRAM FOR OVER-HANGING BEAMS

Beams can have overhangs where one or both ends extend beyond the supports. These overhangs can vary in length and can influence the load distribution and structural behavior of the beam. When analyzing beams with overhangs, the construction of SF and BM diagrams can commence from either the left or the right-hand end, depending on convenience and the specific requirements of the analysis. The following is the discussion of a typical situation of a beam with UDL beam with equal overhanging.

Figure 3.31(a) shows a beam with equal overhanging. Let w be the UDL on the beam.

As the overhangs are equal, $R_A = R_B = \frac{w(L+2a)}{2}$.

(i) S.F diagram

Section DA:

$$F_x = -w \times x \text{ (Linear);}$$

$$F_D = 0; \text{ and } F_A = -w \times a$$

Section AB:

$$F_x = (-w \times x) + \frac{w(L+2a)}{2} \text{ (Linear);}$$

$$F_{A(x=a)} = \frac{w \times L}{2}; F_{B(x=L+a)} = -\frac{w \times L}{2}$$

Section BE:

$$F_x = -(w \times x) + \frac{w(L+2a)}{2} + \frac{w(L+2a)}{2}$$

$$= -(w \times x) + w(L+2a) \text{ (Linear)}$$

$$F_{B(x=L+a)} = (w \times a); F_{E(x=L+2a)} = 0;$$

SF diagram is shown in Fig. 3.31(b).

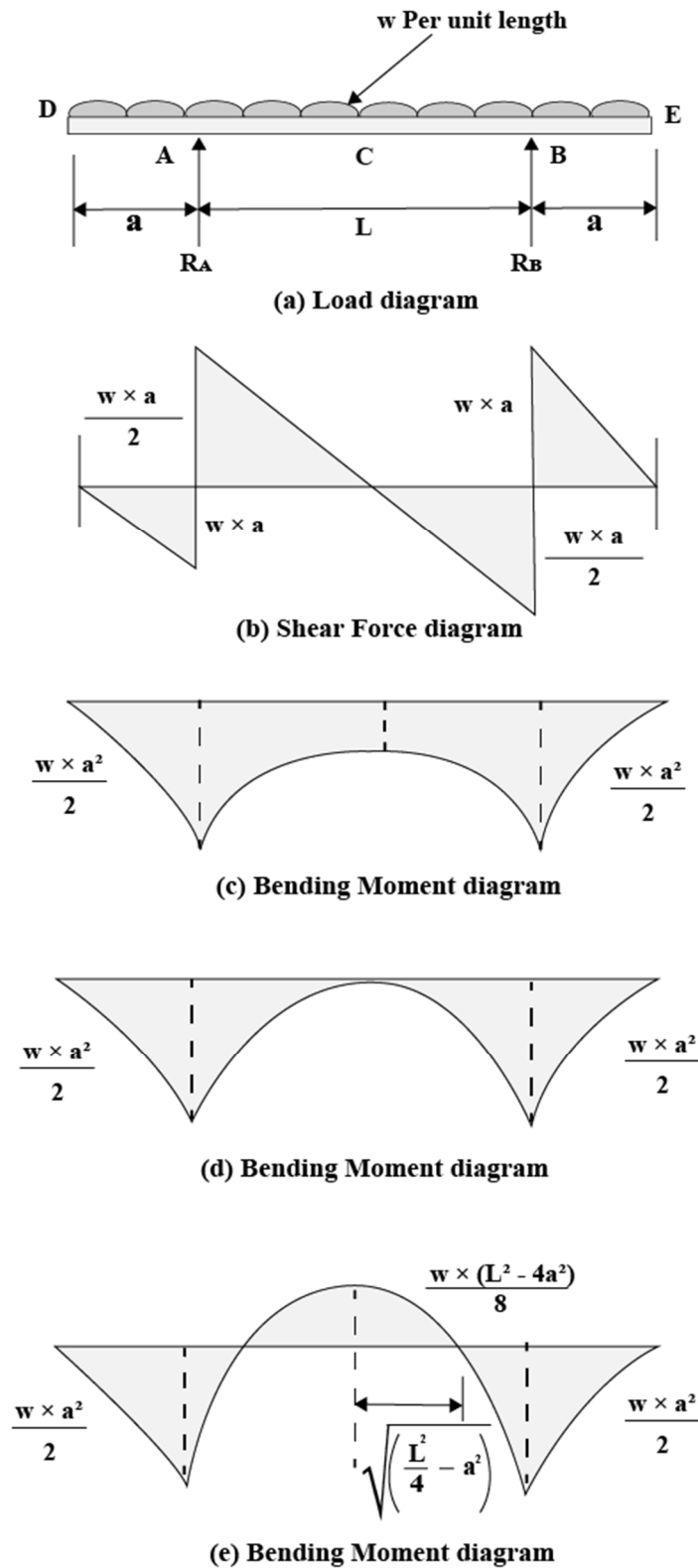


Fig. 3.31(a)-(e): SF and BM diagram for over-hanging beams

(ii) BM diagram**Section DA:**

$$M_x = -\frac{w \times x^2}{2} \text{ (Parabolic);}$$

$$M_D = 0; M_A = -\frac{w \times a^2}{2}$$

Section AB:

$$M_x = -\frac{w \times x^2}{2} + \frac{w \times (L + 2a)}{2} (x - a) \text{ (Parabolic)}$$

$$M_A = -\frac{w \times a^2}{2}; M_{B(x=L+a)} = -\frac{w \times a^2}{2}$$

Section BE:

The BM at the free end E is zero. In this case, it is convenient to start considering the BM. from end E and proceed toward the supports.

$$\text{Then } M_x = -\frac{w \times x^2}{2}.$$

At mid-point C,

$$\begin{aligned} M_{C(x=a+\frac{L}{2})} &= -\frac{w \left(a + \frac{L}{2}\right)^2}{2} + \left(\frac{w(L + 2a)}{2} \times \frac{L}{2}\right) \\ &= -\frac{w}{2} \left(a^2 + \frac{L^2}{4} + aL - \frac{L^2}{2} - aL\right) \\ M_{C(x=a+\frac{L}{2})} &= \frac{w}{8} (L^2 - 4a^2) \end{aligned}$$

The shape of the BM curve between points A to B will depend on the value of $(L^2 - 4a^2)$.

- If $(L^2 - 4a^2) < 0$, i.e., when $L < 2a$, M_c is negative, which implies that the BM will be negative throughout Fig. 3.31(c).

- If $(L^2 - 4a^2) = 0$, i.e., when $L = 2a$, M_c is zero, which implies that the BM will touch the span at the mid-point in Fig. 3.31(d).
- If $(L^2 - 4a^2) > 0$, i.e., when $L > 2a$, M_c indicates two points of contra flexure between points A and B. These points can be determined by setting the expression for the BM between A to B and B equal to zero, as shown in Fig. 3.31(e).

$$-\left(\frac{w \times x^2}{2}\right) + \left(\frac{w(L + 2a)}{2}\right)(x - a) = 0$$

$$x^2 - (2a + L)x = -a(2a + L)$$

Adding $\left(a + \frac{L}{2}\right)^2$ on both sides,

$$x^2 - 2\left(a + \frac{L}{2}\right)x + \left(a + \frac{L}{2}\right)^2 = -a(2a + L) + \left(a + \frac{L}{2}\right)^2$$

Or

$$\left(x - \left(a + \frac{L}{2}\right)\right)^2 = \frac{L^2}{4} - a^2$$

$$\left(x - \left(a + \frac{L}{2}\right)\right) = \pm \sqrt{\left(\frac{L^2}{4} - a^2\right)}$$

As $a + \frac{L}{2}$ is the distance DC, the points of contra flexure are located at a distance of

$\pm \sqrt{\left(\frac{L^2}{4} - a^2\right)}$ from the mid-point of the beam.

3.10.1 POINT OF CONTRAFLEXURE

The point at which BM becomes zero after switching from a negative to a positive sign or vice versa.

Problem 3.9

Draw SF and BM diagram for the loaded beam, as shown in Figure 3.32. Find the point of contraflexure within the span EF.

Solution:

First, evaluate the reaction forces R_E and R_F .

Considering the moments about X, we obtain

$$(R_F \times 12) + (1000 \times 5) = (3000 \times 8) + 1200 (12 + 4)$$

$$12 R_F + 5000 = 24000 + 19200$$

$$12 R_F = 24000 + 19200 - 5000$$

$$R_F = \frac{38200}{12} = 3183.3 \text{ N}$$

$$R_F = 3183.3 \text{ N}$$

$$R_E = \text{Overall load on the beam} - R_F$$

$$R_E = 5200 - 3183.3$$

$$R_E = 2016.7 \text{ N}$$

(i) SF diagram

- S.F at point A = -1000 N
- SF between points A and E remains -1000 N
- SF at E = -1000 + R_E = -1000 + 2016.7 = +1016.7 N
- SF between points E and B remains +1016.7 N
- SF at B = + 1016.7 – 3000 = - 1983.3 N
- SF between points B and F remains – 1983.3 N
- SF at F = – 1983.3 + 3183.3 = 1200 N
- SF between points F and C remains 1200 N

Fig. 3.32 shows the SF diagram

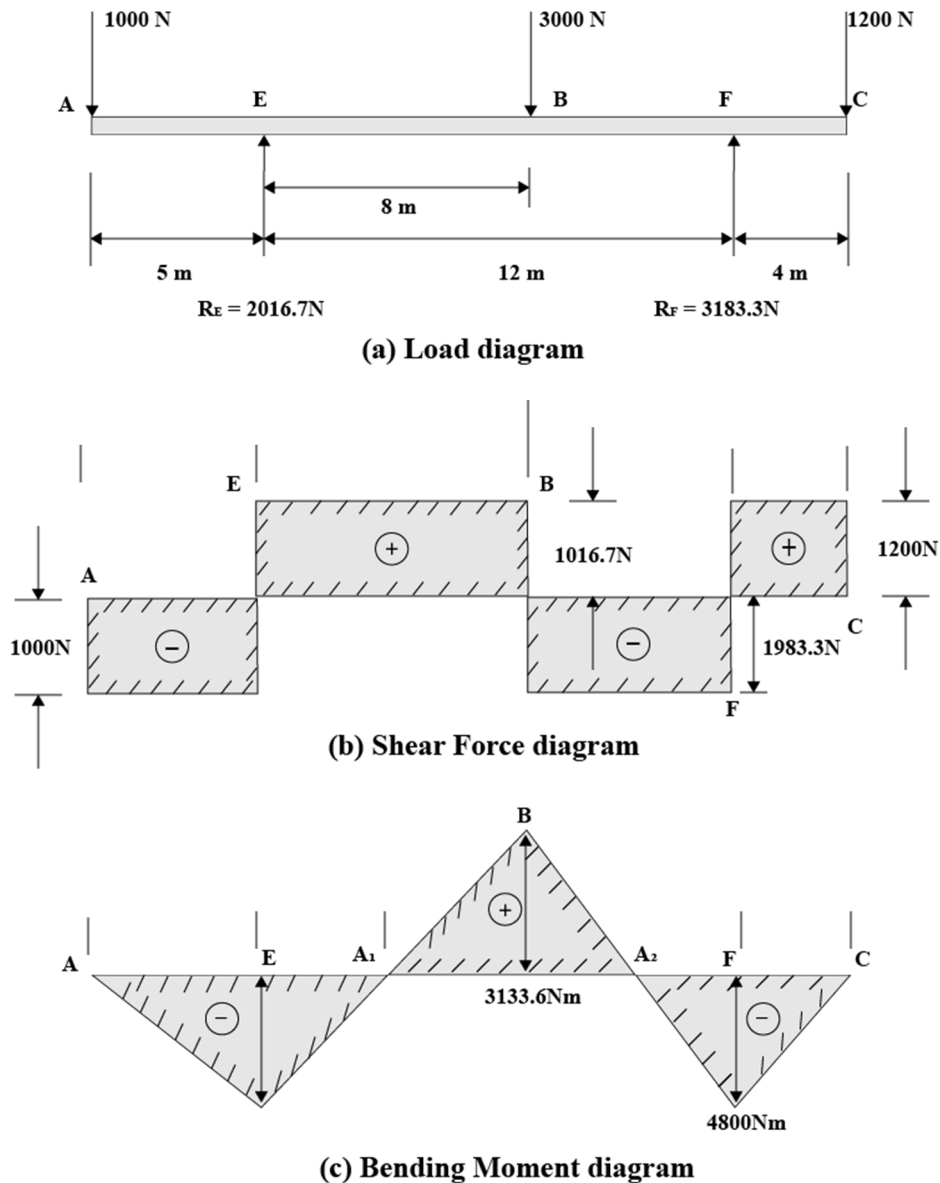


Fig. 3.32(a)-(c): SF and BM diagram for the loaded beam

(ii) B.M diagram

- B.M at point A = 0
- BM at point E = $-(1000 \times 5) = -5000 \text{ Nm}$
- BM at point B = $-(1000) \times (5 + 8) + (R_E \times 8) = -1000(13) + (2016.7 \times 8)$
 $= -13000 + 16133.6 = 3133.6 \text{ Nm}$

- BM at point F = $-1200 \times 4 = -4800$ Nm
- BM at point C = 0

(iii) Point of contraflexure

- After switching its sign, BM will have two points of contraflexure, A_1 and A_2 . Point A_1 is between E and B, whereas point A_2 is between B and F.

- Let point A_1 be x meter from E.

$$\text{Then BM at } A_1 = -1000(5 + x) + (R_E \times x) = -5000 - 1000x + 2016.7x$$

$$A_1 = -5000 + 1016.7x$$

But BM at A_1 is zero.

$$0 = -5000 + 1016.7x$$

$$5000 = 1016.7x$$

$$x = \frac{5000}{1016.7} = 4.917 \text{ m}$$

- Let point A_2 be x meter from F.

$$\text{Then BM at } A_2 = 1200(4 + x) - (R_F \times x) = 4800 + 1200x - 3183.3x$$

$$A_2 = 4800 - 1983.3x$$

But BM at A_2 is zero.

$$0 = 4800 - 1983.3x$$

$$1983.3x = 4800$$

$$x = \frac{4800}{1983.3} = 2.42 \text{ m}$$

3.11 BEAM SUBJECTED TO CONSTANT MOMENT

The beam is subject to a constant moment at a section, and the BM of the section changes abruptly in magnitude depending upon the value of the constant moment.

Problem 3.10

A 10 m long simply supported beam is hinged at A and B. At a 3 m distance from the left A end, it is exposed to a clockwise moment of 30 kNm. Draw the SF and BM diagrams.

Solution:

Fig. 3.33(a) shows the simply supported beam hinged at points A and B. The clockwise couple at point C will attempt to raise the beam at support A and lower the beam at point B support. Therefore, the reaction at A and B will be downward and upward respectively, as shown in Fig. 3.33(b).

Take moment about A, to determine the reactions of R_B .

$$\therefore R_B \times 10 - 30 = 0$$

($\because$ The moment produced by R_B is anti-clockwise whereas the moment at C is clockwise)

$$R_B \times 10 = 30$$

$$R_B = \frac{30}{10} = 3 \text{ kN}$$

$$R_B = 3 \text{ kN}$$

The absence of any external vertical load on the beam means that the reaction at point A will equal the reaction at point B but in the opposite direction.

$$\therefore R_A = \text{Total load on the beam} - R_B \quad (\because \text{Total load on beam} = 0)$$

$$R_A = - R_B$$

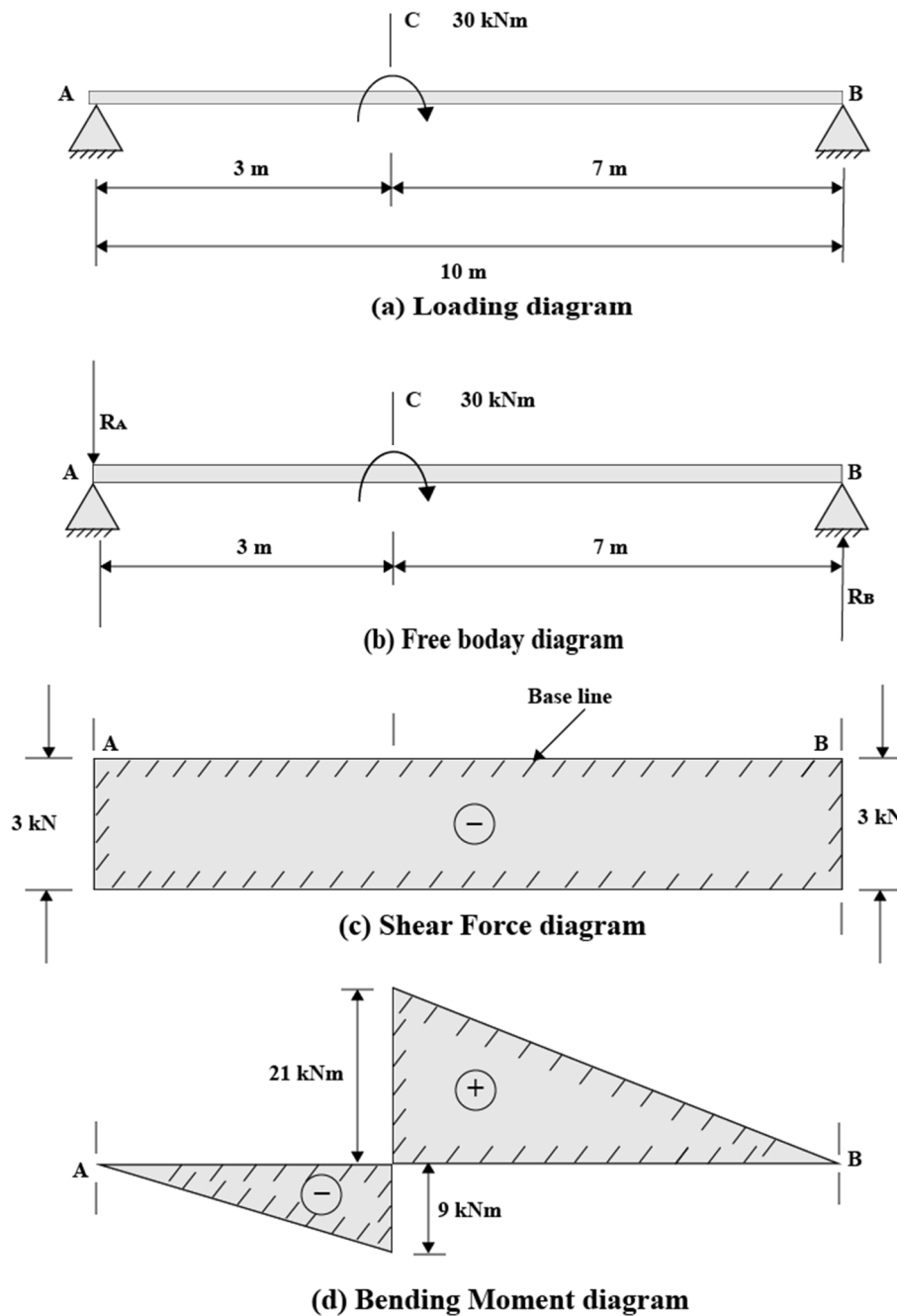


Fig 3.33(a)-(d): SF, BM and axial force diagram

(i) SF diagram

SF at A = $R_A = -3$ kN

SF is constant (i.e., equal to -3 kN) between A and B

Fig. 3.33(c) shows the SF diagram

(ii) BM diagram

BM at A = 0

BM near to the left of C = $R_A \times 3 = -3 \times 3 = -9 \text{ kNm}$

BM near to the right of C = $R_B \times 7 = 3 \times 7 = 21 \text{ kNm}$

The moment made a sudden change in BM at C.

BM at B = 0

Fig. 3.33(d) shows the BM diagram.

3.12 BEAM SUBJECTED TO INCLINED LOAD

SF is the algebraic sum of the forces on either side of any given section of the beam. When a beam supports inclined loads, these loads are divided into horizontal and vertical components for analysis. Only the vertical components will be considered for SF and BM.

Horizontal components of inclined loads exert an axial force, commonly called thrust, on the beam. The changes in axial force along different beam sections can be shown using an axial force diagram or thrust diagram.

In many instances, one end of the beam is typically hinged, while rollers support the other end. Roller supports do not offer any horizontal reaction, meaning only the hinged end will support the horizontal force.

Problem 3.11

The given case involves a beam AC with a length of 8 m, where point E is hinged and point F is supported by rollers. Figure 3.34 illustrates the presence of three inclined point loads being carried out by the beam. Draw the BM and SF and axial force diagram.

Solution:

Initially, determine the vertical components (VC) and horizontal components (HC) of the inclined loads.

- VC of force at point A = $6 \sin 30^\circ = 6 \times 0.5 = 3 \text{ kN}$
- HC of force at point A = $6 \cos 30^\circ = 6 \times 0.866 = 5.196 \text{ kN}$
- VC of force at point B = $10 \sin 60^\circ = 10 \times 0.866 = 8.66 \text{ kN}$
- HC of force at point B = $10 \cos 60^\circ = 10 \times 0.5 = 5 \text{ kN}$
- VC of force at point C = $8 \sin 45^\circ = 8 \times 0.707 = 5.656 \text{ kN}$
- HC of force at point C = $8 \cos 45^\circ = 8 \times 0.707 = 5.656 \text{ kN}$

The HC and VC of all inclined loads are shown in Fig. 3.34

The hinged end will represent the horizontal reaction (HR).

$\therefore$ H.R at E,

$$H_E = -5.196 + 5 + 5.656$$

$$H_E = 5.46 \text{ kN}$$

To determine a vertical reaction (VR), R_E and R_F considering the moments of all forces about E,

$$R_F \times 16 = (4 \times 3) + (8 \times 8.66) + (12 \times 5.656)$$

$$R_F = 9.321 \text{ kN}$$

$$R_E = \text{Overall vertical loads} - R_F$$

$$R_E = (3 + 8.66 + 5.656) - 9.321$$

$$R_E = 7.995 \text{ kN}$$

(i) AF diagram

A.F is due to HC including horizontal reaction

- A.F at E = $+ H_E = + 5.46 \text{ kN}$
- A.F remains $+ 5.46 \text{ kN}$ between E and A
- A.F at A = $H_A = + 5.46 + 5.196 = + 10.656 \text{ kN}$
- A.F remains $+ 10.656 \text{ kN}$ between A and B

- A.F at B = $H_B = + 10.656 - 5 = + 5.656$ kN
- A.F remains + 5.656 kN between B and C
- A.F at C = $H_C = + 5.656 - 5.656 = + 0$ kN
- A.F remains zero between C and F

Fig. 3.34(c) shows the AF diagram

(ii) SF diagram

Vertical loads on the beam

- SF at E = $R_E = + 7.995$ kN
- SF remains + 7.995 kN between E and A
- SF at A = $R_A = + 7.995 - 3 = + 4.995$ kN
- SF remains + 4.995 kN between A and B
- SF at B = $R_B = + 4.995 - 8.66 = - 3.665$ kN
- SF remains - 3.665 kN between B and C
- SF at C = $R_C = - 3.665 - 5.656 = - 9.321$ kN
- SF remains - 9.321 kN between C and F

Fig. 3.34(d) shows the SF diagram

(iii) BM diagram

BM due to transverse loads

- BM at E = 0
- BM at A = $(7.995 \times 4) = 31.98$ kNm
- BM at B = $(7.995 \times 8) - (3 \times 4) = 51.96$ kNm
- BM at C = $(7.995 \times 12) - (3 \times 8) - (8.66 \times 4) = 37.30$ kNm
- BM at F = 0

Fig. 3.34(e) shows the BM diagram

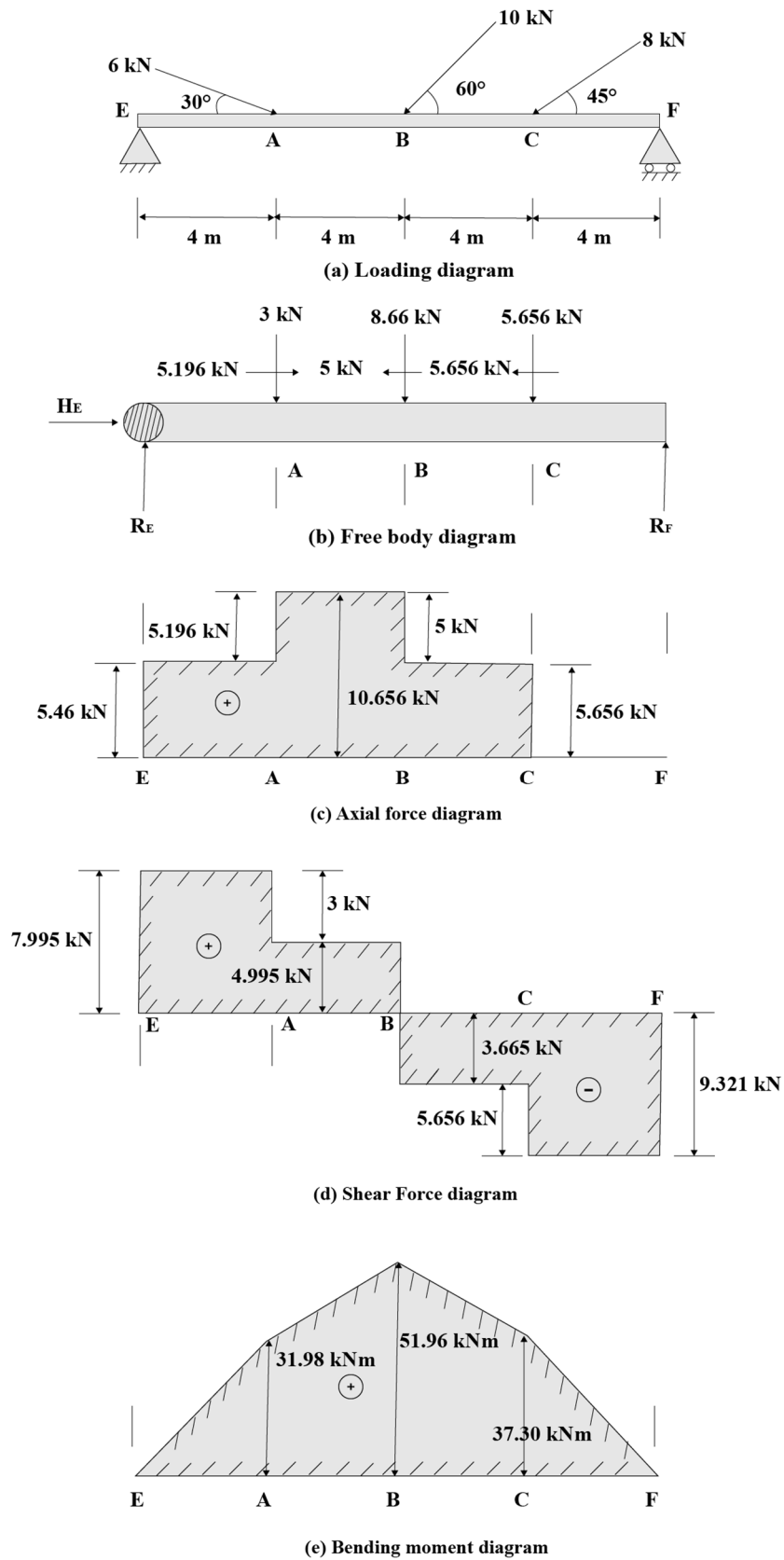


Fig 3.34: SF, BM and axial force diagram

UNIT SUMMARY

- The "Shear Force and Bending Moment" unit in this book is an essential component for understanding the mechanics of structures subjected to external loads.
- This unit begins by introducing the fundamental concepts of shear force and bending moment, elucidating their significance in structural analysis.
- Readers will explore the principles governing the distribution of these forces along the length of structural members, laying the groundwork for further analysis. Through detailed explanations and step-by-step procedures, readers will learn how to calculate shear force and bending moment at any point along a structural element, enabling them to assess the internal forces acting within the structure accurately.
- Furthermore, this unit provides comprehensive coverage of constructing shear force and bending moment diagrams, essential graphical tools for visualizing and interpreting the distribution of internal forces within a structure.
- By solving the actual problems, readers will develop proficiency in analyzing and interpreting shear force and bending moment diagrams, facilitating the design and optimization of various structural systems.
- This unit serves as a cornerstone for students and professionals in civil and mechanical engineering disciplines, equipping them with the knowledge and skills necessary for effective structural analysis and design.

EXERCISE

A. Multiple Choice Questions

- 3.1 For cantilever beam, bending moment will be zero at
(a) $1/4^{\text{th}}$ from free end (c) Center of beam
(b) Fixed end (d) Free end
- 3.2 The load which is distributed uniformly over the length of the beam, then the load is called
(a) Point load (c) Uniformly distributed load
(b) Distributed load (d) Bending loads
- 3.3 Area moment of inertia of the cross-section of the beam resist
(a) The applied normal force (c) The applied torque
(b) Both the torque and bending moment (d) The applied bending moment of the beam
- 3.4 A cantilever 9m long has uniformly distributed load over the entire length. The maximum bending moment is 8100 N-m, the rate of loading is:
(a) 200 N/m (c) 400N/m
(b) 100 N/m (d) 900 N/m
- 3.5 If a beam is fixed at both its ends, it is called a
(a) Fixed beam (c) Encastered beam
(b) Build in beam (d) All the above
- 3.6 If a beam is supported with more than two supports, it is called a
(a) Built in beam (c) Simply supported beam
(b) Continuous beam (d) Encastered beam
- 3.7 A simply supported beam of span (l) carries a point load (W) at the centre of the beam. The shear force diagram will be
(a) A rectangle (c) Two equal and opposite rectangles
(b) A triangle (d) Two equal and opposite triangles
- 3.8 A simply supported beam of span (l) carries a uniformly distributed load over the whole span. The shear force diagram will be
(a) A rectangle (c) Two equal and opposite rectangles
(b) A triangle (d) Two equal and opposite triangles

- 3.9 The bending moment on a section is maximum where shearing force is
(a) Minimum (c) Zero
(b) Maximum (d) Equal
(e) Changing sign (f) None of the above
- 3.10 The point of contraflexure depends on loading in
(a) Continuous beams (c) Overlapping beams
(b) Cantilevers beams (d) Simply supported beams
- 3.11 The bending moment at a section, where shear force is zero, will be
(a) Zero (c) Minimum
(b) Maximum (d) Either maximum or minimum
- 3.12 A beam of uniform strength is one which has same
(a) bending stress at every section
(b) Deflection throughout the beam
(c) Bending moment throughout the beam
(d) Shear force throughout the beam
- 3.13 A cantilever of length (l) carries a uniformly distributed load w N per unit length for the whole length. The shear force at the free end will be
(a) wl (c) $wl/2$
(b) $wl^2/2$ (d) Zero
- 3.14 The downward moment at the fixed end is
(a) 0 (c) Wl
(b) $Wl^2/2$ (d) $Wl/2$
- 3.15 The point of zero bending moment, where the continuous curve of bending moment changes sign is called
(a) The point of contraflexure (c) The point of virtual hinge
(b) The point of inflation (d) All of the above

Answer to Multiple-Choice Questions

3.1 (d), 3.2 (c), 3.3 (d), 3.4 (a), 3.5 (d), 3.6 (b), 3.7 (c), 3.8 (d), 3.9 (e), 3.10 (c), 3.11 (d), 3.12 (a), 3.13 (d), 3.14 (b), 3.15 (d)

B. Subjective Questions

- 3.1 What are the sign conventions for shear force and bending moment in general?
- 3.2 What do you mean by point of contraflexure? Is the point of contraflexure and point of inflexion different?
- 3.3 Explain the terms: (i) shear force, (ii) bending moment, (iii) shear force diagram and (iv) bending moment diagram.
- 3.4 How will you draw the S.F and B.M diagrams for a beam which is subjected to inclined loads?
- 3.5 What are the different types of beams? Differentiate between a point load and a uniformly distributed load.
- 3.6 How will you determine the maximum bending moment in a simply supported beam?
- 3.7 Explain briefly the relationship between shear force and bending moment at a section.

C. Numerical Problems

1. A 2m long cantilever beam carries a point load of 1 kN at its free end and of 2 kN load at a 1 m distance from the other free end. Draw S.F and B.M diagram.

(Ans: $F_{\max} = +3 \text{ kN}$; $M_{\max} = -4 \text{ kNm}$)

2. A long 4 m cantilever beam carries point loads of 1 kN, 2 kN, and 3 kN at a distance of 1 m, 2 m, and 4m respectively, from the fixed end. Draw S.F and B.M diagram.

(Ans: $F_{\max} = +6 \text{ kN}$; $M_{\max} = -17 \text{ kNm}$)

3. A 5 m long cantilever beam carries a UDL of 2 kN/m over its entire length and a point load of 4 kN at its free end. Draw S.F and B.M diagram.

(Ans: $F_{\max} = +14$ kN; $M_{\max} = -45$ kNm)

4. A 6 m long cantilever beam carries a UDL that ranges from zero at the free end to 2 kN/m at the fixed end. Draw S.F and B.M diagram.

(Ans: $F_{\max} = +6$ kN; $M_{\max} = -12$ kNm)

5. A 8 m long SSB carries point loads of 4 kN and 6 kN at a 2 m and 4 m distance from the left side end, respectively. Draw SF and BM diagram.

(Ans: $M_{\max} = +20$ kNm)

6. An SSB of length 8m rests on supports of 6m apart, the right-side end is overhanging by 2 m. The beam carries a UDL of 1500 N/m over the entire length. Draw S.F and B.M diagram and calculate the point of contraflexure.

(Ans: $M_{\max} = 5.33$ kNm)

7. An SSB of length 10 m carries point loads of 30 kN and 50 kN at a distance of 3 m and 7 m from the left end, respectively. Draw the S.F. and B.M diagrams for the beam.

(Ans: $M_{\max} = +132$ kNm)

KNOW MORE

- *Structural Analysis:* Shear force and bending moment diagrams are used to analyze the internal forces and moments in structural members such as beams and frames. By plotting these diagrams, engineers can determine the critical locations where the shear force and bending moment are maximum, aiding in the design process.

- *Design Verification:* Engineers use shear force and bending moment diagrams to verify the adequacy of a structural design. By comparing the calculated internal forces and moments to the design requirements and material properties, they can ensure that the structure will safely withstand the applied loads.
- *Selection of Structural Members:* Shear force and bending moment diagrams help in selecting appropriate structural members for a given application. By analyzing the distribution of internal forces and moments along the length of a beam, engineers can choose beams with suitable cross-sectional dimensions and material properties to meet the design requirements.
- *Load Distribution:* Shear force diagrams provide information about how external loads are distributed along the length of a beam. Engineers can use this information to optimize load distribution and minimize the risk of structural failure due to uneven loading.
- Overall, shear force and bending moment diagrams play a crucial role in structural engineering by providing valuable insights into the behavior of structural members under various loading conditions and guiding the design, analysis, and rehabilitation of structures.

REFERENCE AND SUGGESTED READINGS

1. J.L. Meriam and L.G. Kraige, Engineering mechanics: Dynamic, Wiley Publisher, New York
2. D.S. Bedi, “Engineering Mechanics”; Khanna publications, New Delhi.
3. Bansal RK, “A text book of Engineering Mechanics”; Laxmi publications, New Delhi
4. Bedi D.S., Strength of Materials, Khanna Publishing House, Delhi, Ed. 2018

5. Rattan S.S., Strength of Materials, McGraw Hill Education; New Delhi.
6. Subramaniam R, Strength of Materials, Oxford University Press.

Dynamic QR Code for Further Reading

Shear Force and Bending
Moment diagram



Experiment on Shear Force
and Bending Moment



Types of beams and loading



Slope and deflection of beams



Bending Moment
sign convention



Point to keep in mind for
drawing SF and BM diagram



Relation between shear force,
bending moment, and loading rate



Concept of shear force and
bending moment



4

BENDING AND SHEAR STRESSES IN BEAM

UNIT SPECIFICS

Upon completion of this chapter, the reader will have acquired the following abilities:

Describe the behaviour of beams under lateral loads,

- *Derive the bending equation and discuss the significance of assumptions made in deriving the equation,*
- *Draw the bending stress distribution diagram and calculate the maximum normal stresses in a given section,*
- *Explain the behaviour of composite beams and determine the stresses in such beams,*
- *Derive the equation for shear stress distribution and calculate the maximum shear stress,*
- *Explain the concept of unsymmetrical bending and determine the stresses in such cases.*

The "Bending and Shear Stresses in Beams" unit in this book offers a comprehensive exploration of the mechanical behavior of beams under bending and shear loads. Readers will delve into the fundamental principles governing the distribution of bending and shear stresses in the beam structures, gaining insight into how these stresses influence beam performance and design. The unit covers the

calculation and graphical representation of bending and shear stress distributions, providing readers with practical tools for analyzing and designing beam structures. Apart from this at the end of the unit, a succinct recapitulation of the overall broad concepts is provided in the form of a unit summary. Besides, a large number of multiple-choice questions as well as descriptive type questions with Bloom's taxonomy action verbs are included. It is to be noted that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge. Video resources along with QR codes are given for getting more information on various topics of interest which can be surfed or scanned through mobile phones for viewing.

RATIONALE

The concepts of bending and shear stresses in beams are foundational in structural engineering, crucial for analyzing the behavior of beams under various loading conditions. Bending stress, also known as flexural stress, arises due to the bending moment induced by external loads applied to the beam. It causes tensile and compressive forces along the cross-section of the beam, leading to elongation on one side (tensile stress) and contraction on the other (compressive stress). Shear stress, on the other hand, on a plane perpendicular to the longitudinal axis of the beam and is caused by shear forces acting parallel to the cross-section. Bending stress and shear stress work in tandem to determine the overall structural integrity of a beam. Engineers use mathematical models and formulas, such as the Euler-Bernoulli beam theory to calculate bending and shear stresses at different sections along the length of the beam. Moreover, this knowledge is vital in various engineering applications.

PRE-REQUISITES

Before reading this unit, the students are advised to revisit the following:

Engineering Mechanics: Sem I

UNIT OUTCOMES

After completing this unit, readers will be able to:

U4-O1: Understand the concepts of bending stress and shear stress in beams.

U4-O2: Identify the origins and distributions of bending and shear stresses in beam structures.

U4-O3: Learn how bending and shear stresses influence beam deformation and failure.

U4-O4: Explore mathematical models and formula for analyzing bending and shear stresses in beams.

U4-O5: Examine the factors affecting bending and shear stresses, including geometry, material properties, and applied loads.

Unit-4 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES <i>(1-Weak Correction; 2- Medium Correction; 3- Strong Correction)</i>				
	CO-1	CO-2	CO-3	CO-4	CO-5
U4-O1	3	1	1	2	3
U4-O2	3	2	2	2	2
U4-O3	3	1	1	2	1
U4-O4	3	3	2	2	3
U4-O5	3	2	1	2	2

4.1 INTRODUCTION

When an external load is applied to a beam, it induces shear forces and bending moments at various sections along the length of the beam. These forces and moments lead to deformations in the beam, and the material of the beam offers resistance to these deformations in the form of stresses. The stresses resulting from the bending moment are referred to as bending stresses. This chapter will delve into the principles of pure bending, the derivation of the formula for calculating bending stresses, bending stress analysis for unsymmetrical and symmetrical beam sections, and the structural strength of the beam.

4.2 SIMPLE BENDING OR PURE BENDING

If a beam experiences a constant bending moment along its length without any shear force, it will develop stresses only due to the bending moment within the length of the beam. This condition is referred to as pure bending or simple bending. The stresses that emerge within this beam section are specifically termed as bending stresses.

Fig. 4.1(a) depicts a beam supported at points A and B, with equal overhanging lengths on each side of the supports. The point loads P is applied at each overhanging section. The corresponding shear force and bending moment diagrams for the beam are shown in Fig. 4.1(b) and 4.1(c), respectively. These diagrams reveal that there is no shear force between points A and B, while the bending moment remains constant within this span. This indicates that the beam is under a constant bending moment between points A and B, which defines the condition known as "pure bending" or "simple bending" for the beam between A and B.

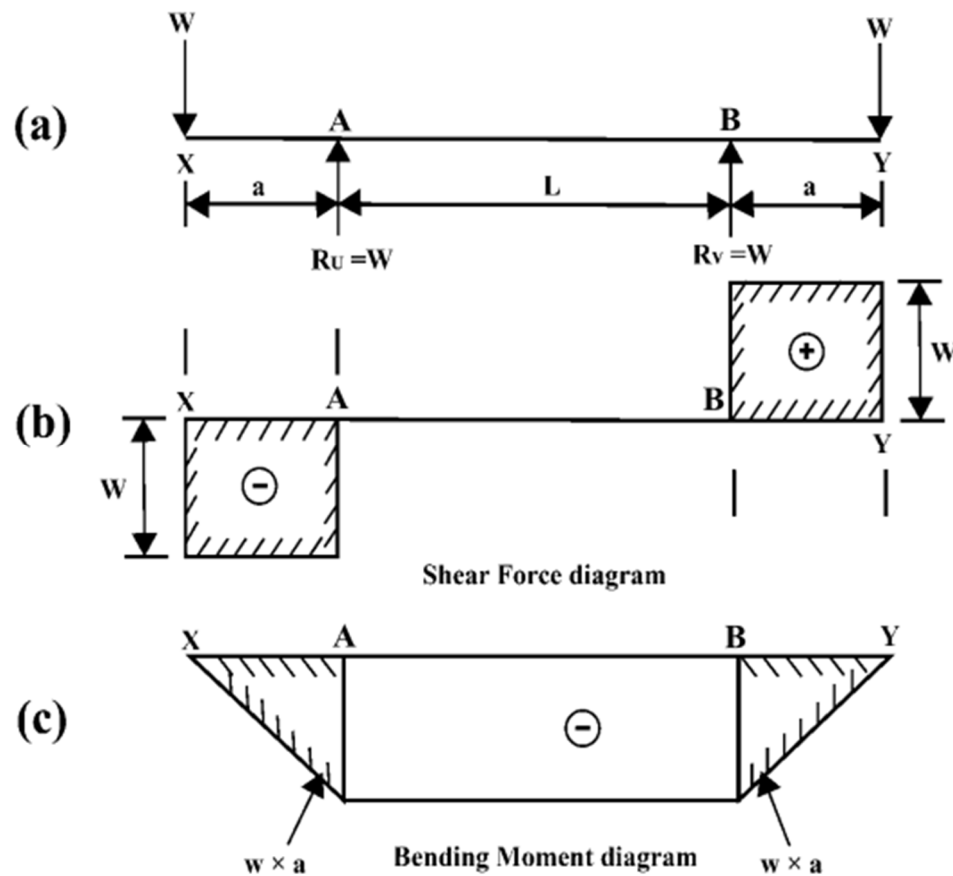


Fig. 4.1(a)-(c): Specific loading condition where the beam undergoes pure bending

4.3 THEORY OF SIMPLE BENDING ASSUMPTIONS

The theory of simple bending makes the following assumptions:

- The material of the beam is perfectly isotropic and homogenous.
- The material remains within its elastic limit and follows the Hooke's law.
- Young's modulus of elasticity (E) is same for tension and compression.
- Transverse sections, which were initially plane before bending, remain plane even after bending.
- The beam is in a state of equilibrium, i.e., no net forces are causing pulling or pushing within the beam section.

4.4 THEORY OF SIMPLE BENDING

Consider a small length of a simply supported beam subjected to a bending moment, as shown in Fig. 4.2(a). Now consider two sections, EF and GH, which are normal to the axis of the beam XY. Due to the action of the bending moment, the beam as a whole will bend, as shown in Fig. 4.2(b).

Consider a segment ' dx ' which forms a circular beam curvature. Upon closer consideration, it becomes evident that the layers of the beam, which were initially of equal length, no longer maintain uniform lengths. The uppermost layer of the beam EG has experienced compression, resulting in its reduction to $E'G'$. As we proceed downward through the lower layers of the beam, it is evident that they have also experienced compression through a decreasing degree. This pattern continues until we reach layer XY, which, despite bending into the shape $X'Y'$, has retained its original length without any change.

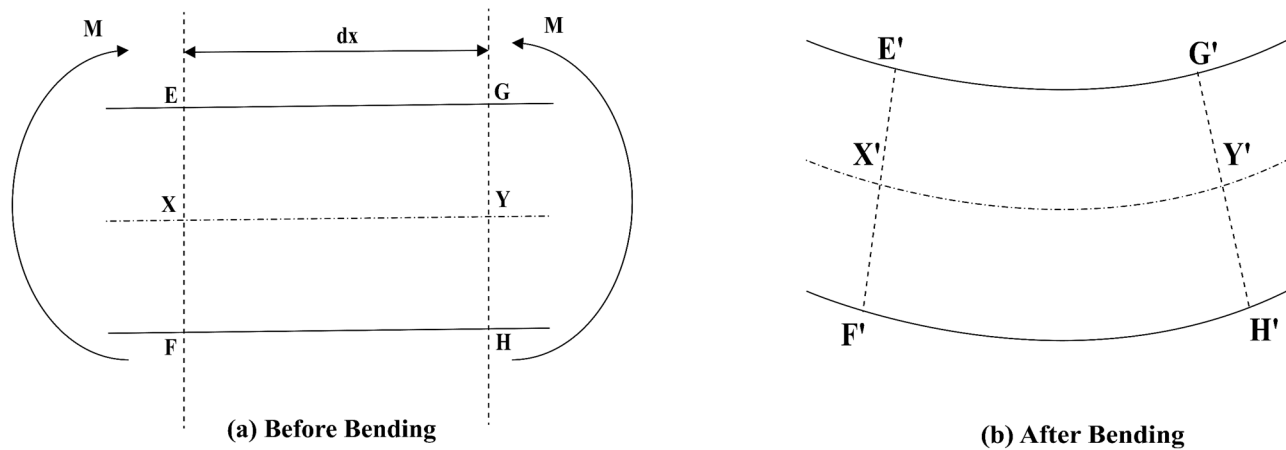


Fig. 4.2(a)-(b): Figure showing the simple bending

When proceed deeper into the lower layers, it observed that these layers have been subjected to tension, leading to elongation. The degree of extension increases as it progresses downwards until it reaches the lowest layer, FH, which has undergone significant stretching and transforming into F'H'.

It is observed that the layers above XY have experienced compression, while those below XY have undergone stretching. The extent of compression or stretching depends on the layer's position relative to XY. The layer XY, which remains unaffected by compression or stretching, is referred to as the neutral layer. This theory of bending is known as the simple bending theory.

4.5 BENDING STRESS EXPRESSION

Consider a small beam of length ' $d\mathbf{x}$ ' which is exposed to a bending moment, as shown in Fig. 4.3(a). Due to the influence of this moment, this segment undergoes deformation, bending into an arc with its center at point O, as illustrated in Fig. 4.3(b).

Let, M represents the applied bending moment on the beam.

θ denotes the angle formed at the centre by the arc formed during bending.

R represents the radius of curvature of the beam.

y represents the distance of any point from the neutral axis.

Now, consider a layer UV positioned at a distance y from the neutral axis XY of the beam. Following the bending process, this layer undergoes compression and transforms into the shape U'V', as shown in Fig. 4.3(b).

The reduction in the length of this layer is given by

$$\delta l = UV - U'V'$$

$$\therefore \text{Strain } \epsilon = \frac{\text{Change in length}}{\text{Original Length}} = \frac{\delta l}{l} = \frac{UV - U'V'}{UV}$$

By analyzing the geometry of the curved beam, it can be seen that the two sections, $OU'V'$ and $OX'Y'$, exhibit similarity.

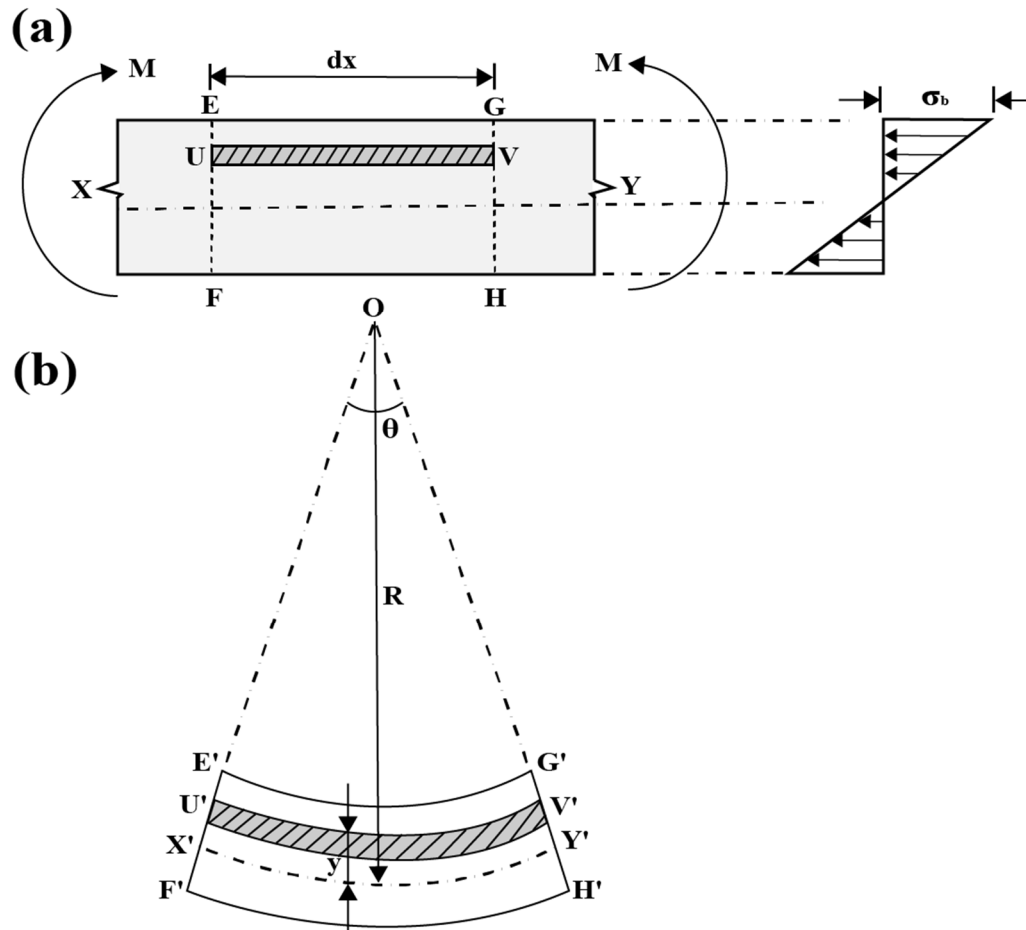


Fig. 4.3(a)-(b): Figure showing the Bending stress

$$\therefore \frac{U'V'}{X'Y'} = \frac{R - y}{R}$$

$$\text{or} \quad 1 - \frac{U'V'}{X'Y'} = 1 - \frac{R - y}{R}$$

$$\text{or} \quad \frac{X'Y' - U'V'}{X'Y'} = \frac{y}{R}$$

$$\frac{UV - U'V'}{UV} = \frac{y}{R} \quad (UV = X'Y' = \text{Neutral axis})$$

$$\epsilon = \frac{y}{R} \quad \therefore \epsilon = \frac{UV - U'V'}{UV}$$

Therefore, it becomes evident that the strain (ϵ) within a layer is directly proportional to its distance from the neutral axis. Also the bending stress,

$$\sigma = \text{Strain} \times \text{Elasticity} = \epsilon \times E$$

$$\sigma = \left(\frac{y}{R}\right) \times E \quad \left(\because \epsilon = \frac{y}{R}\right)$$

Since both R and E remain constant in this equation, the stress at any given point is directly proportional to y , which is the distance of the point from the neutral axis.

This expression can also be expressed as follows.

$$\frac{\sigma}{y} = \frac{E}{R} \quad \text{or} \quad \sigma = \frac{E}{R} \times y \quad \dots (4.1)$$

SOLVED PROBLEM

Problem 4.1

Determine the minimum radius of a drum around which a 3mm diameter copper wire must be wound to ensure that the maximum bending stress in the wire does not exceed 100 MPa, given the modulus of elasticity of copper as 150 GPa.

Solution:

Diameter of the copper wire (d) = 3mm;

Maximum bending stress in the wire (σ) = 100 MPa = 100N/mm²

Modulus of elasticity of copper (E) = 150GPa = 150 × 10³ N/mm²

We know that the distance between the neutral axis of a wire and its extreme fibre

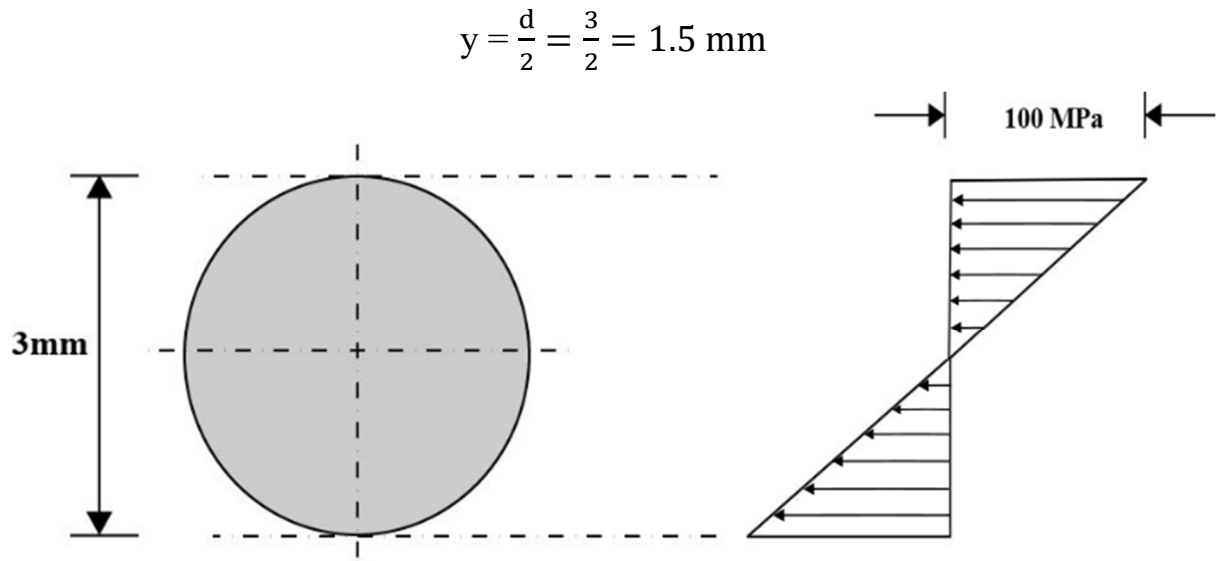


Fig. 4.4 Drum wound with copper wire

∴ Minimum radius (R) of the drum

$$R = \frac{y}{\sigma} \times E$$

$$\therefore \left(\frac{\sigma_b}{y} = \frac{E}{R} \right)$$

$$R = \frac{1.5}{100} \times 150 \times 10^3$$

$$R = 2.25 \times 10^3 \text{ mm} = 2.25 \text{ m}$$

Problem 4.2

Find the value of Young's modulus for the material of a metallic rod, which has a diameter of 15mm and is bent into a circular shape with a radius of 8m, given that the maximum bending stress in the rod is 150 MPa.

Solution:

Diameter of the metallic rod (d) = 15mm;

Radius of the circular shape (R) = 8m = 8×10^3 mm

Maximum bending stress in the rod (σ) = 150 MPa = 150N/mm²

We know that the distance between the neutral axis of a wire and its extreme fibre

$$y = \frac{d}{2} = \frac{15}{2} = 7.5 \text{ mm}$$

∴ Young's modulus of the rod material

$$E = \frac{\sigma}{y} \times R \quad \because \left(\frac{\sigma}{y} = \frac{E}{R} \right)$$

$$E = \frac{150}{7.5} \times (8 \times 10^3)$$

$$E = 160 \times 10^3 \text{ N/mm}^2 = 160 \text{ GPa}$$

4.6 POSITION OF THE NEUTRAL AXIS

In any normal cross-section of a beam, the neutral axis of a section is defined as the line where the neutral layer intersects. As discussed in section 4.4, one side of this neutral axis experiences compressive stress, while the other side undergoes tensile stress. There is no stress in the neutral axis. There is no extension and contraction in the neutral axis.

Consider the beam section in Fig. 4.5, where NA is the neutral axis of the section. Consider a small layer EF of the beam section which is located at a distance y from the neutral axis, as shown in Fig. 4.5.

Let, δa be the area of the layer EF

As discussed in section 4.5, Eqn (4.1) the stress intensity in the layer EF is,

$$\sigma = \frac{E}{R} \times y$$

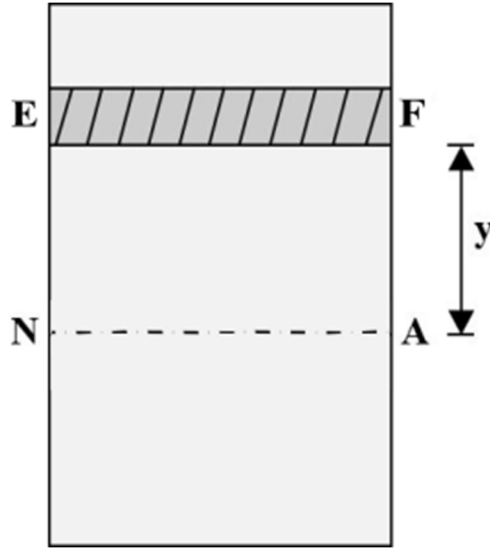


Fig. 4.5: Position of the neutral axis

∴ The load on the layer EF = Area × Intensity of stress

$$\delta F = \frac{E}{R} \times y \times \delta a$$

The load in the beam section.

$$F = \sum y \times \frac{E}{R} \times \delta a \quad \dots (4.2)$$

As the section remains in equilibrium, the total force over the entire section is .

$$\therefore \frac{E}{R} \times \sum y \times \delta a = 0$$

$$\text{Or} \quad \sum y \times \delta a = 0 \quad \because \left(\frac{E}{R} \text{ not equal to zero} \right)$$

Upon closer consideration, it becomes clear that $y \times \delta a$ represents the moment of the area about the neutral axis, and $\sum y \times \delta a$ signifies the moment of the entire cross-sectional area about the neutral axis. Consequently, the neutral axis of the section is positioned in such a way that the moment of the whole area about this axis equals zero.

The moment of any area about an axis through its central axis always intersects its centroid. Therefore, to determine a neutral axis of a section, first calculate its centroid and then draw a line passing through this centroid perpendicular to the plane of bending. This line represents the neutral axis of the section.

4.7 MOMENT OF RESISTANCE

As discussed earlier in section 4.4, one side of the neutral axis experiences compressive stress, while the other side experiences tensile stress. These stresses create a couple which is equal to the external moment (M). This moment, which opposes the external bending moment, is referred to as the moment of resistance.

Consider the beam section with NA as the neutral axis of the section (refer Fig. 4.5). The length EF is located at a distance y from the neutral axis.

∴ The load on the layer EF = Area $\times$ Intensity of stress

$$\delta F = \frac{E}{R} \times y \times \delta a$$

The moment of this load about the neutral axis NA is,

$$\delta M = \frac{E}{R} \times y \times \delta a \times y$$

$$\delta M = \frac{E}{R} \times y^2 \times \delta a \quad \dots (4.3)$$

The total moment of the section about the neutral axis is

$$M = \frac{E}{R} \times \int y^2 \times \delta a = \frac{E}{R} \times \int y^2 \times \delta a \quad \dots (4.4)$$

The expression $I = \int y^2 \times \delta a$ signifies the moment of inertia of the area of the cross section about the neutral axis thus,

$$M = \frac{E}{R} \times I \quad \text{(Where I = Moment of inertia)}$$

$$\text{or} \quad \frac{M}{I} = \frac{E}{R} \quad \dots (4.5)$$

From Eqn (4.1) and (4.5), we get that,

$$\begin{aligned} \frac{\sigma}{y} &= \frac{E}{R} \\ \frac{M}{I} &= \frac{\sigma}{y} = \frac{E}{R} \end{aligned} \quad \dots (4.6)$$

It is the important equation from the simple bending theory which is most commonly used in the design of beams.

4.8 SECTION MODULUS

We have already discussed in section 4.7 the relationship for finding the bending stress on the extreme fibre of a section, which is given by

$$\frac{M}{I} = \frac{\sigma}{y} \quad (\text{or}) \quad M = \sigma \times \frac{I}{y} \quad \dots (4.7)$$

From this relation, we find that the stress in a section is proportional to its distance from the centre of gravity. If $y_{\max}$ is the distance between the neutral axis and the extreme position of the section, then

$$\text{Where,} \quad M = \sigma_{\max} \times \frac{I}{y_{\max}}$$

$$M = \sigma_{\max} \times Z$$

From these both equations, we get

$$\therefore \quad Z = \frac{I}{y_{\max}}$$

which is known as the section modulus. The general practice of writing the above equation is

$$M = \sigma \times Z \quad \dots (4.8)$$

Where,

σ denotes the maximum stress, which can be either tensile or compressive.

4.9 SECTION MODULUS FOR VARIOUS BEAM SECTIONS

We know that if the section of a beam is symmetrical, its centre of gravity and the neutral axis will lie in the middle of its depth. We shall now consider the section modulus of the following sections:

- Rectangular section
- Hollow rectangular section
- Circular section
- Hollow circular section

4.9.1 RECTANGULAR SECTION

The moment of inertia of a rectangular section about the axis through its C.G is given by,

$$I = \frac{bd^3}{12} \quad \dots (4.9)$$

Where ‘d’ is the depth of the section and the ‘b’ is the width of the section.

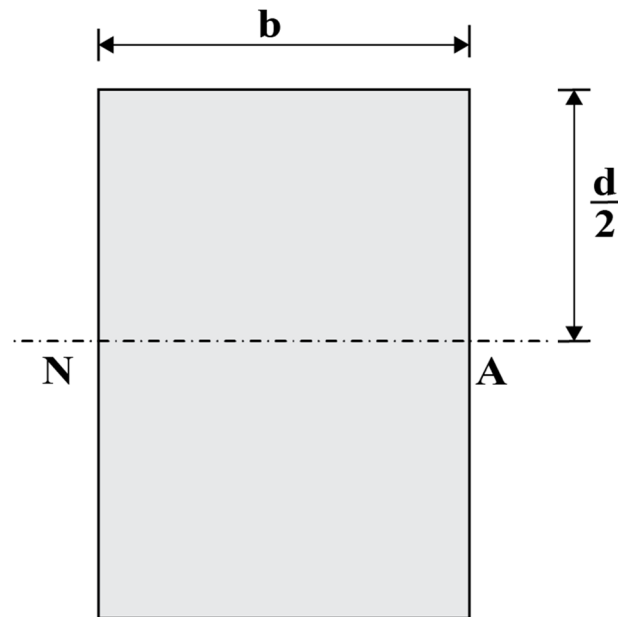


Fig. 4.6: Section modulus of the rectangular section

The distance of the outermost layer from the neutral axis (NA) is given by,

$$y_{\max} = \frac{d}{2} \quad \dots (4.10)$$

∴ Section modulus is given by,

$$Z = \frac{I}{y_{\max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)} = \frac{bd^3}{12} \times \frac{2}{d}$$

$$Z = \frac{bd^2}{6} \quad \dots (4.11)$$

4.9.2 HOLLOW RECTANGULAR SECTION

The moment of inertia of a hollow rectangular section about the axis through its C.G is given by,

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

Thus, The depth of the inner section as 'd' and the width of the inner section is 'b'.

$$I = \frac{1}{12} [BD^3 - bd^3] \quad \dots (4.12)$$

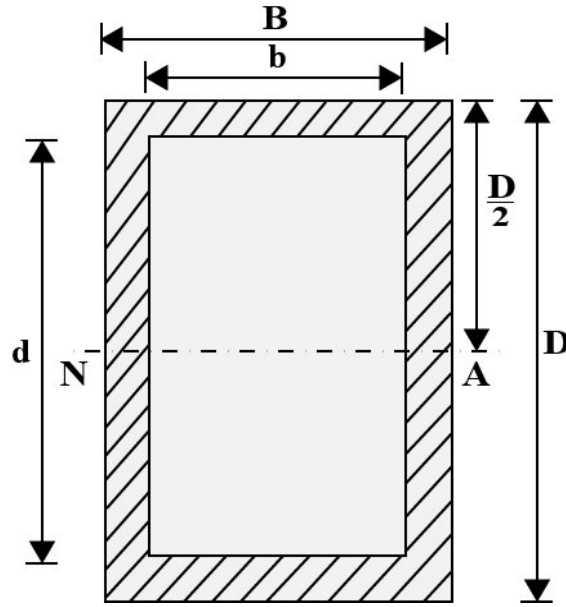


Fig. 4.7: Section modulus of the hollow rectangular section

The distance of the outermost layer from the neutral axis (NA) is given by,

$$y_{\max} = \frac{D}{2} \quad \dots (4.13)$$

∴ Section modulus is given by,

$$Z = \frac{I}{y_{\max}} = \frac{[BD^3 - bd^3]/12}{\left(\frac{D}{2}\right)} = \frac{[BD^3 - bd^3]}{12} \times \frac{2}{D}$$

$$Z = \frac{[BD^3 - bd^3]}{6D} \quad \dots (4.14)$$

4.9.3 CIRCULAR SECTION

The moment of inertia of a circular section about the axis through its C.G given by,

$$I = \frac{\pi}{64} d^4 \quad \dots (4.15)$$

Where 'd' diameter of the circular section

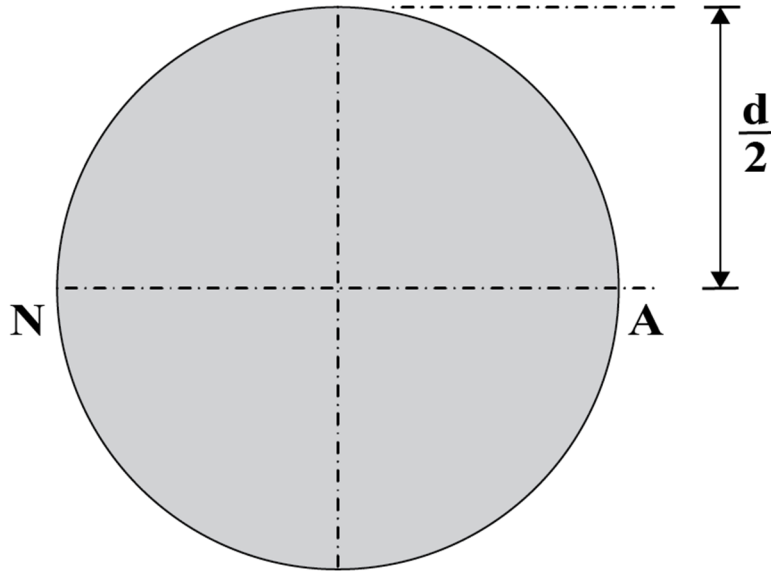


Fig. 4.8: Section modulus of the circular section

The distance of the outermost layer from the neutral axis (NA) is given by,

$$y_{\max} = \frac{d}{2} \quad \dots (4.16)$$

∴ Section modulus is,

$$Z = \frac{I}{y_{\max}} = \frac{\pi d^4 / 64}{\left(\frac{d}{2}\right)} = \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$Z = \frac{\pi d^4}{32} \quad \dots (4.17)$$

4.9.4 HOLLOW CIRCULAR SECTION

The moment of inertia of a hollow circular section about the axis through its C.G is given by,

$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

Where D is outer diameter and d is the inner diameter of the circular section.

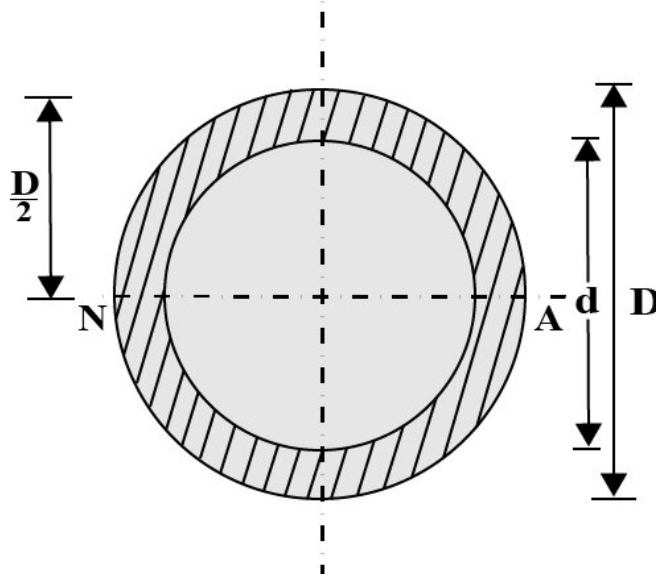


Fig. 4.9: Section modulus of the hollow circular section

$$I = \frac{\pi}{64} [D^4 - d^4] \quad \dots (4.18)$$

The distance of the outermost layer from the neutral axis is given by,

$$y_{\max} = \frac{D}{2} \quad \dots (4.19)$$

∴ Section modulus is given by,

$$Z = \frac{I}{y_{\max}} = \frac{\pi[D^4 - d^4]}{64 \times \left(\frac{D}{2}\right)} = \frac{\pi[D^4 - d^4]}{64} \times \frac{2}{D}$$

$$Z = \frac{\pi[D^4 - d^4]}{32D} \quad \dots (4.20)$$

Problem 4.3

A rectangular beam of 300 mm deep and 500 mm wide is simply supported over a span of 10 m. Find the uniformly distributed load per metre that the beam may carry if bending stress does not exceed 130 N/mm^2 .

Solution

Depth of the beam, $d = 300 \text{ mm}$

Width of the beam, $b = 500 \text{ mm}$

Span length of the beam, $L = 10 \text{ m}$

Maximum bending stress in the beam $\sigma_{\max} = 130 \text{ N/mm}^2$

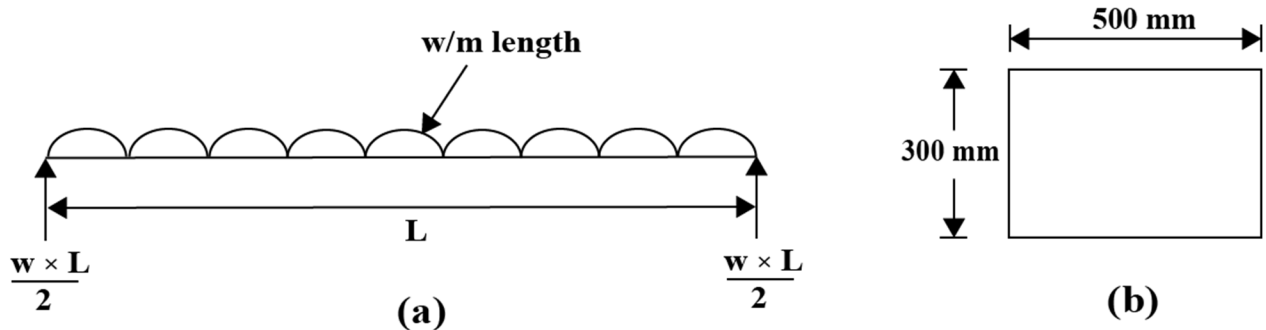


Fig. 4.10(a)-(b): Rectangular beam simply supported over a span

Let, w = uniformly distributed load per m length over the beam

The section modulus of the rectangular section is

$$Z = \frac{bd^2}{6} = \frac{500 \times 300^2}{6} = 7500000 \text{ mm}^3$$

The maximum bending moment for a simply supported beam carrying the uniformly distributed load, as shown in Fig. 4.10, is the centre of the beam. It is given by

$$M = \frac{w \times L^2}{8} = \frac{w \times 10^2}{8} = \frac{w \times 100}{8} \quad (\because L = 10 \text{ m})$$

$$M = 12.5 \times w = 12.5 w \times 1000$$

$$M = 12500 w \text{ N.mm}$$

Now, using the section modulus equation, we get,

$$M = \sigma_{\max} \times Z$$

$$12500 w = 130 \times 7500000$$

$$w = \frac{130 \times 7500000}{12500}$$

$$w = 78000 \text{ N/mm}$$

Problem 4.4

A beam is simply supported and carries a uniformly distributed load of 50 kN/m over the whole span. The cross-section of the beam is rectangular, having a depth of 600 mm. If the maximum stress in the material of the beam is 130 N/mm² and the moment of inertia of the section is $8 \times 10^8 \text{ mm}^4$, find the length of the beam.

Solution:

Uniformly distributed load (w) = 50 kN/m

Depth of the beam (d) = 600 mm

Maximum bending stress in the beam ($\sigma_{\max}$) = 130 N/mm²

Moment of inertia of the section (I) = $8 \times 10^8 \text{ mm}^4$

The section modulus is given by

$$Z = \frac{I}{y_{\max}}$$

Where,

$$y_{\max} = \frac{d}{2} = \frac{600}{2} = 300 \text{ mm}$$

$$\therefore Z = \frac{8 \times 10^8}{300} = 26.67 \times 10^5 \text{ mm}^3$$

The maximum bending moment for a simply supported beam, carrying a uniform distributed load over the whole span is at the centre of the beam, which is equal to

$$\frac{wL^2}{8}$$

$$M = \frac{wL^2}{8} = \frac{50000 \times L^2}{8}$$

$$M = 6250 L^2 \text{ N.m} = 6250 L^2 \times 1000 \text{ N.mm}$$

Now, using the given equation, we get,

$$M = \sigma_{\max} \times Z$$

$$6250 L^2 \times 1000 = 130 \times 26.67 \times 10^5$$

$$L^2 = \frac{130 \times 26.67 \times 10^5}{6250 \times 1000} = 54.08$$

$$L = \sqrt{55.47}$$

$$L = 7.448 \text{ m}$$

Problem 4.5

A timber beam of a rectangular section of length 10m is simply supported at both the ends. The beam carries a uniform distributed load of 15 kN/m over the entire length and a point load of 12 kN at 4 m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 10 N/mm², find the suitable dimensions of the section.

Solution:

Length of the rectangular section beam (L) = 10 m

Uniform distributed load over the entire beam (w) = 15 kN/m = 15000 N/m

Point load at 4m from the left support (w) = 12 kN = 12000 N

Beam depth = 2 × width of beam

$$\therefore d = 2b$$

Maximum bending stress in the beam, ($\sigma_{\max}$) = 10 N/mm²

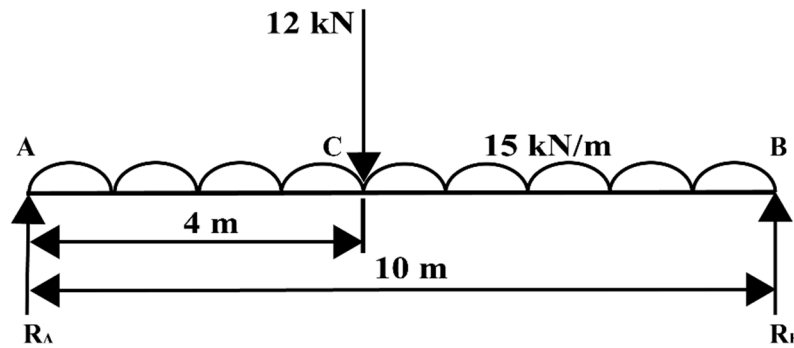


Fig. 4.11: Simply supported timber beam under uniform and point loads

First, calculate the section where the bending moment is maximum, the shear force will be zero or change its sign. Now, the equation of pure bending can be used to find the cross-section dimensions. Calculate the reaction R_A and R_B of the beam.

Taking moments about A, we get

$$R_B \times 10 = (15000 \times 10 \times 5) + (12000 \times 4)$$

$$R_B = \frac{(750000) + (48000)}{10}$$

$$R_B = 79800 \text{ N}$$

$$R_A = \text{Total load} - R_B$$

$$R_A = (15000 \times 10 + 12000) - 79800 = 82200 \text{ N}$$

Now

$$\text{shear force on the left of C} = 82200 - (15000 \times 4) = 22200 \text{ N}$$

$$\text{shear force on the right of C} = 22200 - 12000 = 10200 \text{ N}$$

$$\text{shear force at B} = R_B = 79800 \text{ N}$$

The shear force is changing sign between section BC; hence, the shear force is zero at some point between B and C.

Let the shear force be zero at x meter from B.

Equating the shear force at this section to zero, we have

$$15000 \times x - R_B = 0$$

$$15000 \times x - 79800 = 0$$

$$15000x = 79800$$

$$x = \frac{79800}{15000}$$

$$x = 5.32 \text{ m}$$

∴ Maximum bending moment is at 5.32m from B.

$$\therefore \text{Maximum bending moment (M)} = (R_B \times 5.32) - \left(15000 \times 5.32 \times \frac{5.32}{2}\right)$$

$$M = (79800 \times 5.32) - (212268)$$

$$M = (424536) - (212268)$$

$$M = 212268 \text{ Nm} = 212268 \times 1000 \text{ N.mm}$$

Section modulus for the rectangular beam is given by,

$$Z = \frac{b d^2}{6} = \frac{b \times (2b)^2}{6} = \frac{2 b^3}{3}$$

Now, using the given equation, we get,

$$M = \sigma_{\max} \times Z$$

$$212268 \times 1000 = 10 \times \frac{2 b^3}{3}$$

$$b^3 = \frac{212268 \times 1000 \times 3}{20}$$

$$b^3 = 31840200$$

$$b = 316.95 \text{ mm}$$

And

$$d = 2 \times b = 2 \times 316.95 = 633.90 \text{ mm}$$

Problem 4.6

A rolled steel joist of the I section is shown in Fig. 4.12. This beam carries a uniform distributed load of 50 kN/m over span of 15 m supported at the edges, calculate the maximum stress produced due to bending.

Solution:

Uniform distributed load over the span (P) = 50 kN/m = 50000 N/m

Length of the span (L) = 15 m

Distance of the outermost layer from the neutral axis, ($y_{\max}$) = $\frac{500}{2} = 250$ mm

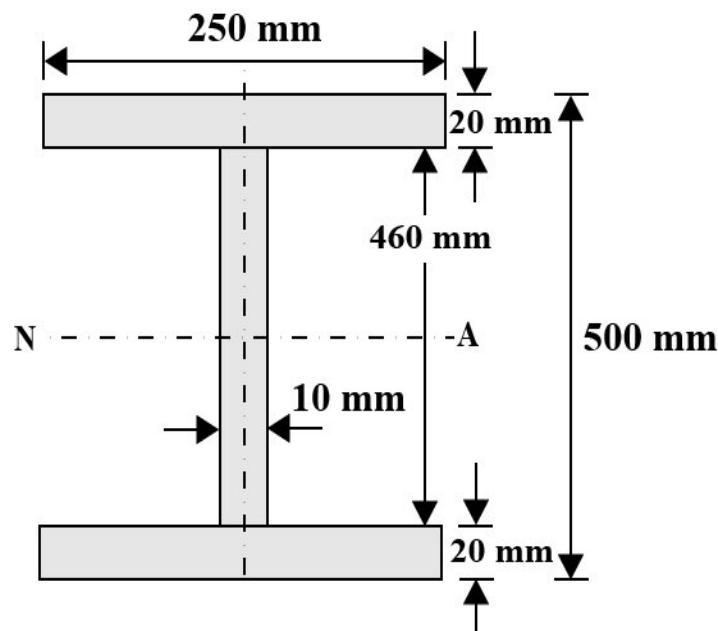


Fig. 4.12: Rolled steel joist (I-Section) under uniformly distributed load

Moment of inertia about the neutral axis.

$$\begin{aligned}
 &= \frac{250 \times 500^3}{12} - \frac{(250 - 10) \times 460^3}{12} \\
 &= 2604166666.66 - 1946720000 \\
 &= 657446666.66 \text{ mm}^4
 \end{aligned}$$

Maximum bending moment is given by,

$$M = \frac{P \times L^2}{8} = \frac{50000 \times 15^2}{8}$$

$$M = 1406250 \text{ Nm}$$

$$M = 1406250 \times 1000 \text{ Nmm}$$

Now, using the relation,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$\sigma = \frac{1406250 \times 1000}{657446666.66} \times 250$$

$$\sigma = 534.739 \text{ N/mm}^2$$

Problem 4.7

A cylinder of 600 mm internal diameter and 30 mm thick is carrying water in full. It is made up of cast iron and is supported at two points 15 m apart. Find the maximum stress in the metal. The cast iron and water weigh 75000 N/m^3 and 15000 N/m^3 , respectively.

Solution:

Internal diameter of the cylinder (D_i) = 600 mm = 0.6 m

Thickness of the cylinder (t) = 30 mm

$$\begin{aligned}\therefore \text{Outer diameter of the cylinder } (D_o) &= D_i + (2 \times t) = 600 + (2 \times 30) \\ &= 660 \text{ mm} = 0.66 \text{ m}\end{aligned}$$

Weight density of cast iron = 75000 N/m^3

Weight density of water = 15000 N/m^3

$$\text{Internal area of pipe, } (A_i) = \frac{\pi}{4} D_i^2 = \frac{\pi}{4} \times 0.6^2 = 0.2827 \text{ m}^2$$

It is also equal to the area of the section with water.

$$\therefore \text{Area of section with water} = 0.2827 \text{ m}^2$$

Outer area of pipe, $(A_o) = \frac{\pi}{4} D_o^2 = \frac{\pi}{4} \times 0.66^2 = 0.3421 \text{ m}^2$

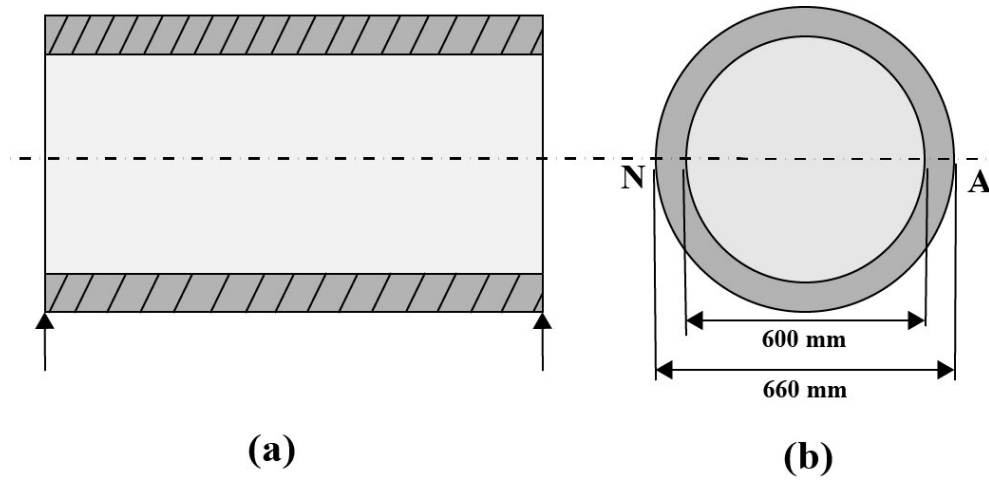


Fig. 4.13(a)-(b) Thin cylinder supported at two points filled with water

$$\begin{aligned} \therefore \text{Area of cross section of the pipe} &= \frac{\pi}{4} D_o^2 - \frac{\pi}{4} D_i^2 \\ &= \frac{\pi}{4} [D_o^2 - D_i^2] \\ &= \frac{\pi}{4} [0.66^2 - 0.60^2] = 0.05937 \text{ m}^2 \end{aligned}$$

Moment of inertia of the section about its centroid,

$$I = \frac{\pi}{64} [D_o^4 - D_i^4] = \frac{\pi}{64} [660^4 - 600^4] = 2.952 \times 10^9 \text{ mm}^4$$

To find the weight of the pipe and the weight of water for the one-meter length.

Weight of the pipe of one-meter length = weight density of cast iron $\times$ volume of pipe

$$\begin{aligned} &= 75000 \times [\text{area of pipe section} \times \text{length}] \\ &= 75000 \times 0.05937 \times 1 \quad (\because \text{Length} = 1 \text{ m}) \\ &= 4452.75 \text{ N} \end{aligned}$$

Weight due to distribution of water for one-meter length = weight density of water
 × volume of water

$$= 15000 \times [\text{area of water section} \times \text{length}]$$

$$= 15000 \times 0.2827 \times 1$$

$$= 4240.5 \text{ N}$$

$$\therefore \text{Total weight on the pipe for one-meter run} = 4452.75 + 4240.5 = 8693.25 \text{ N}$$

Hence, the above weight is uniformly distributed over the pipe. The maximum bending moment due to the U.D.L is $\left(\frac{wL^2}{8}\right)$, whereas P is U.D.L which is 8693.25 N/m.

$\therefore$ Maximum bending moment due to U.D.L.,

$$M = \frac{wL^2}{8} = \frac{8693.25 \times 15^2}{8} \quad (\because L = 15 \text{ m})$$

$$M = 244497.65 \text{ Nm} = 244497.65 \times 10^3 \text{ N.mm}$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M}{I} \times y$$

The stress will be maximum, where y is maximum. But the maximum value is

$$y = \frac{D_o}{2} = \frac{660}{2} = 330 \text{ mm}$$

$$y_{\max} = 330 \text{ mm}$$

$\therefore$ Maximum stress,

$$\sigma_{\max} = \frac{M}{I} \times y_{\max}$$

$$\sigma_{\max} = \frac{244497.65 \times 10^3}{2.952 \times 10^9} \times 330$$

$$\sigma_{\max} = 27.332 \text{ N/mm}^2$$

4.10 STRENGTH OF THE SECTION

It is also termed the flexural strength of the section, which means the moment of resistance offered by the section.

We know that

$$\frac{M}{I} = \frac{\sigma}{y} \quad (\text{Refer 4.6})$$

It is thus obvious that the moment of resistance depends upon the moment of inertia (or section modulus) of the section. A little consideration will show that the moment of inertia of the beam section depends upon its disposition in relation to the neutral axis.

We know that in the case of a beam subjected to transverse loading, the bending stress at a point is directly proportional to its distance from the neutral axis. It is thus obvious that a larger area near the neutral axis of the beam is uneconomical. This idea is put into practice by having beam sections, such that the flanges alone take the bending stress.

4.11 BENDING STRESS IN SYMMETRICAL SECTIONS

We know that in the symmetrical sections (circular, square or rectangular), the centroid of the section lies at the geometrical centre of the section as shown in Fig. 4.14. The neutral axis of a section passes through its centroid. The outermost layer or extreme fibre is at a distance of $d/2$ from its geometrical centre, where d is the diameter (in circular section) or depth (in square or rectangular section).

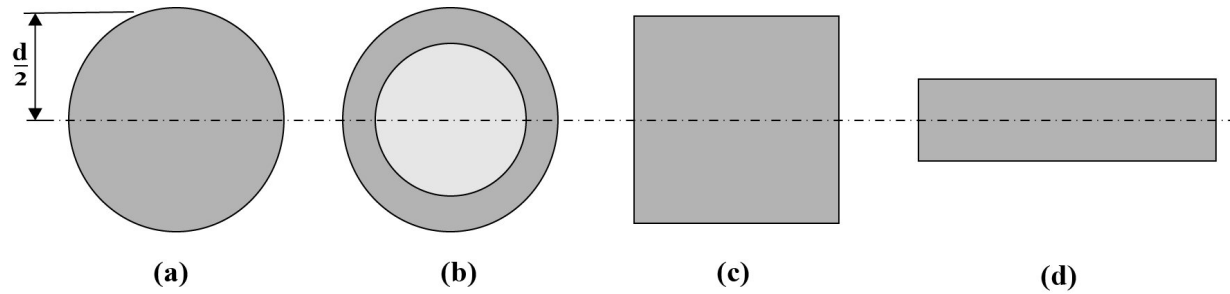


Fig. 4.14(a)-(d): Different symmetrical cross-section

Problem 4.8

A rectangular beam 80 mm wide and 160 mm deep is simply supported over a span of 8 m. If the beam is subjected to be point load of 15 kN at its centre, find the maximum bending stress induced in the beam section.

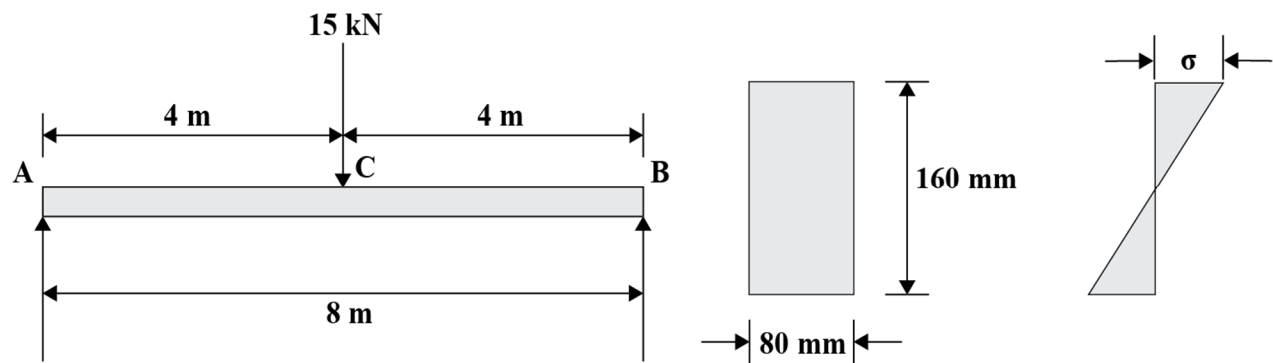


Fig. 4.15: Simply supported rectangular beam under central point load

Solution:

Width of the rectangular beam (b) = 80 mm;

Deep of the rectangular beam (d) = 160 mm;

Length of the rectangular beam $L = 8 \times 10^3$ mm and

Central point load (P) = 15 kN = 15×10^3 N

We know that the maximum bending moment at the centre of a simply supported beam subjected to point load at the mid section is given as,

$$M_{\max} = \frac{P \times L}{4} = \frac{(15 \times 10^3) \times (8 \times 10^3)}{4} = 30 \times 10^6 \text{ Nmm}$$

Section modulus of the rectangular section

$$Z = \frac{bd^2}{6} = \frac{80 \times 160^2}{6} = 341 \times 10^3 \text{ mm}^3$$

$\therefore$ Maximum bending stress,

$$\sigma_{\max} = \frac{M}{Z} = \frac{30 \times 10^6}{341 \times 10^3} = 87.97 \text{ MPa}$$

Problem 4.9

A rectangular beam 400 mm deep is simply supported over a span of 5 meter. What will be the uniformly distributed load that the beam can carry, if the bending stress is not to exceed 140 MPa. Take $I = 230 \times 10^6 \text{ mm}^4$

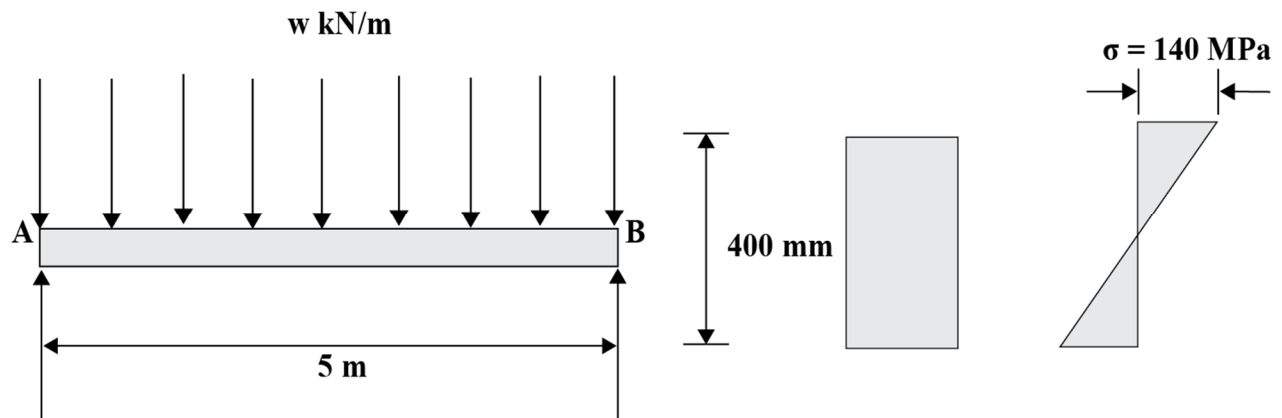


Fig. 4.16: Simply supported rectangular beam under uniformly distributed load

Solution:

Deep of the rectangular beam (d) = 400 mm

Length of the rectangular beam (L) = 5m = 5×10^3 mm

Bending stress ($\sigma_{\max}$) = 140 MPa

Moment of inertia (I) = $230 \times 10^6 \text{ mm}^4$

We know that the distance between the neutral axis of the section and the extreme fibre,

$$y = \frac{d}{2} = \frac{400}{2} = 200 \text{ mm}$$

Section modulus of the rectangular section.

$$Z = \frac{I}{y} = \frac{230 \times 10^6}{200} = 1.15 \times 10^6 \text{ mm}^3$$

∴ Moment of resistance

$$M = \sigma_{\max} \times Z$$

$$M = 140 \times (1.15 \times 10^6) = 161 \times 10^6 \text{ N.mm}$$

We know that maximum bending moment occurs at the centre of a simply supported beam subjected to a uniformly distributed load (w).

$$\begin{aligned} M &= \frac{wL^2}{8} \\ 161 \times 10^6 &= \frac{w \times (5 \times 10^3)^2}{8} \\ w &= \frac{(161 \times 10^6) \times 8}{(5 \times 10^3)^2} \\ w &= 51.52 \text{ N/mm} = 51.52 \text{ kN/m} \end{aligned}$$

Problem 4.10

A cantilever beam of the rectangular cross-section having 100 mm width and 140 mm depth, subjected to a point load of 8 kN at the free end and the bending stress is not to exceed 50 MPa, find the span of the cantilever beam.

Solution:

Width of the rectangular beam (b) = 100 mm

Depth of the rectangular beam (d) = 140 mm

Cantilever beam subjected to point load (P) = 8 kN

Bending stress ($\sigma_{\max}$) = 50 MPa = 50 N/mm².

L = Length of the cantilever beam

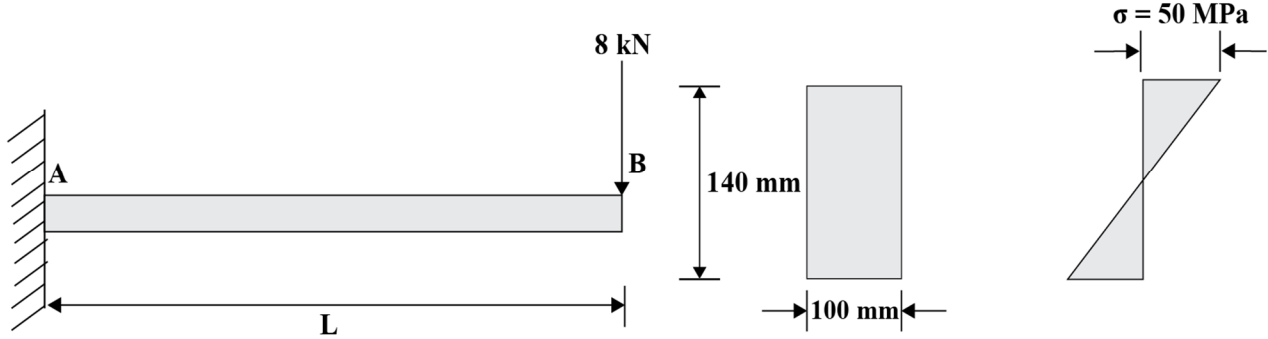


Fig. 4.17 Cantilever beam under point load at free end

We know that section modulus of the rectangular section is given as,

$$Z = \frac{bd^2}{6} = \frac{100 \times (140)^2}{6} = 326 \times 10^3 \text{ mm}^3$$

Maximum bending moment at the fixed end of the cantilever subjected to a point load at the free end.

$$M = P \times L = (8 \times 10^3) \times L$$

Maximum bending stress [$\sigma_{\max}$]

$$\sigma_{\max} = \frac{M}{Z}$$

$$50 = \frac{(8 \times 10^3) \times L}{326 \times 10^3}$$

$$50 = \frac{4 \times L}{163}$$
$$L = \frac{163 \times 50}{4} = 2037.5 \text{ mm}$$
$$L = 2037.5 \text{ mm} = 2.04 \text{ m}$$

4.12 BENDING STRESS IN UNSYMMETRICAL SECTIONS

In the case of symmetrical section, the neutral axis passes through the geometrical centre of the section. But in unsymmetrical sections such as L and T sections, it is difficult to locate the position of neutral axis. Hence the value of y from the topmost layer or bottom layer of the section from neutral axis will not be same. To find the bending stress in the beam, the higher value of y is used. As the neutral axis passes through the centre of gravity of the section, in unsymmetrical sections, first the centre of gravity is calculated in manner as explained in chapter 1.

Problem 4.11

A cast iron bracket subjected to bending has the cross-section of I-form with unequal flanges. The dimensions of the section are shown in Fig. 4.18. Find the position of the neutral axis and the moment of inertia of the section about the neutral axis. If the maximum bending moment on the section is 60 MNmm, determine the maximum bending stress. What is the nature of the stress?

Solution:

Maximum bending moment, $M = 60 \text{ MN mm} = 60 \times 10^6 \text{ N}\cdot\text{mm}$

Let us first calculate the centroid of the section. Let $\bar{y}$ is the distance of the centroid from the bottom face. The section is symmetrical about y axis and hence $\bar{y}$ is to be calculated. Thus,

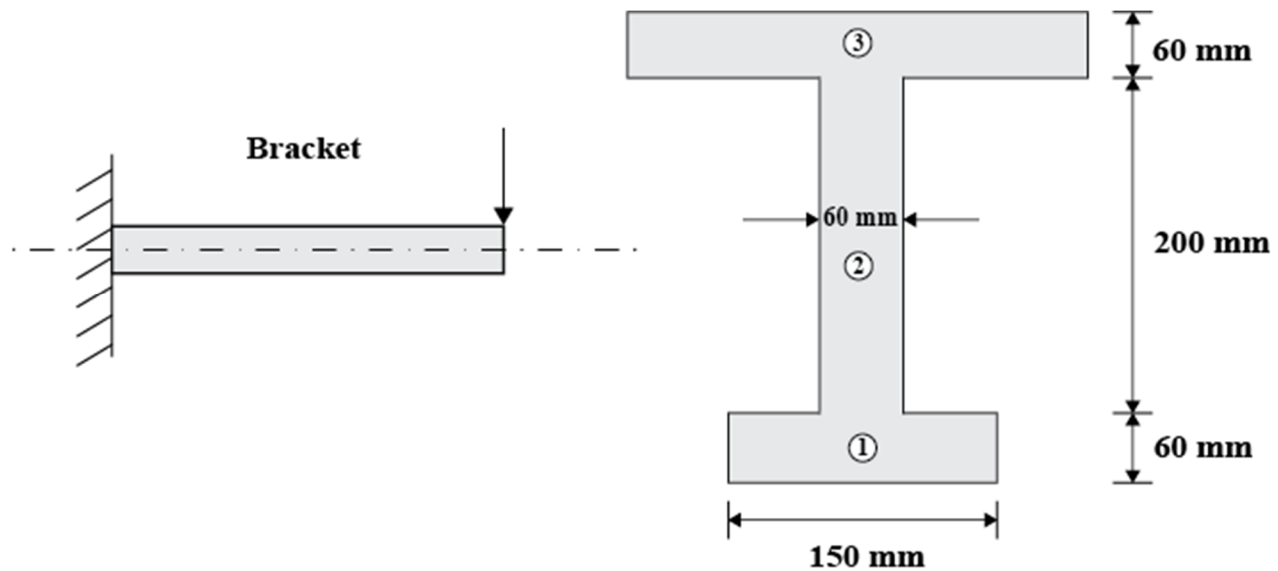


Fig. 4.18: Cross-section of cast iron bracket with I-form

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$

$$A_1 = \text{Area of bottom flange} = 150 \times 60 = 9000 \text{ mm}^2$$

y_1 = Distance of centroid of the bottom flange from the bottom.

$$= \frac{60}{2} = 30 \text{ mm}$$

$$A_2 = \text{Area of web} = 200 \times 60 = 12000 \text{ mm}^2$$

y_2 = Distance of centroid of the web from the bottom

$$= 60 + \frac{200}{2} = 160 \text{ mm}$$

$$A_3 = \text{Area of top flange} = 200 \times 60 = 12000 \text{ mm}^2$$

y_3 = Distance of centroid of the top flange from the bottom face

$$= 60 + 200 + \frac{60}{2} = 290 \text{ mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$

$$\bar{y} = \frac{(9000 \times 30) + (12000 \times 160) + (12000 \times 290)}{(9000 + 12000 + 12000)}$$

$$\bar{y} = \frac{5670000}{33000}$$

$$\bar{y} = 171.81 \text{ mm}$$

Hence the neutral axis is at a distance of 171.81 mm from the bottom face.

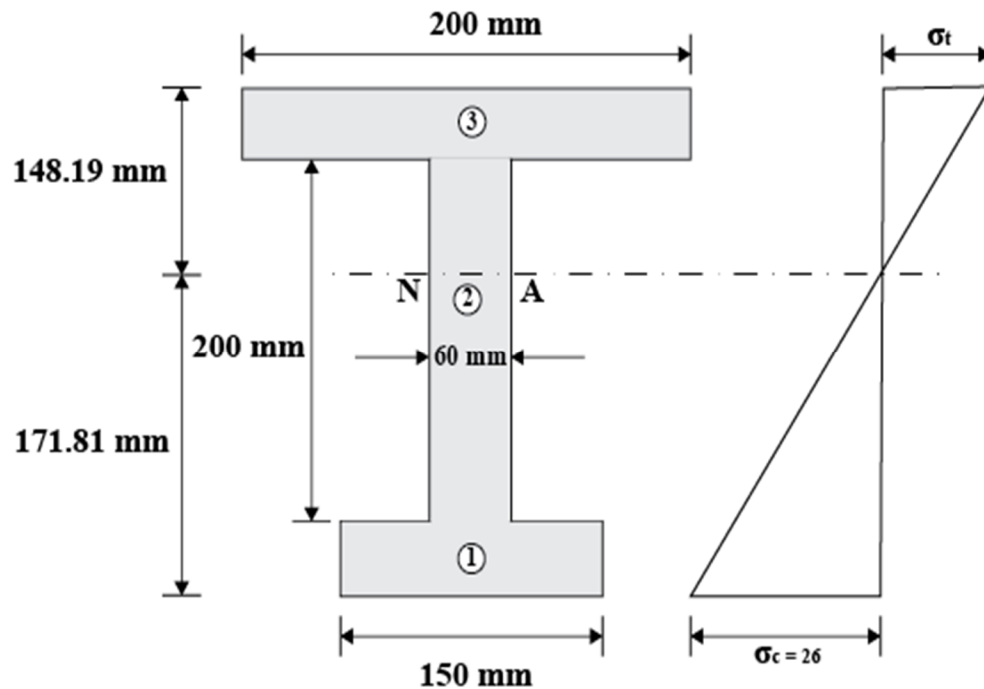


Fig. 4.19: Position of neutral axis from the bottom face in I-section

Moment of inertia of the section about the neutral axis

$$I = I_1 + I_2 + I_3$$

Where I_1 = Moment of inertia of bottom flange about neutral axis

= Moment of inertia of bottom flange about an axis passing through its

C.G. + $A_1 \times (\text{Distance of its C.G. from neutral axis})^2$

$$= \left(\frac{150 \times 60^3}{12} \right) + 9000 \times (171.81 - 30)^2$$

$$= 2700000 + 180990684.9$$

$$= 183690684.9 \text{ mm}^4$$

Similarly

I_2 = Moment of inertia of web about neutral axis

$$= \left(\frac{60 \times 200^3}{12} \right) + 12000 \times (171.81 - 160)^2$$

$$= 40000000 + 1673713.2$$

$$I_2 = 41673713.2 \text{ mm}^4$$

I_3 = Moment of inertia of top flange about neutral axis

$$= \left(\frac{200 \times 60^3}{12} \right) + 12000 \times (290 - 171.81)^2$$

$$= 3600000 + 167626513.2$$

$$I_3 = 171226513.2 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 183690684.9 + 41673713.2 + 171226513.2 = 396590911.3 \text{ mm}^4$$

Now distance of C.G from the upper top layer = $320 - \bar{y}$

$$= 320 - 171.81 = 148.19 \text{ mm}$$

And the distance of C.G from the bottom layer $\bar{y} = 171.81 \text{ mm}$

Hence, we shall take the value of $y = 171.81 \text{ mm}$ for maximum bending stress.

Now using the equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y = \frac{60 \times 10^6}{396590911.3} \times 171.81 = 26 \text{ N/mm}^2$$

$\therefore$ Maximum bending stress = 26 N/mm^2

Problem 4.12

The T-section cast iron beam as shown in Fig. 4.20, is simply supported over a 10 m span. The beam bears a uniform distributed load of 2 kN/m along the entire span. Calculate the maximum tensile and compressive stresses in the beam.

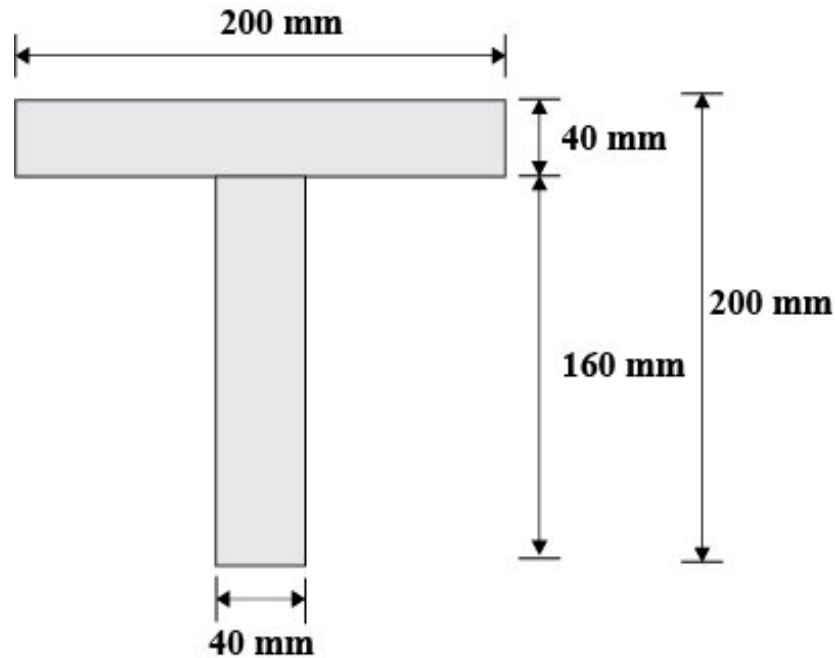


Fig. 4.20: T-Section cast iron beam under uniform distributed load

Solution:

Length of the beam $L = 10$ m

Uniformly distributed load (w) = 2 kN/m = 2000 N/m

To find the position of the neutral axis, the C.G of the section is to be obtained first.

The C.G will be lying on the y-y axis.

Let $\bar{y}$ be the C.G of the section from the bottom

$$\begin{aligned}\bar{y} &= \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(200 \times 40) \times \left(160 + \frac{40}{2}\right) + (160 \times 40) \times \left(\frac{160}{2}\right)}{8000 + 6400} \\ &= \frac{1952000}{14400} = 135.56 \text{ mm}\end{aligned}$$

Neutral axis lies at the distance of 135.56 mm from the bottom face or $200 - 135.56$ at 64.44 mm from the top face.

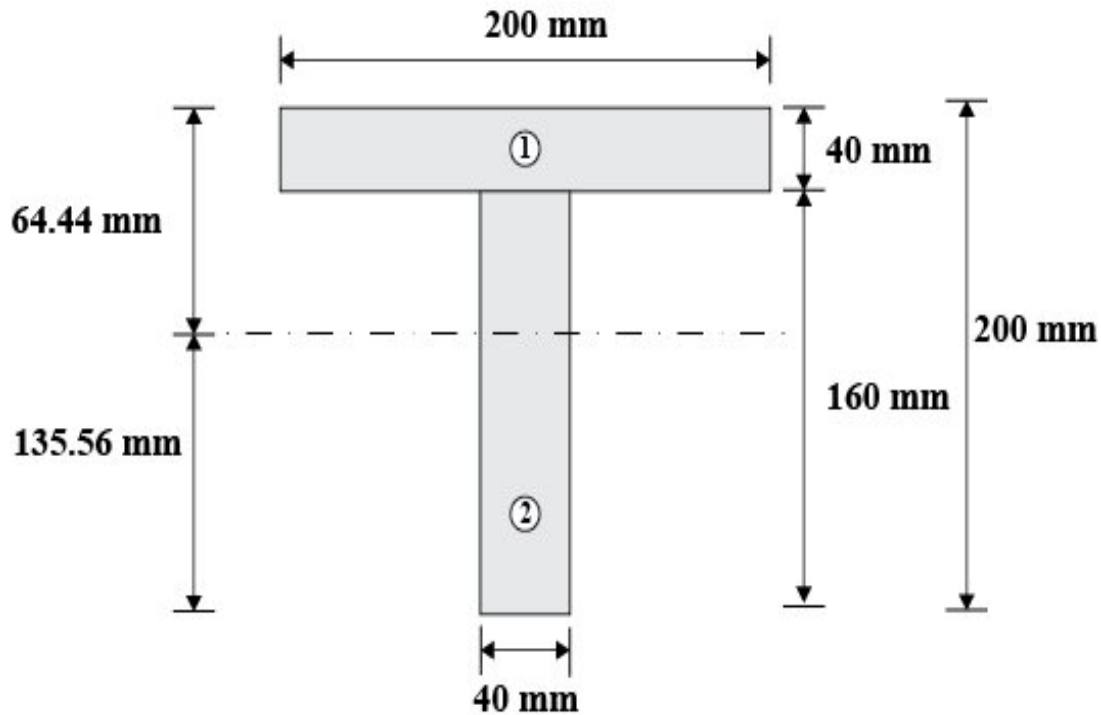


Fig. 4.21: Neutral position from the bottom face in T-section

Now the moment of inertia of the section about the neutral axis is given by,

$$I = I_1 + I_2$$

Where,

I_1 = Moment of inertia of top flange about the neutral axis.

= MI of top flange about its C.G + $A_1 \times (\text{Distance of its C.G from neutral axis})^2$

$$= \left(\frac{200 \times 40^3}{12} \right) + (200 \times 40) \times (64.44 - 20)^2$$

$$= 1066666.67 + 15799308.8$$

$$= 16865975.47 \text{ mm}^4$$

I_2 = Moment of inertia of web about the neutral axis.

$$= \text{MI of web about its C.G} + A_2 \times (\text{Distance of its C.G from the neutral axis})^2$$

$$= \left(\frac{40 \times 160^3}{12} \right) + (40 \times 160) \times (135.56 - 80)^2$$

$$= 13653333.33 + 19756247.04$$

$$= 33409580.37 \text{ mm}^4$$

$$I = I_1 + I_2 = 16865975.47 + 33409580.37 = 50275555.84 \text{ mm}^4$$

For a simply supported beam, the maximum tensile stress will be at the extreme bottom layer and maximum compressive stress will be at the extreme top layer.

Maximum Bending moment is given by,

$$M = \frac{w \times L^2}{8} = \frac{2000 \times 10^2}{8} = 25000 \text{ N/m}$$

$$M = 25000 \times 1000 = 25000000 \text{ N/mm}$$

Now using the Eqn (4.6)

$$\frac{M}{I} = \frac{\sigma}{y}$$

(i) For maximum tensile stress,

y = The distance of the bottom layer from the neutral axis = 135.56 mm

$$\sigma = \frac{M}{I} \times y = \frac{25000000}{50275555.83} \times 135.56 = 67.41 \text{ N/mm}^2$$

(ii) For maximum compressive stress,

y = The distance of the top layer from the neutral axis = 64.64 mm

$$\sigma = \frac{M}{I} \times y = \frac{25000000}{50275555.83} \times 64.44 = 32.043 \text{ N/mm}^2$$

4.13 COMPOSITE BEAMS (FLITCHED BEAMS)

A composite section may be defined as a section made up of two or more different materials, joined together in such a manner that they act as a single material and, each material bends to the same radius of curvature. If the material of lower strength, is used alone, it requires a larger area; which does not suit if there is a space limitation. A material is then reinforced with some other material, of higher strength, to reduce the cross-sectional area of the beam and to meet the space requirements as is done in the case of reinforced cement concrete beams.

In such cases, the total moment of resistance equals the sum of the moments of the individual sections.

The beam shown in Fig. 4.22 is comprised of a composite section consisting of two distinct materials.

Let

E_1 = Modulus of elasticity of part 1,

I_1 = Moment of inertia of the part 1,

M_1 = Moment of resistance of the part 1,

σ_1 = Stress in part 1,

Z_1 = Modulus of section for part 1,

$E_2, I_2, M_2, \sigma_2, Z_2$ = Corresponding values for the part 2 and

R = Radius of curvature of the bent specimen.

We know that the moment of resistance for beam 1,

$$M_1 = \sigma_1 \times Z_1 \quad \dots (4.21)$$

Similarly,

$$M_2 = \sigma_2 \times Z_2 \quad \dots (4.22)$$

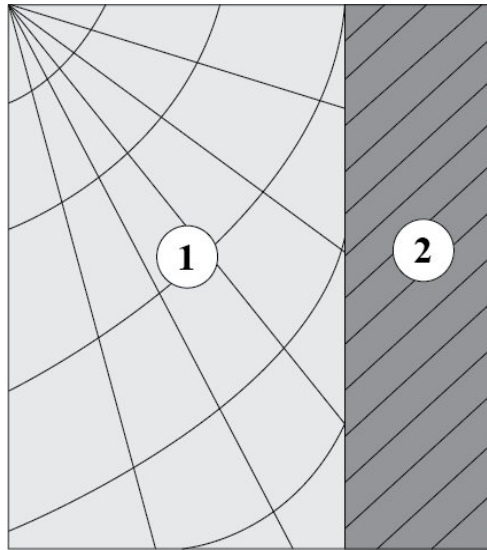


Fig. 4.22: Composite beam (Flitched beam)

We also know that at any distance from the neutral axis, the strain in both the materials will be the same.

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad \text{or} \quad \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 = m \times \sigma_2$$

$$\sigma_1 = m\sigma_2 \quad \dots (4.23)$$

Where $m = \frac{E_1}{E_2}$ is Modulus ratio.

Using the relation

$$\frac{M}{I} = \frac{\sigma}{y}$$

We get,

$$M = \frac{\sigma}{y} \times I$$

Hence moment of resistance of part 1 and part 2 are given by,

$$M_1 = \frac{\sigma_1}{y} \times I_1 \quad \text{and} \quad M_2 = \frac{\sigma_2}{y} \times I_2 \quad \dots (4.24)$$

∴ Total moment of resistance of the composite section,

$$M = M_1 + M_2$$

$$M = \left(\frac{\sigma_1}{y} \times I_1 \right) + \left(\frac{\sigma_2}{y} \times I_2 \right)$$

$$M = \left(\frac{m\sigma_2}{y} \times I_1 \right) + \left(\frac{\sigma_2}{y} \times I_2 \right) \quad (\because \sigma_1 = m\sigma_2 \text{ from Eqn 4.23})$$

$$M = \frac{\sigma_2}{y} [mI_1 + I_2] \quad \dots (4.25)$$

In Eqn (4.25) $mI_1 + I_2$ can be treated as an equivalent moment of inertia of the cross section, as it is entirely composed of material 2 (i.e., composite), providing equivalent resistance to bending the composite beam.

Let this be denoted by I .

$$\therefore \quad I = mI_1 + I_2 \quad \dots (4.26)$$

$$\text{Then, } M = \frac{\sigma_2}{y} \times I$$

The equivalent section is created by combining the moments of inertia: $I = mI_1 + I_2$. This involves scaling the dimensions of material (part 1) parallel to the neutral axis by a factor of m . The resulting equivalent figure aids in determining the position of the neutral axis and the equivalent moment of inertia.

Problem 4.13

A flitched beam consists of a wooden joist 100 mm wide and 300 mm deep strengthened by two steel plates 20 mm thick and 300 mm deep as shown in Fig. 4.23. If the maximum stress in the wooden joist is 10 N/mm^2 , find the corresponding maximum stress attained in steel. Also find the moment of resistance of the composite section. Take Young's modulus for steel as $2 \times 10^5 \text{ N/mm}^2$ and for wood as $1 \times 10^4 \text{ N/mm}^2$.

Solution:

Let part 1 represents steel and part 2 represents wooden joist.

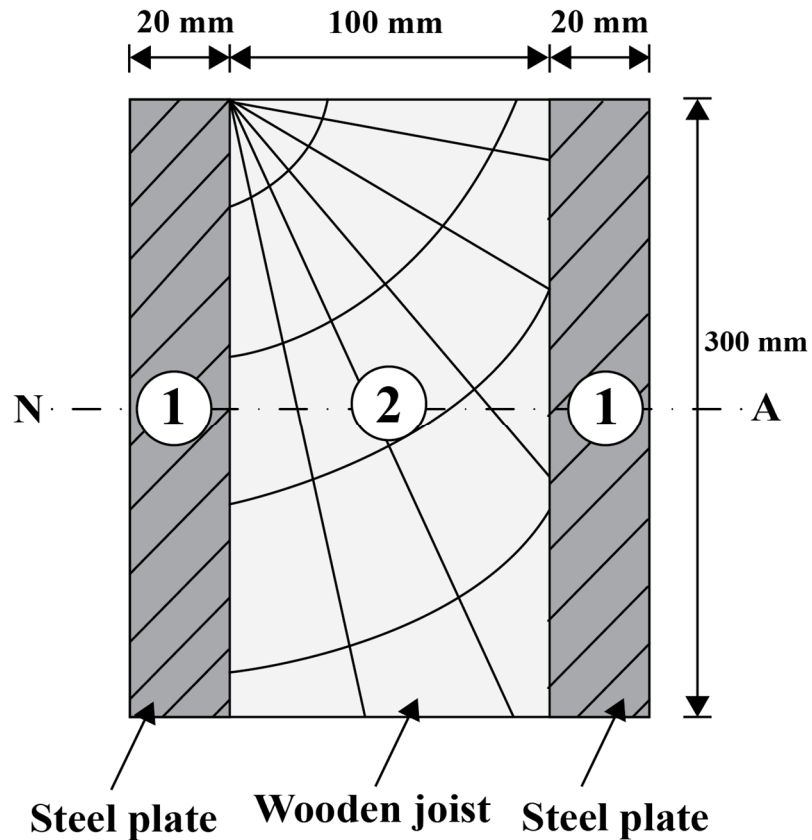


Fig. 4.23: Flitched beam with wooden joist and steel plates

Width of the wooden joist, $b_2 = 100 \text{ mm}$

Depth of the wooden joist, $d_2 = 300 \text{ mm}$

Width of the one steel plate, $b_1 = 20 \text{ mm}$

Depth of the one steel plate, $d_1 = 300 \text{ mm}$

Number of steel plates = 2

Maximum stress in wooden joist, $\sigma_2 = 10 \text{ N/mm}^2$

Young's modulus of steel $E_1 = 2 \times 10^5 \text{ N/mm}^2$

Young's modulus of wooden joist $E_2 = 1 \times 10^4 \text{ N/mm}^2$

Now moment of inertia of wooden joist about the neutral axis,

$$I_2 = \frac{b_2 d_2^3}{12} = \frac{100 \times 300^3}{12} = 225000000 \text{ mm}^4$$

Now moment of inertia of two steel plates about the neutral axis,

$$I_1 = \left(2 \times \frac{b_1 d_1^3}{12} \right) = 2 \times \frac{20 \times 300^3}{12} = 90000000 \text{ mm}^4$$

Now modulus ratio between the wooden joist and steel is given by,

$$m = \frac{E_1}{E_2} = \frac{2 \times 10^5}{1 \times 10^4} = 20$$

The equivalent moment of inertia (I) is given by Eqn (4.25)

$$I = mI_1 + I_2$$

$$I = (20 \times 90000000) + 225000000 = 202500 \times 10^4 \text{ mm}^4$$

Moment of resistance of the composite section is given by the Eqn (4.23)

$$\therefore M = \frac{\sigma_2}{y} \times I$$

$$M = \frac{202500 \times 10^4 \times 10}{150} \quad (\because y = 100 \text{ mm})$$

$$M = 202500 \times 10^3 \text{ N.mm} = 202500 \text{ Nm}$$

Maximum stress in steel

Let, σ_1 = Maximum stress in the steel

Now using the Eqn (4.23), we get

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\therefore \sigma_1 = \frac{E_1}{E_2} \times \sigma_2 \quad \left(\because \frac{E_1}{E_2} = 20, \sigma_2 = 10 \text{ N/mm}^2 \right)$$

$$\sigma_1 = 20 \times 10 = 200 \text{ N/mm}^2$$

Problem 4.14

A flitched timber beam made up of steel and timber has a cross section as shown in Fig. 4.24. Determine the moment of resistance of the beam. Take $\sigma_s = 150$ MPa and $\sigma_t = 10$ MPa.

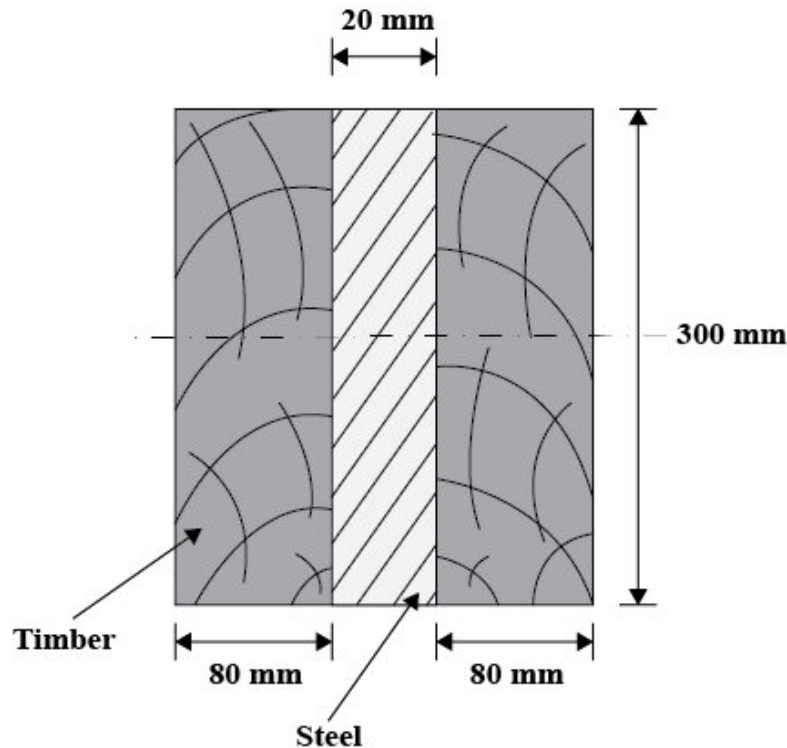


Fig. 4.24: Flitched timber beam with steel and timber sections

Solution:

Width of each timber section (b_t) = 80 mm

Depth of each timber section (d_t) = 300 mm

Stress in timber (σ_t) = 10 MPa = 10 N/mm²

Width of steel section (b_s) = 20 mm

Depth of steel section (d_s) = 300 mm

Stress in steel (σ_s) = 150 MPa = 150 N/mm²

We know that the section modulus of a rectangular body,

$$Z = \frac{bd^2}{6}$$

∴ Section modulus of the timber section,

$$Z_t = 2 \times \left[\frac{80 \times (300)^2}{6} \right] = 240 \times 10^4 \text{ mm}^3 \quad \dots \text{(For two sections)}$$

Similarly, section modulus for the steel section

$$Z_s = \left[\frac{20 \times (300)^2}{6} \right] = 30 \times 10^4 \text{ mm}^3$$

We also know that the moment of resistance for timber,

$$M_t = \sigma_t \times Z_t = 10 \times 240 \times 10^4 = 24 \times 10^6 \text{ N/mm}$$

Similarly,

$$M_s = \sigma_s \times Z_s = 150 \times 30 \times 10^4 = 45 \times 10^6 \text{ N/mm}$$

∴ Total moment of resistance of the beam,

$$M = M_t + M_s = (24 \times 10^6) + (45 \times 10^6) = 69 \times 10^6 \text{ N/mm}$$

$$M = 69 \times 10^6 \text{ N/mm}$$

4.14 SHEAR STRESS AT A SECTION

Consider a small section ABCD of length dx of a beam loaded with uniformly distributed load as shown in Fig. 4.25(a).

We know that when a beam is loaded with a uniformly distributed load, the shear force and bending moment vary at every point along the length of the beam.

Let

M = Bending moment at the section AB,

$M + dM$ = Bending moment at the section CD,

F = Shear force on AB,

$F + dF$ = Shear force on CD, and

L = Moment of inertia of the section about its neutral axis.

Now consider an elementary strip at a distance y from the neutral axis as shown in Fig. 4.25(b).

Now, let σ - Bending stress across AB at distance y from the neutral axis and dA - Cross-sectional area of the strip.

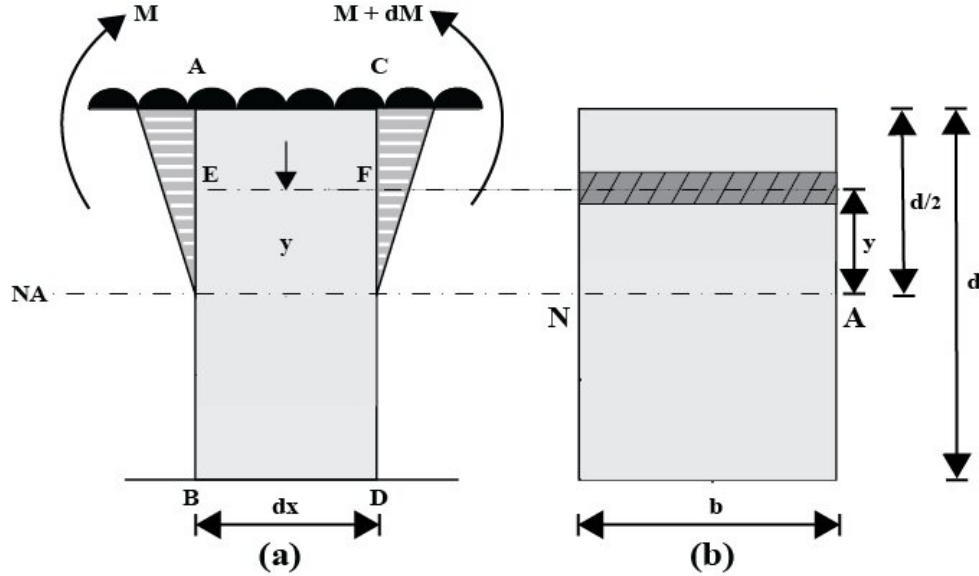


Fig. 4.25(a)-(b) Figure showing the shear stress at a section

We know that

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad \sigma = \frac{M}{I} \times y$$

Similarly,

$$\sigma + d\sigma = \frac{M + dM}{I} \times y$$

Where $\sigma + d\sigma$ - Bending stress along CD

The force acting on the elementary strip across AB

= Stress $\times$ Area of elemental strip

= $(\sigma \times dA)$

= $\left(\frac{M}{I} \times y \times dA\right)$ (4.27)

The force acting on the elementary strip across CD is Stress $\times$ Area

= $(\sigma + d\sigma) \times dA = \left(\frac{M + dM}{I} \times y \times dA\right)$ (4.28)

∴ Net unbalanced force on the strip

$$= \left(\frac{M + dM}{I} \times y \times dA \right) - \left(\frac{M}{I} \times y \times dA \right) = \frac{dM}{I} \times y \times dA$$

The total unbalanced force (F) above the neutral axis may be found out by integrating the above equation between 0 and d/2

$$= \int_0^{\frac{d}{2}} \frac{dM}{I} \times y \times dA = \frac{dM}{I} \int_0^{\frac{d}{2}} y \times dA \quad \dots (4.29)$$

Where dA - Area of the section above the neutral axis, and $\bar{y}$ is the distance between the centre of gravity of the area and the neutral axis.

We know that the shear stresses,

$$\tau = \frac{\text{Total force}}{\text{Area}} = \frac{\frac{dM}{I} \cdot A\bar{y}}{dx \cdot b} \quad (\text{Where } b \text{ is the width of beam})$$

$$\tau = \frac{dM}{dx} \times \frac{A \cdot \bar{y}}{Ib} \quad \left(\because \text{Shear force } F = \frac{dM}{dx} \right)$$

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib} \quad \dots (4.30)$$

4.15 SHEAR STRESS DISTRIBUTION FOR DIFFERENT SECTIONS

The following are the important sections over which the shear stress distribution is to be obtained

- Rectangular section,
- Triangular section,
- Circular Section,
- I-Section,
- T-Sections

4.15.1 RECTANGULAR SECTION

Consider a beam of rectangular section ABCD of width and depth as shown in Fig. 4.26(a). We know that the shear stress on a layer EF of a beam, at a distance y from the neutral axis.

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib} \quad \dots (\text{Refer 4.30})$$

Where F - Shear force at the section,

A - Area of section above y (shaded area AEFD),

$\bar{y}$ - Distance of the centre of gravity of the shaded area from the neutral axis,

$A \cdot \bar{y}$ - Moment of the shaded area about the neutral axis,

I - Moment of inertia of the whole section about its neutral axis, and

b - Width of the section.

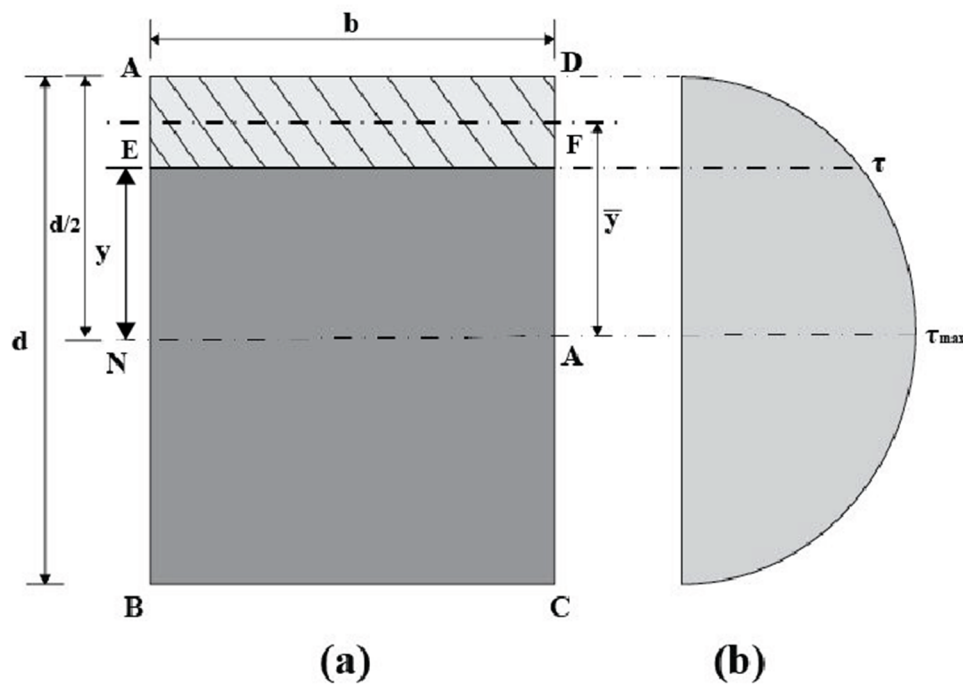


Fig. 4.26(a)-(b): Shear stress distribution in rectangular section

We know that the area of the shaded section AEFD,

$$A = b \left(\frac{d}{2} - y \right) \quad \dots (4.31)$$

$$\begin{aligned}\therefore \bar{y} &= y + \frac{1}{2} \left(\frac{d}{2} - y \right) = y + \frac{d}{4} - \frac{y}{2} \\ \bar{y} &= \frac{d}{4} + \frac{y}{2} = \frac{1}{2} \left(y + \frac{d}{2} \right) \quad \dots (4.32)\end{aligned}$$

Substituting the above values of A and $\bar{y}$ in Eqn (4.30),

$$\begin{aligned}\tau &= F \times \frac{A \cdot \bar{y}}{Ib} = F \times \frac{b \left(\frac{d}{2} - y \right) \times \frac{1}{2} \left(y + \frac{d}{2} \right)}{Ib} \\ \tau &= \frac{F}{2I} \left(\frac{d^2}{4} - y^2 \right) \quad \dots (4.33)\end{aligned}$$

We see, from the above equation, that as the value of y decreases the value of τ increases. At the point, where $y = d/2$, $\tau = 0$; and where $y = 0$, τ is maximum. We also see that the variation of τ with respect to y is a parabola.

At the neutral axis, the value of τ is maximum. Thus, substituting $y = 0$ and $I = \frac{bd^3}{12}$ in the above equation,

$$\begin{aligned}\tau_{\max} &= \frac{F}{\left(2 \times \frac{bd^3}{12} \right)} \left(\frac{d^2}{4} \right) = \frac{3F}{2bd} = 1.5 \tau_{\text{av}} \quad \dots \left(\tau_{\text{av}} = \frac{F}{\text{Area}} = \frac{F}{bd} \right) \\ \tau_{\max} &= 1.5 \tau_{\text{av}} \quad \dots (4.34)\end{aligned}$$

The shear stress distribution diagram as shown in Fig. 4.26(b).

Problem 4.15

A wooden beam 150 mm wide, 300 mm deep and 5 m long is carrying a uniformly distributed load of 50 kN/m. Determine the maximum shear stress and sketch the variation of shear stress along the depth of the beam.

Solution:

Width of the wooden beam (b) = 150 mm

Depth of the wooden beam (d) = 300 mm

Length of the wooden beam (L) = 5 m = 5×10^3 mm

Uniformly distributed load (w) = 50 kN/m = 50 N/mm

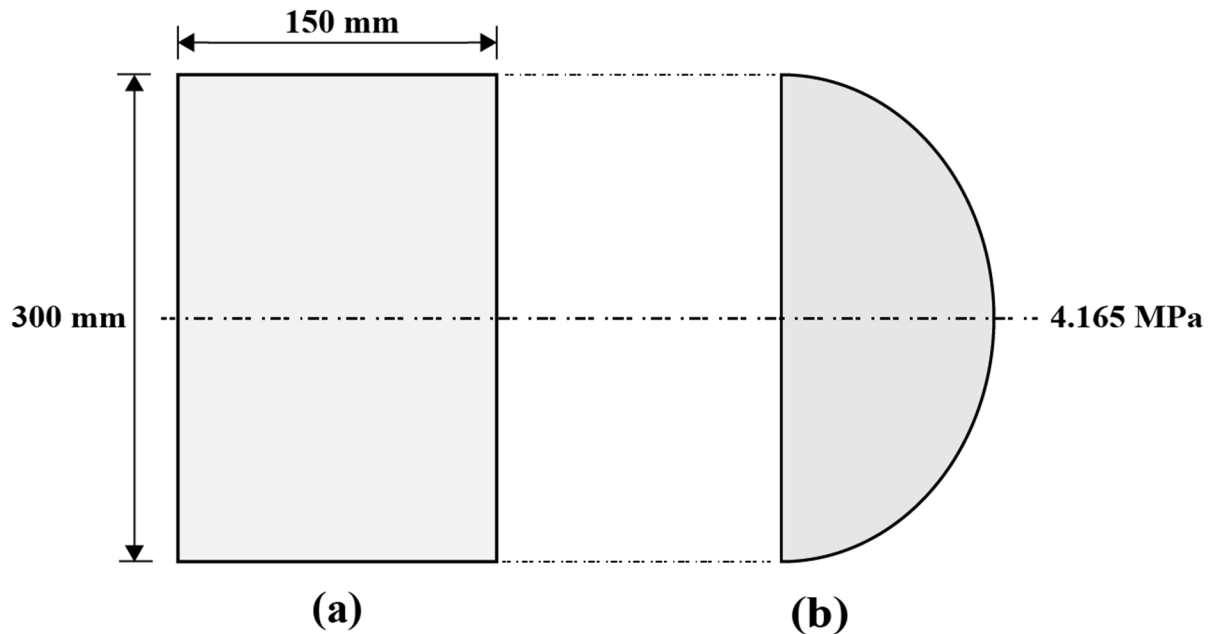


Fig. 4.27(a)-(b): Wooden beam under uniformly distributed load

We know that shear force at one end of the beam,

$$F = \frac{w \times L}{2} = \frac{50 \times (5 \times 10^3)}{2} = \frac{250000}{2} = 125000 \text{ N}$$

Area of the beam section

$$A = b \cdot d = 150 \times 300 = 45000 \text{ mm}^2$$

$\therefore$ Average shear stress across the section,

$$\tau_{av} = \frac{F}{\text{Area}} = \frac{125000}{45000} = 2.777 \text{ N/mm}^2 = 2.777 \text{ MPa}$$

Maximum shear stress,

$$\tau_{\max} = 1.5 \times \tau_{av} = 1.5 \times 2.777 = 4.1655 \text{ MPa}$$

The diagram showing the variation of shear stress along the depth of the beam is shown in Fig. 4.27(b)

4.15.2 TRIANGULAR SECTION

Consider a beam of triangular cross-section EJK of base B and height H as shown in Fig. 4.28(a).

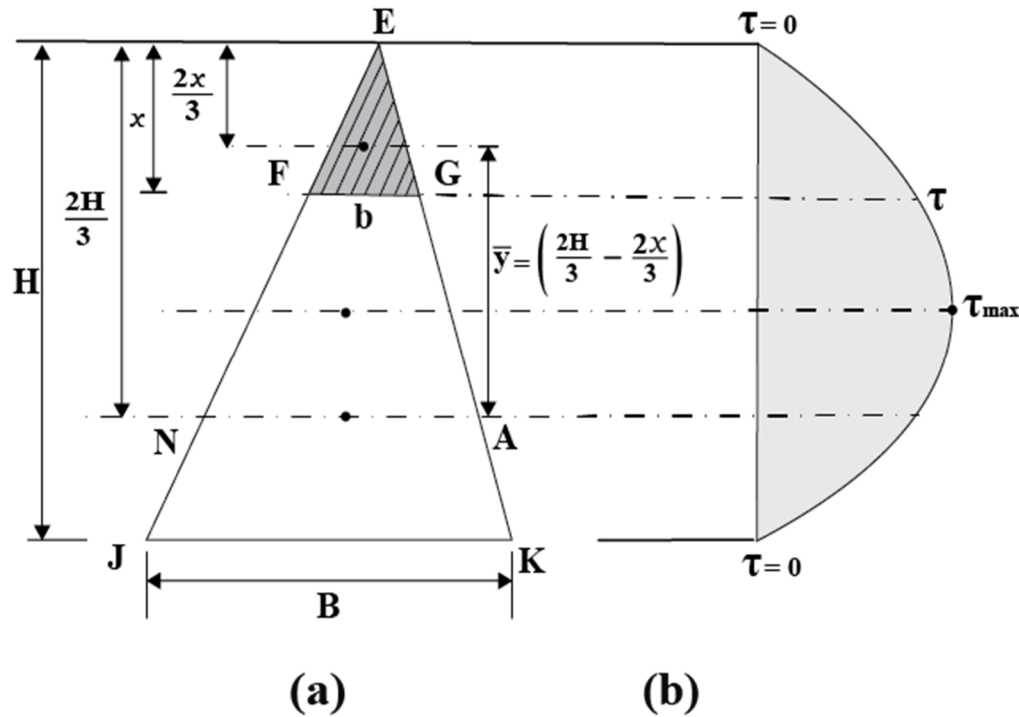


Fig. 4.28(a)-(b): Shear stress distribution in triangular section

We know that the shear stress on a layer FG at a distance y from the neutral axis,

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib} \quad \dots \text{(Refer Eqn 4.30)}$$

where F = Shear force at the section,

$A \cdot \bar{y}$ = Moment of the shaded area about the neutral axis and

I = Moment of inertia of the triangular section about its neutral axis

We know the width of the small triangle EFG,

$$b = \frac{B \cdot x}{H} \quad \dots (4.35)$$

$$\left(\because \text{Two Similar } \triangle EFG \text{ and } \triangle EJK \left(\frac{b}{B} = \frac{x}{H} \right) \right)$$

$\therefore$ Area of the shaded section GFE,

$$A = \frac{1}{2} b \cdot x = \frac{1}{2} \left(\frac{B \cdot x}{H} \cdot x \right) = \frac{Bx^2}{2H}$$

And $\bar{y} = \frac{2H}{3} - \frac{2x}{3} = \frac{2}{3} (H - x) \quad \dots (4.36)$

Substituting the values of b , A and $\bar{y}$ in Eqn (4.30)

$$\begin{aligned} \tau &= F \times \frac{\left(\frac{Bx^2}{2H} \right) \times \frac{2}{3} (H - x)}{I \times \frac{Bx}{H}} \\ \tau &= \frac{F}{3I} \times [x (H - x)] \\ \tau &= \frac{F}{3I} \times [Hx - x^2] \quad \dots (4.37) \end{aligned}$$

Thus, we see that the variation of τ with respect to x is parabola. We also see that as a point where $x = 0$ or $x = H$, $\tau = 0$. At neutral axis, where $x = \frac{2H}{3}$

$$\begin{aligned} \tau &= \frac{F}{3I} \left[\left(H \times \frac{2H}{3} \right) - \left(\frac{2H}{3} \right)^2 \right] \\ \tau &= \frac{F}{3I} \left[\frac{2H^2}{9} \right] = \frac{2FH^2}{27I} \quad \dots \left(\because I = \frac{BH^3}{36} \right) \\ \tau &= \frac{2FH^2}{27 \left(\frac{BH^3}{36} \right)} = \frac{8F}{3BH} \end{aligned}$$

$$\tau = \frac{4}{3} \times \frac{F}{\text{Area}} = 1.33\tau_{av} \quad \dots\dots \left(\because \tau_{av} = \frac{F}{\text{Area}} = \frac{2F}{BH} \right)$$

$$\tau = 1.33\tau_{av} \quad \dots (4.38)$$

For maximum shear stress, differentiating the Eqn (4.37) and equating to zero

$$\frac{d}{dx} \left[\frac{F}{3I} \times (Hx - x^2) \right] = 0$$

$$\therefore H - 2x = 0 \text{ or } x = \frac{H}{2}$$

Substituting this value of x in Eqn (4.37),

$$\tau_{\max} = \frac{F}{3I} \left[\left(H \times \frac{H}{2} \right) - \left(\frac{H}{2} \right)^2 \right] = \frac{FH^2}{12I}$$

$$\tau_{\max} = \frac{FH^2}{12 \left(\frac{BH^3}{36} \right)} \quad \dots\dots \left(\because I = \frac{BH^3}{36} \right)$$

$$\tau_{\max} = \frac{3F}{BH} = \frac{3}{2} \times \frac{F}{\text{Area}} = 1.5 \tau_{av}$$

$$\tau_{\max} = 1.5 \tau_{av} \quad \dots (4.39)$$

The shear stress distribution diagram is as shown in Fig. 4.28(b).

Problem 4.16

A beam of triangular cross section having base width of 150 mm and height of 300 mm is subjected to a shear force of 15 kN. Find the value of maximum shear stress and sketch the shear stress distribution along the depth of beam.

Solution

Base width (b) = 150 mm

Height (h) = 300 mm and shear force (F) = 15 kN = 15×10^3 N

We know that area of beam section,

$$A = \frac{b \cdot h}{2} = \frac{150 \times 300}{2} = 22500 \text{ mm}^2$$

∴ Average shear stress across the section,

$$\tau_{av} = \frac{F}{A} = \frac{15 \times 10^3}{22500} = 0.666 \frac{\text{N}}{\text{mm}^2} = 0.666 \text{ MPa}$$

Maximum shear stress,

$$\tau_{max} = 1.5 \times 0.666 = 0.999 \text{ MPa}$$

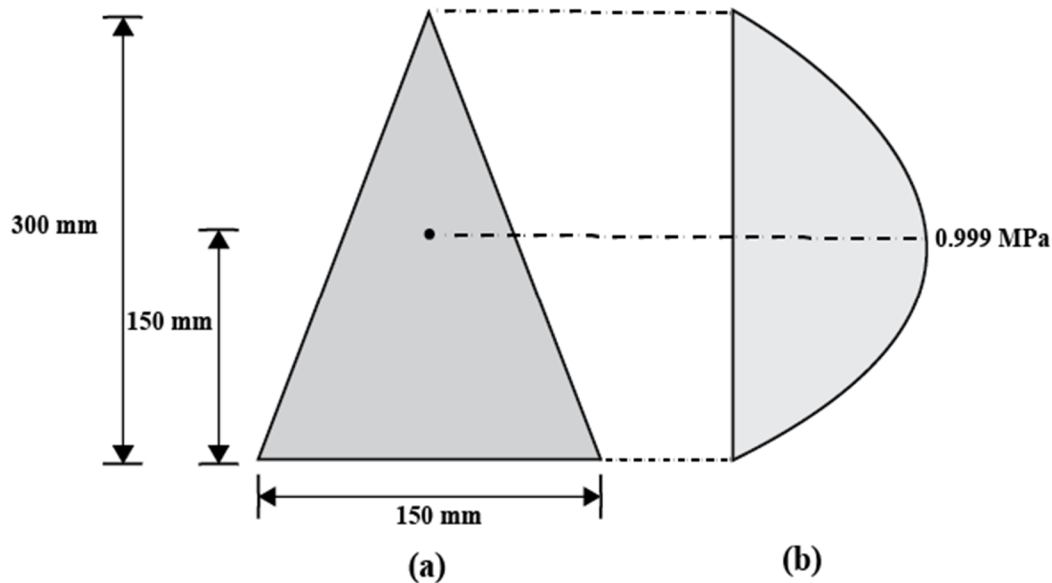


Fig. 4.29(a)-(b): Beam of triangular cross-section under shear force

The diagram showing the variation of shear stress along the depth of the beam is shown in Fig. 4.29(b).

4.15.3 CIRCULAR SECTION

Consider a circular section of diameter d as shown in Fig. 4.30(a), We know that the shear stress on a layer CD at a distance y from the neutral axis.

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib} \quad \dots \text{(Refer 4.30)}$$

where F = Shear force at the section

$A \cdot \bar{y}$ = Moment of the shaded area about the neutral axis,

r = Radius of the circular section,

I = Moment of inertia of the circular section and

b = Width of the strip CD

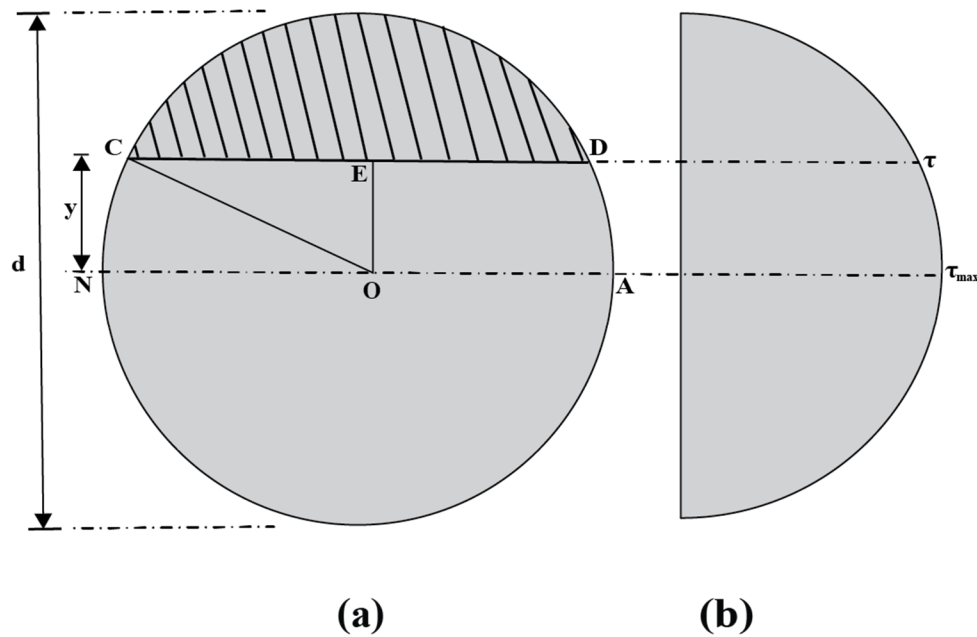


Fig. 4.30(a)-(b): Shear stress distribution in circular section

We know that in a circular section,

Consider a strip CD of thickness dy at a distance y from the neutral axis.

Then,

$$\begin{aligned} \therefore \quad b &= CD \\ b &= 2 \times CE & (\because CD = 2 \times CE) \\ b &= 2 \sqrt{r^2 - y^2} \end{aligned}$$

(In right angle triangle OCE , side $CE = \sqrt{r^2 - y^2}$)

And area of the shaded strip,

$$dA = 2 \sqrt{r^2 - y^2} . dy \quad \dots (4.40)$$

∴ Moment of this area dA about the neutral axis

$$\begin{aligned} &= y \times dA && \because (dA = 2 \sqrt{r^2 - y^2} . dy) \\ &= 2 y \sqrt{r^2 - y^2} . dy \end{aligned}$$

Now moment of the whole shaded area about the neutral axis may be found out by integrating the above equation between the limits y and r,

$$\begin{aligned} A . \bar{y} &= \int_y^r 2y \sqrt{r^2 - y^2} . dy \\ A . \bar{y} &= \int_y^r b . y . dy && \left(\because b = 2\sqrt{r^2 - y^2} \right) \quad \dots (4.41) \end{aligned}$$

We know that width of the strip CD,

$$b = 2\sqrt{r^2 - y^2}$$

$$\text{or} \quad b^2 = 4 (r^2 - y^2) \quad (\text{Squaring on both the sides}) \quad \dots (4.42)$$

Differentiating both sides of the above Eqn (4.42),

$$2b . db = 4 (- 2y) dy$$

$$b . db = - 4 y . dy$$

$$\text{or} \quad y . dy = - \frac{1}{4} b . db \quad \dots (4.43)$$

Substituting the value of y . dy in Eqn (4.41)

$$A . \bar{y} = \int_y^r b . \left(- \frac{1}{4} b . db \right)$$

$$A \cdot \bar{y} = -\frac{1}{4} \int_y^r b^2 \cdot db \quad \dots (4.44)$$

We know that at y , width is b and when $y = r$, width $b = 0$. Therefore, the limits of integration may be changed from y to r , from zero to b in Eqn (4.44),

Or
$$A \cdot \bar{y} = \frac{1}{4} \int_0^b b^2 \cdot db$$

$$A \cdot \bar{y} = \frac{1}{4} \int_0^b b^2 \cdot db$$

$$A \cdot \bar{y} = \frac{1}{4} \left[\frac{b^3}{3} \right]_0^b$$

$$A \cdot \bar{y} = \frac{b^3}{12} \quad \dots (4.45)$$

Now substituting this value of $A \cdot \bar{y}$ in our original formula for the shear stress,

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib}$$

$$\tau = F \times \frac{\left(\frac{b^3}{12}\right)}{Ib}$$

$$\tau = F \times \frac{b^2}{12I} \quad \dots (4.46)$$

$$\tau = F \times \frac{(2\sqrt{r^2 - y^2})^2}{12I} \quad \dots \dots (\because b = 2\sqrt{r^2 - y^2})$$

$$\tau = F \times \frac{r^2 - y^2}{3I} \quad \dots (4.47)$$

Thus, we again see that τ increases as y decreases. At the point, where $y = r$, $\tau = 0$, where y is zero, τ is maximum. Thus, the variation of τ with respect to y is a parabolic curve, with a τ is maximum at the neutral axis.

Substituting $y = 0$ and $I = \left(\frac{\pi}{64} \times d^4\right)$ in the above equation,

$$\tau = F \times \frac{\left(\frac{d}{2}\right)^2}{3 \left(\frac{\pi}{64} \times d^4\right)}$$

$$\tau = \frac{4 F}{3 \left(\frac{\pi}{4} \times d^2\right)} = \frac{4}{3} \times \frac{F}{\text{Area}} \quad \therefore \left(\tau_{av} = \frac{F}{\text{Area}} = \frac{F}{\left(\frac{\pi}{4} \times d^2\right)} \right)$$

$$\tau = 1.33 \tau_{av} \quad \dots (4.48)$$

The shear stress distribution diagram is shown in Fig. 4.30(b).

Problem 4.17

A circular beam of 150 mm diameter is subjected to a shear force of 50 kN. Calculate the value of maximum shear stress and sketch the variation of shear stress along the depth of the beam.

Solution

Diameter (d) = 150 mm and

Shear force (F) = 50 kN = 50×10^3 N

We know that area of the beam section,

$$A = \frac{\pi}{4} (d)^2 = \frac{\pi}{4} (150)^2 = 17671.45 \text{ mm}^2$$

∴ Average shear stress across the section

$$\tau_{av} = \frac{F}{A} = \frac{50 \times 10^3}{17671.45} = 2.829 \text{ N/mm}^2 = 2.829 \text{ MPa}$$

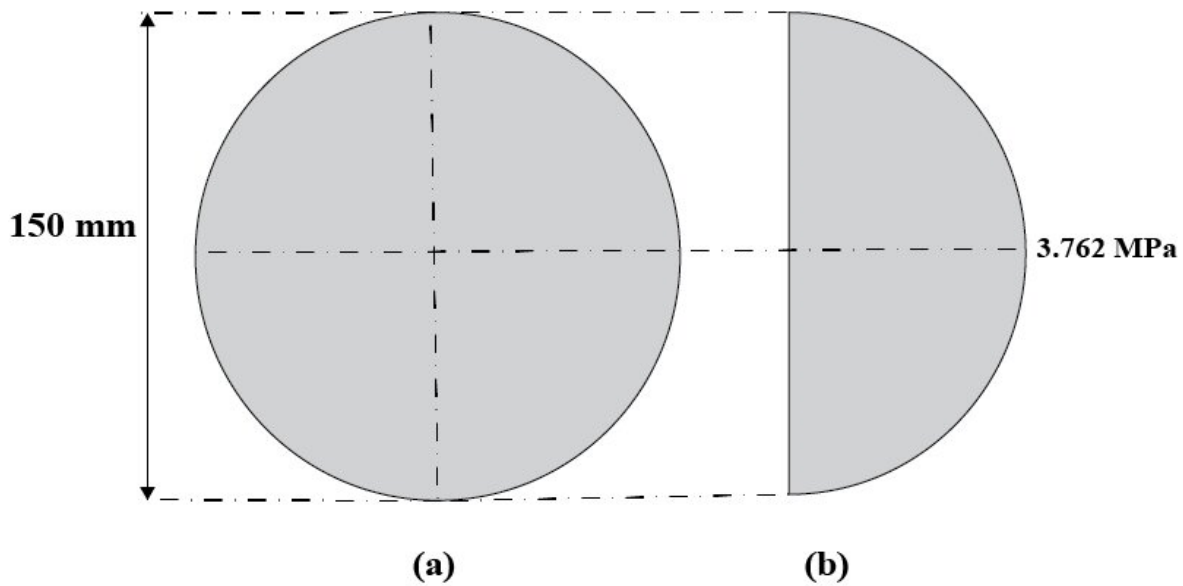


Fig. 4.31(a)-(b): Circular beam under shear force

The maximum shear stress,

$$\tau_{max} = 1.33 \times \tau_{av} = 1.33 \times 2.829 = 3.762 \text{ MPa}$$

The diagram showing the variation of shear stress along the depth of the beam is shown in Fig. 4.31(b).

4.15.4 I – SECTION

Consider a beam of an I – section as shown in Fig. 4.32(a).

Let

B = Overall width of the section

D = Overall depth of the section

d = Depth of the web, and

b = Thickness of the web.

We know that the shear stress of a layer at a distance y from the neutral axis,

$$\tau = F \times \frac{A \cdot \bar{y}}{I \cdot b} \quad \dots (\text{Refer 4.30})$$

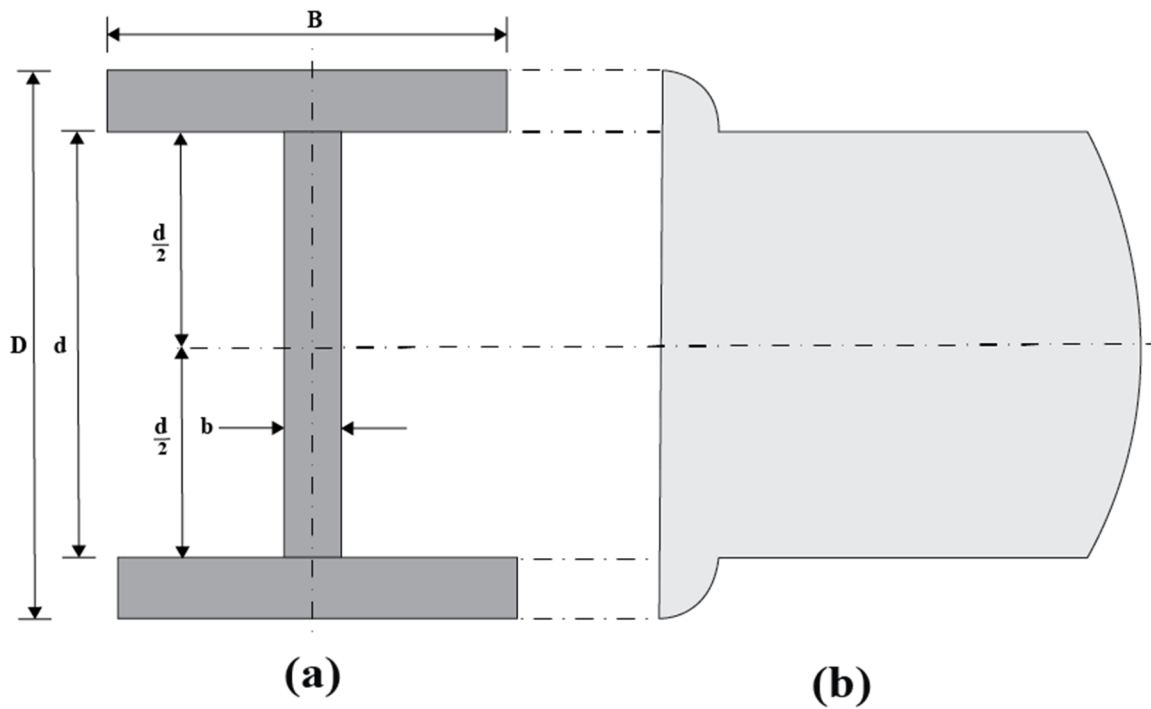


Fig. 4.32(a)-(b): Shear stress distribution in I – section

In this case the shear stress distribution in the web and the shear stress distribution in the flange are to be calculated separately. Let us first calculate the shear stress distribution in the flange.

(i) Shear stress distribution in the flange

Consider a section at a distance y from neutral axis in the flange as shown in Fig. 4.33.

Width of the section = B

Shaded area of the flange, $A = B \left(\frac{D}{2} - y \right)$

$$\bar{y} = y + \frac{1}{2} \left(\frac{D}{2} - y \right)$$

$$\bar{y} = y + \left(\frac{D}{4} - \frac{y}{2} \right)$$

$$\bar{y} = \frac{D}{4} + \frac{y}{2} = \frac{1}{2} \left(\frac{D}{2} + y \right) \quad \dots (4.49)$$

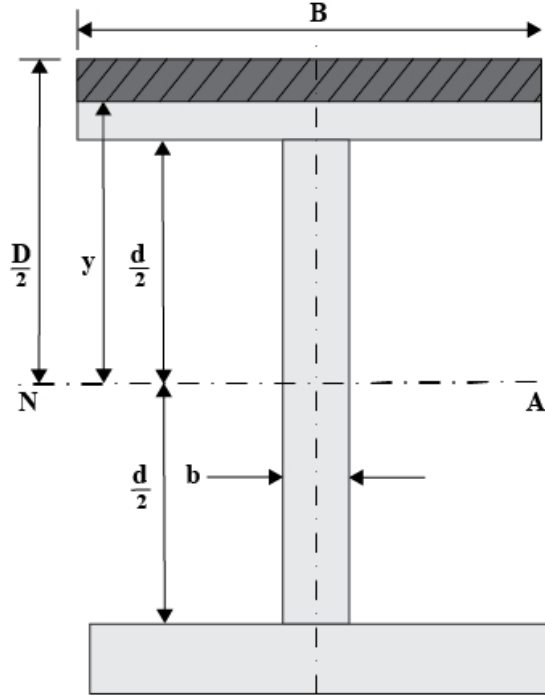


Fig. 4.33: Shear stress distribution in the flange

Hence shear stress in the flange becomes,

$$\tau = F \times \frac{A \cdot \bar{y}}{I B} \quad (\because \text{width} = B)$$

$$\tau = F \times \frac{B \left(\frac{D}{2} - y \right) \cdot \frac{1}{2} \left(\frac{D}{2} + y \right)}{I \cdot B}$$

$$\tau = \frac{F}{2I} \left[\left(\frac{D}{2} \right)^2 - y^2 \right]$$

$$\tau = \frac{F}{2I} \left[\frac{D^2}{4} - y^2 \right] \quad \dots (4.50)$$

Hence the variation of shear stress (τ) with respect to y in the flange is parabolic. It is also clear from Eqn (4.50) that with the increase of y , shear stress decrease.

For the top edge of the flange

$$y = \frac{D}{2} \quad \dots (4.51)$$

Hence shear stress,

$$\begin{aligned} \tau &= \frac{F}{2I} \left[\frac{D^2}{4} - \left(\frac{D}{2} \right)^2 \right] = 0 \\ \tau &= 0 \quad \dots (4.52) \end{aligned}$$

For the bottom edge of the flange

$$y = \frac{d}{2} \quad \dots (4.53)$$

Hence shear stress,

$$\begin{aligned} \tau &= \frac{F}{2I} \left[\frac{D^2}{4} - \left(\frac{d}{2} \right)^2 \right] = \frac{F}{2I} \left[\frac{D^2}{4} - \frac{d^2}{4} \right] \\ \tau &= \frac{F}{8I} \cdot (D^2 - d^2) \quad \dots (4.54) \end{aligned}$$

(ii) Shear stress distribution in the web

Consider a section at a distance y in the web from the neutral axis as shown in Fig. 4.34.

Width of the section = b

Here $A \cdot \bar{y}$ is made up of two parts as moment of the flange area about the neutral axis plus moment of the shaded area of the web about the neutral axis.

$\therefore A \cdot \bar{y}$ = Moment of the flange area about the neutral axis + moment of the shaded area of the web about the neutral axis.

$$\begin{aligned}
 A \cdot \bar{y} &= B \left(\frac{D}{2} - \frac{d}{2} \right) \times \left[\frac{d}{2} + \frac{1}{2} \left(\frac{D}{2} - \frac{d}{2} \right) \right] + \left[b \left(\frac{d}{2} - y \right) \times \frac{1}{2} \left(\frac{d}{2} + y \right) \right] \\
 A \cdot \bar{y} &= B \left(\frac{D}{2} - \frac{d}{2} \right) \times \left[\frac{1}{2} \left(\frac{D+d}{2} \right) \right] + \left[\frac{b}{2} \left(\frac{d}{2} - y \right) \times \left(\frac{d}{2} + y \right) \right] \\
 A \cdot \bar{y} &= B \frac{(D^2 - d^2)}{8} + \left[\frac{b}{2} \left(\frac{d^2}{4} - y^2 \right) \right] \quad \dots (4.55)
 \end{aligned}$$

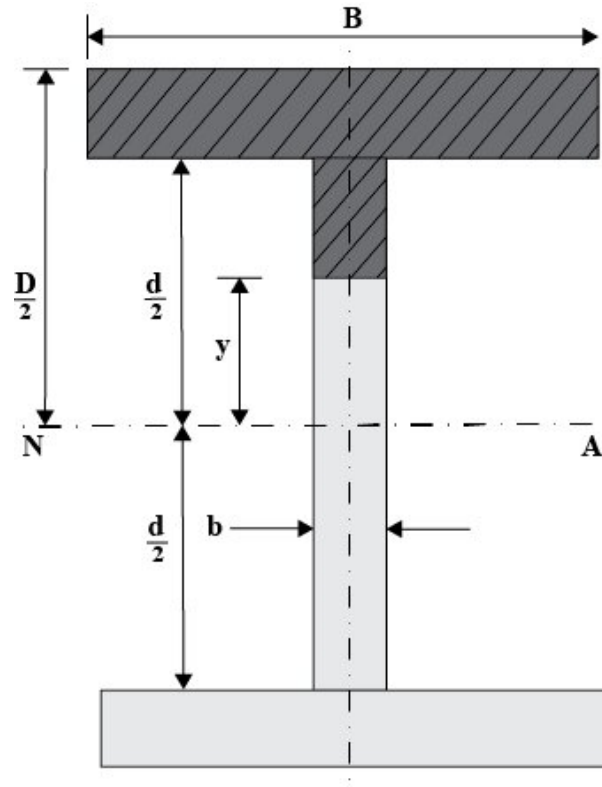


Fig 4.34: Shear stress distribution in the web

Hence the shear stress in the web becomes as

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib} = \frac{F}{IB} \times \left[\frac{B}{8} (D^2 - d^2) + \frac{b}{2} \left(\frac{d^2}{4} - y^2 \right) \right] \quad \dots (4.56)$$

From Eqn (4.56) it is clear that the variation of τ with respect to y is parabolic. Also, with the increase of y , τ decreases.

At the neutral axis, $y = 0$ the shear stress is maximum.

$$\therefore \tau_{\max} = \frac{F}{Ib} \times \left[\frac{B}{8} (D^2 - d^2) + \left(\frac{b}{2} \times \frac{d^2}{4} \right) \right] \quad \dots \dots (\text{Substituting } y = 0)$$

$$\tau_{\max} = \frac{F}{Ib} \times \left[\frac{B}{8} (D^2 - d^2) + \left(\frac{bd^2}{8} \right) \right] \quad \dots (4.57)$$

At the junction of top of the web and bottom of flange,

$$y = \frac{d}{2} \quad \dots (4.58)$$

Hence shear stress is given by,

$$\tau = \frac{F}{Ib} \times \left[\frac{B}{8} (D^2 - d^2) + \frac{b}{2} \left(\frac{d^2}{4} - \left(\frac{d}{2} \right)^2 \right) \right] \quad \dots \dots (\text{Substituting } y = \frac{d}{2})$$

$$\tau = \frac{F \times B(D^2 - d^2)}{8 I \times b} \quad \dots (4.59)$$

Problem 4.18

An I-section beam $400 \text{ mm} \times 300 \text{ mm}$ has a web thickness of 15 mm and a flange thickness of 30 mm . It carries a shearing force of 300 kN at a section. Sketch the shear stress distribution across the section.

Solution

Depth (d) = 400 mm

Flange width (B) = 300 mm

Width of web = 15 mm

Flange thickness = 30 mm and

Shear force (F) = 300 kN = 300×10^3 N

We know that moment of inertia of the I-section about its C.G. and parallel to x - x axis,

$$I_{xx} = \left(\frac{300 \times 400^3}{12} \right) - \left(\frac{285 \times 340^3}{12} \right) = 66653 \times 10^4 \text{ mm}^4$$

We also know that shear stress at the upper edge of the upper flange is zero. The shear stress at the both of the upper flange is (Refer Eqn 4.54)

$$\tau = \frac{F}{8I} [D^2 - d^2] = \frac{300 \times 10^3}{8 \times (66653 \times 10^4)} [(400)^2 - (340)^2]$$

$$\tau = 2.498 \text{ N/mm}^2 = 2.498 \text{ MPa}$$

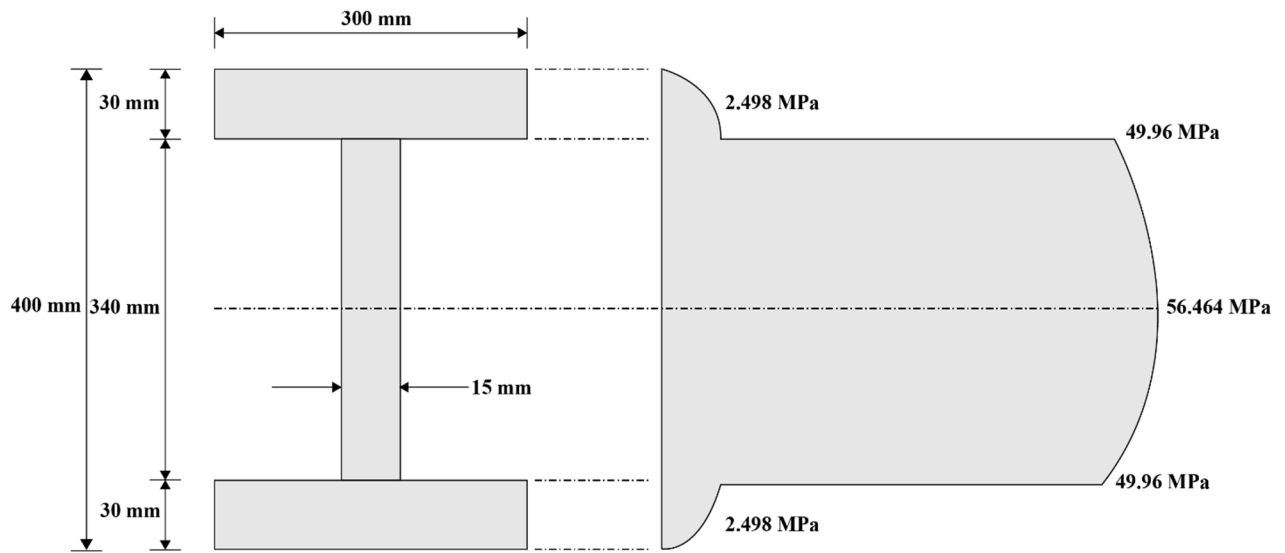


Fig. 4.35(a)-(b): Shear stress distribution in I-Section beam due to shear force

The shear stress at the junction suddenly increases from 2.498 MPa to $\left(2.498 \times \frac{300}{15} \right) = 49.96$ MPa.

We also know that the maximum shear stress, (Refer Eqn 4.59)

$$\tau_{\max} = \frac{F}{Ib} \times \left[\frac{B}{8} (D^2 - d^2) + \left(\frac{bd^2}{8} \right) \right]$$

$$\tau_{\max} = \frac{300 \times 10^3}{(66653 \times 10^4) \times 15} \left[\frac{300}{8} (400^2 - 340^2) + \left(\frac{15 \times 340^2}{8} \right) \right]$$

$$\tau_{\max} = 56.464 \text{ N/mm}^2 \text{ or } 56.464 \text{ MPa}$$

Now complete the shear stress distribution diagram across the section as shown in Fig. 4.35(b).

4.15.5 T - SECTION

The shear stress distribution over a T-section is obtained in the same manner as over an I-section. But in this case the position of neutral axis (i.e., position of C.G.) is to be obtained first, as the section is not symmetrical about x-x axis. The shear stress distribution diagram will also not be symmetrical.

Problem 4.19

A T- shaped cross section of a beam shown in Figure 4.36 is subjected to a vertical shear force of 50 kN. Calculate the shear stress at important points and draw shear stress distribution diagram. Moment of inertia about the horizontal neutral axis is $314.221 \times 10^4 \text{ mm}^4$.

Solution

Shear Force (F) = 50 kN = $50 \times 10^3 \text{ N}$

Moment of inertia about the horizontal neutral axis is

$$I = 314.221 \times 10^4 \text{ mm}^4$$

First calculate the position of neutral axis. This can be obtained if we know the position of C.G. of given T-section. The given section is symmetrical about the axis Y-Y and hence the C.G. of the section will lie on Y-Y axis.

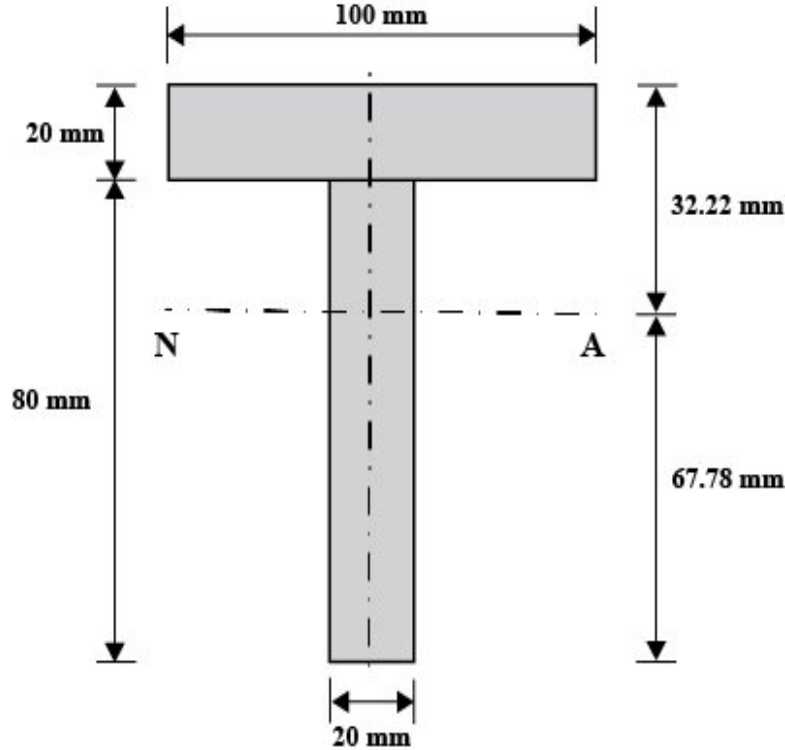


Fig. 4.36: T-Shaped cross section beam under vertical shear force

Let y = Distance of the C.G of the section from the top of the flange.

$$\begin{aligned} \text{Then } y &= \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} \\ y &= \frac{[(100 \times 20) \times 10] + [(20 \times 80) \times (20 + \frac{80}{2})]}{(100 \times 20) + (20 \times 80)} \\ y &= \frac{20000 + 96000}{2000 + 1600} = 32.222 \text{ mm} \end{aligned}$$

Hence neutral axis will be at a distance of 32.222 mm from the top of the flange as shown in Fig. 4.36.

Where $A = 20 \times 100 = 2000 \text{ mm}^2$

$\bar{y}$ = Distance of C. G of the area of flange from neutral axis

$$\bar{y} = 32.22 - \frac{20}{2} = 22.22 \text{ mm}$$

b = Width of flange = 100 mm

$$\therefore \tau = \frac{50000 \times 2000 \times 22.22}{314.221 \times 10^4 \times 100} = 7.07 \text{ N/mm}^2$$

Shear stress distribution in the web

The shear stress in the web just at the junction of the web and flange will suddenly increase from 7.07 N/mm^2 to $7.07 \times \frac{100}{20} = 35.35 \text{ N/mm}^2$.

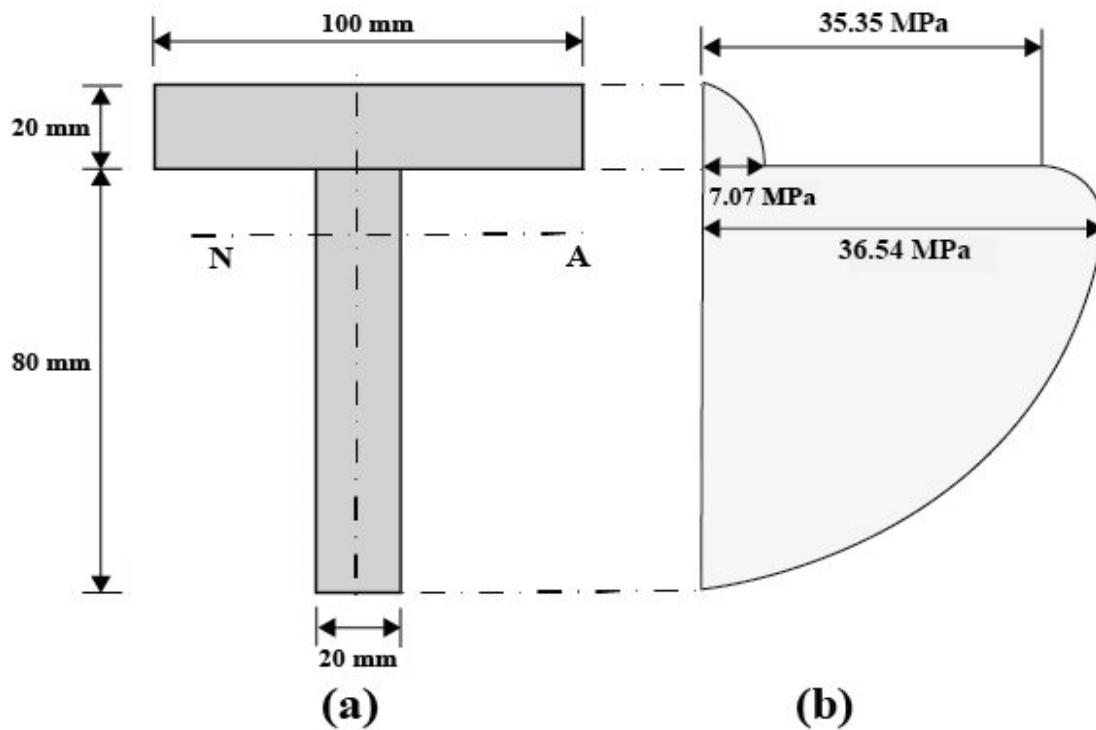


Fig. 4.37 (a)-(b): Shear stress distribution diagram of T-section

The shear stress will be maximum at neutral axis. Hence the shear stress at the neutral axis is given by,

$$\tau = F \times \frac{A \cdot \bar{y}}{Ib}$$

Where $A \cdot \bar{y}$ = Moment of the area above the neutral axis.

= Moment of area of flange about the neutral axis + Moment of area of web above the neutral axis.

$$= (20 \times 100) \times (32.22 - 10) + 20 \times (32.22 - 20) \times \frac{12.22}{2}$$

$$= 44440 + 1493.284 = 45933.284 \text{ mm}^2$$

Where $b = 20 \text{ mm}$

$$\therefore \tau = \frac{50000 \times 45933.284}{314.22 \times 10^4 \times 20} = 36.54 \text{ N/mm}^2$$

Now the shear stress distribution diagram can be drawn as shown in Fig.4.37(b).

UNIT SUMMARY

- The unit on bending and shear stress in beams provides students with a comprehensive understanding of how beams respond to applied loads, with a focus on the distribution of bending and shear stresses.
- Beginning with an exploration of different types of supports and beams, students learn to identify and classify various structural elements and their behaviors under different loading conditions.
- They then delve into the types of loads commonly applied to beams, including uniformly distributed, uniformly varying, and concentrated loads, gaining insights into their effects on beam structures.
- Moving on, students explore the concepts of shear force and bending moment, essential for analyzing the internal forces and moments within beams. Through practical examples and problem-solving exercises, students learn to construct shear force and bending moment diagrams for different types of beams and loading configurations, enabling them to visualize and understand the distribution of forces and moments along the length of the beam.
- By the end of the unit, students will have acquired a solid foundation in bending and shear stress analysis, equipping them with the skills necessary to excel in fields such as structural engineering, mechanical engineering, and civil engineering.

EXERCISE

A. Multiple Choice Questions

- 4.1 For steel, the ultimate strength in shear as compared to ultimate strength in tension is
(a) Same (c) $1/4$
(b) $1/2$ (d) $2/3$
- 4.2 If section modulus of a beam is increased the bending stress in the beam will
(a) Increase (c) Become zero
(b) Decrease (d) Not change
- 4.3 The strength of beam section mainly depends on
(a) Intensity of stress (c) Section modulus
(b) Moment of resistance (d) Bending moment
- 4.4 The SI units of stress is
(a) kg/sq.cm (c) N/sq.m
(b) N/sq.mm (d) kg/sq.m
- 4.5 Neutral axis is the axis at which stress is
(a) Zero (c) Positive
(b) Negative (d) Infinite
- 4.6 In the case of triangular section, the shear stress is maximum at the
(a) Neutral axis (c) Height of $h/2$
(b) Height of $2h/3$ (d) Centre of gravity
- 4.7 In a simply supported rectangular beam loaded transversely, the maximum tensile bending stress occurs at
(a) Top fiber (c) Neutral axis
(b) Bottom fiber (d) Between top fiber and neutral axis
- 4.8 An inverted T-section is subjected to a shear force F . the maximum shear stress will occur at
(a) Top of the section (c) Neutral axis of the section
(b) Junction of web and flange (d) Bottom of the section

- 4.9 In the theory of simply bending, the bending stress in the beam section varies
(a) Linearly (c) Elliptically
(b) Parabolically (d) None of them
- 4.10 A beam of uniform strength has constant
(a) Shear force (c) Cross-sectional area
(b) bending moment (d) Deflection
- 4.11 When a rectangular section of a beam is subjected to a shearing force, the ratio of maximum shear stress to the average shear stress is
(a) 2.0 (c) 1.5
(b) 1.75 (d) 1.25

Answer to Multiple-Choice Questions

4.1 (d), 4.2 (b), 4.3 (c), 4.4 (c), 4.5 (a), 4.6 (c), 4.7 (b), 4.8 (c), 4.9 (a), 4.10 (b), 4.11 (c)

B. Subjective Questions

- 4.1 Discuss the procedure to find the bending stress in symmetrical section?
- 4.2 State the assumption made in the theory of simple bending?
- 4.3 How will you find the bending stress in a hollow circular section?
- 4.4 Explain the terms: Neutral axis, Section modulus, and moment of resistance?
- 4.5 What do you mean by section modulus? Find the expressions for section moduli for a rectangular, circular and hollow circular sections.
- 4.6 How will you draw the shear stress distribution diagram for composite section?
- 4.7 Derive the expression for the shear stress at any point in a circular section of a beam which is subjected to a shear force F .

- 4.8 Show that for a beam subjected to pure bending, neutral axis coincides with the centroid of the cross-section.
- 4.9 What do you mean by strength of a section?

C. Numerical Problems

1. A rectangular beam 100 mm wide and 150 mm deep is subjected to a shear force of 30 kN. Determine: (i) Average shear stress and (ii) Maximum shear stress.
(Ans: 2 N/mm²; 3 N/mm²)
2. A rectangular beam 100 mm wide is subjected to a maximum shear force of 100 kN. Find the depth of the beam if the maximum shear stress is 6 N/mm².
(Ans: 250 mm)
3. A timber beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The length of the beam is 6 m and depth of beam is 1 m. Determine the maximum bending stress and the maximum shear stress.
(Ans. 12 N/mm²; 1 N/mm²)
4. A beam of circular cross-section with 105 mm diameter is subjected to a shear force of 5 kN. Calculate: (i) average shear stress, and (ii) maximum shear stress. Also sketch the variation of the shear stress along the depth of the beam.
(Ans: (i) 0.577 N/mm² (ii) 0.769 N/mm²)
5. The maximum shear stress in a beam of circular section of diameter 150 mm, is 5.28 N/mm². Find the shear force to which the beam is subjected.
(Ans. 70 kN)
6. An I -section beam consists of flanges 150 mm × 20 mm and a web of 310 mm × 10 mm. Find the magnitude of maximum shear stress when it is subjected to a shear force of 40kN and draw the shear stress distribution diagram over the depth of the section.
(Ans: 13.1 MPa)

7. A T-section beam with $100\text{ mm} \times 15\text{ mm}$ flange and $150\text{ mm} \times 15\text{ mm}$ web is subjected to a shear force of 10 kN at a section. Draw the variation of shear stress across the depth of the beam and obtain the value of maximum shear stress at the section. (Ans: 6.3 MPa)

KNOW MORE

- *Civil Engineering:* In the design and construction of bridges, buildings, and other structures, engineers must consider bending and shear stresses to ensure the safety and stability of the structures. Understanding these concepts helps engineers determine the appropriate size, shape, and materials for structural members to withstand anticipated loads.
- *Mechanical Engineering:* Bending and shear stress analysis is crucial in the design of mechanical components such as beams, shafts, and frames. Engineers use these concepts to optimize the design of machine elements for strength, stiffness, and reliability, ensuring that they can withstand the forces and moments encountered during operation.
- *Aerospace Engineering:* In the aerospace industry, where weight and strength are critical factors, engineers rely on bending and shear stress analysis to design aircraft wings, fuselages, and other structural components. By understanding these concepts, engineers can optimize the weight-to-strength ratio of aerospace structures, improving fuel efficiency and performance.
- *Automotive Engineering:* Bending and shear stress analysis play a vital role in the design of automotive chassis, frames, and suspension systems. Engineers use these concepts to ensure that vehicles can withstand the forces and vibrations experienced during operation, enhancing safety and durability.

REFERENCE AND SUGGESTED READINGS

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2. D.S. Bedi, “Engineering Mechanics”; Khanna publications, New Delhi.
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4. Punmia B C, Strength of Materials, Laxmi Publications (p) Ltd. New Delhi.
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Dynamic QR Code for Further Reading

Shear stress in beam



Shear stress distribution in Rectangular section



Shear stress distribution in Circular section



Shear stress distribution in Hollow rectangular section



Bending stress distribution concept



Bending stress in beam



5 COLUMNS

UNIT SPECIFICS

Upon completion of this chapter, the reader will have acquired the following abilities:

- *Understand the concept of stability of compression members,*
- *Explain the structural behaviour of columns under load,*
- *Derive the Euler buckling load for columns with different end conditions,*
- *Understand the limitation and applicability of Euler's formulae,*
- *Derive the Rankine-Gordon empirical formula for compression members,*
- *Calculate and compare the buckling load for columns using Euler and Rankine formulae,*
- *Derive the secant formula for columns subjected to eccentric loads, and apply it for calculating permissible loads and eccentricities,*
- *Understand the structural action of beam-columns and derive the beam-column formulae, and*
- *Calculate the permissible load on columns with unsymmetrical section*

This unit is designed to familiarize readers with the fundamental principles of column analysis, essential for understanding the behavior of slender structural elements under compressive loads. Readers will explore key concepts such as buckling, effective length, and column stability, gaining insight into how these

factors influence the design and performance of columns in various engineering applications. The unit covers methods for determining critical loads, assessing column stability, and designing columns to resist buckling and other failure modes. Additionally, readers will learn about different types of columns, including short, long, and intermediate columns, and the factors that govern their behavior. Through clear explanations and illustrative examples, this unit provides a comprehensive overview of column analysis, equipping readers with the knowledge and skills necessary to design safe and efficient columns. For students or professionals in civil, and mechanical engineering, this unit serves as an essential guide for mastering the principles of column design and analysis.

Apart from this at the end of the unit, a succinct recapitulation of the overall broad concepts is provided in the form of a unit summary. Besides, a large number of multiple-choice questions as well as descriptive type questions with Bloom's taxonomy action verbs are included. A list of references and suggested readings are given in the unit so that one can go through them for practice. It is important to note that for getting more information on various topics of interest some QR codes have been provided in different sections which can be scanned for relevant supportive knowledge. Video resources along with QR codes are given for getting more information on various topics of interest which can be surfed or scanned through mobile phones for viewing.

RATIONALE

The concept of columns is extensively utilized in structural engineering and architecture, playing a crucial role in the design and construction of buildings, bridges, and other structures. Columns are structural members primarily designed to support axial compressive loads, transferring these loads to the foundation

below. Understanding the behavior of columns is essential for ensuring structural stability, safety, and efficiency. Engineers consider various factors such as column geometry, material properties, loading conditions, and support conditions when designing columns to withstand anticipated loads without failure. Overall, the concept of columns is foundational in structural analysis and design, influencing the planning and construction of a wide range of engineered structures across different industries and disciplines.

PRE-REQUISITES

*Before reading this unit, the students are advised to revisit the following:
Engineering Mechanics: Sem I*

UNIT OUTCOMES

After completing this unit, you will be able to:

- U5-O1: Understand the fundamental concepts of column behavior, including buckling and stability.*
- U5-O2: Identify different types of columns based on their geometrical configurations and loading conditions.*
- U5-O3: Analyze the critical load for columns using appropriate theoretical approaches, such as Euler's formula or the tangent modulus method.*
- U5-O4: Recognize the influence of material properties, column geometry, and boundary conditions on column behavior and performance.*
- U5-O5: Apply knowledge of column analysis to practical engineering problems, including structural design and optimization for various applications.*

Unit-5 Outcomes	EXPECTED MAPPING WITH COURSE OUTCOMES <i>(1-Weak Correction; 2- Medium Correction; 3- Strong Correction)</i>				
	CO-1	CO-2	CO-3	CO-4	CO-5
U5-O1	3	2	1	2	1
U5-O2	3	2	1	2	1
U5-O3	3	2	1	2	1
U5-O4	3	2	1	2	1
U5-O5	3	2	1	2	1

5.1 INTRODUCTION

A column or strut is a structural element that experiences an axial compressive force. If the structural element in civil structures oriented vertically and securely fixed on both ends while subjected to axial compression it is categorized as a **column**, like a vertical pillar that supports a roof and a floor. On the other hand, when the structural element is not oriented vertically and has one or both ends connected by hinges or pins, it is referred to as a strut in mechanical systems. Examples of struts include connecting rods, piston rods, and similar components.

5.2 BEHAVIOUR AND CLASSIFICATION OF COLUMNS

Consider the column member shown in Fig. 5.1(a). When the column member carries an axial load P , its behaviour is governed by the factors such as (i) the length of the member, (ii) the size (cross section), (iii) end conditions, and (iv) material properties. It is to be observed that the strength of a column exhibits an inverse relationship with the square of its length, a direct correlation with its cross-section and the material properties. The end conditions, shown in Fig. 5.1(b), affect the strength because the end restraints imposed affect the stability characteristics.

When a column is short, where 'short' is defined in all design codes based upon the ratio of length and the least lateral dimension or the radius of gyration, and the load is truly axial, the average value can always be calculated as $\sigma = \text{load} / \text{area}$.

The strength of the column or its load-carrying capacity may be determined as $[\text{stress} \times \text{area}]$, where σ can have a suitable value. For example, in the case of steel, σ is the yield stress of the material. Beyond the yield stress, the material will yield, and the strength above this limit will have no practical use. For practical design,

the same formula can be used, where σ_Y being the yield stress for ultimate strength design or yield stress divided by a safety factor for working stress design.

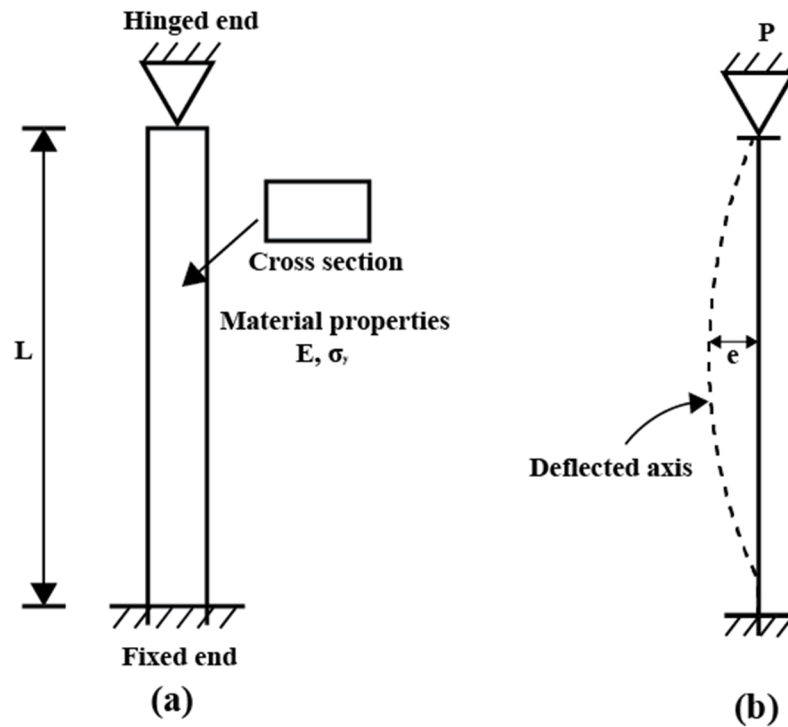


Fig. 5.1: Simply supported column subjected to axial load P

There are many factors, which control the behaviour of columns. The load may not be truly axial, due to the imperfections of the column. This imperfection essentially led to buckling, i.e., lateral deflection of the bar. When such imperfections are less and the column is short, the behaviour of the column is governed by the direct compressive stress as discussed earlier. If the column is long, the effect of buckling becomes significant and becomes the principal cause of failure.

Consider the column shown in Fig. 5.1(b), due to imperfection in material, the accidental eccentricity of the load, or the centroid line being initially not straight, the section is subjected to bending moment due to the compressive load.

For smaller values of load, the column is under compression. Long columns are inherently prone to buckling, and this lateral deflection causes a bending moment. The longer the column, the smaller the load value required to initiate lateral deflection. The column cross-section experiences both direct stress caused by the axial force P and bending stress induced by the bending moment (P_e). The column, therefore, tends to deflect laterally. It makes long columns behave much differently from short columns. For short columns, lateral deflections, and bending stress are small, and the direct stress dominate. For long columns, however, bending stress become significant.

5.2.1 AXIALLY LOADED COMPRESSION MEMBERS

Axially loaded compression members may thus be classified into short, medium, and long columns. Column failure transpires due to the presence of one of the subsequent stress conditions within the columns:

- Compressive stress in the axial direction,
- Buckling or bending stress, and
- Concurrent combination of axial compression and buckling or bending stress.

5.2.1.1 SHORT COLUMN

Consider a short column of uniform cross-sectional area ' A '. This column is subjected to compressive load P , shown in Fig. 5.2. Thus, the resultant compressive stress in the column is expressed as:

$$\sigma = \frac{P}{A}$$

As the column is exposed to increasing loads, it will eventually reach a stage where it experiences the ultimate crushing stress. Beyond this point, the column will fail due to crushing. The load at which this crushing occurs is referred to as the crushing load.

Let

P_c - Crushing load,

σ_c - Crushing stress

A - Cross-sectional area.

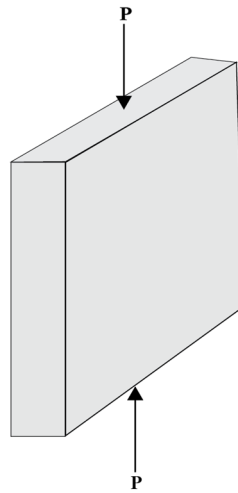


Fig. 5.2: Figure showing the short column, aspect ratio (h/b) less than 2.0

Then
$$\sigma_c = \frac{P_c}{A} \quad \text{.....(5.1)}$$

Crushing is the main cause of failure in all short columns.

5.2.1.2 LONG COLUMN

Unlike short columns, long columns do not fail solely due to crushing but rather undergo bending, a phenomenon known as buckling. Consider a long column with a uniform cross-sectional area ' A ' and a length ' L ' as shown in Fig. 5.3. A column is classified as "**long column**" when its length in relation to its lateral dimensions is significantly large.

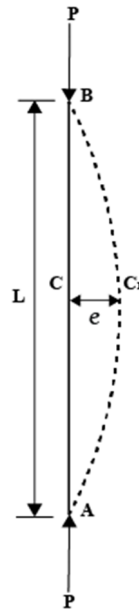


Fig. 5.3: Figure showing the long column

Let

L - Length of the column.

P - Compressive load

A - Cross-sectional area of the column.

e - Maximum deflection at the centre caused by bending.

σ_o - Stress caused by a direct load, calculated as $\frac{P}{A}$

σ_b - Stress caused by bending at the column's center, determined as $\frac{P \cdot e}{Z}$

Where, the section modulus is represented by Z.

The following are the stress in the midsection:

Minimum stress = $\sigma_o - \sigma_b$; Maximum stress = $\sigma_o + \sigma_b$

Column failure occurs when the maximum value of stress ($\sigma_o + \sigma_b$) exceeds the crushing stress σ_c . Long column fail mainly due to bending stresses and compressive load have negligible role in failure.

5.2.1.3 END CONDITIONS OF COLUMNS

In practice, columns are used with different end conditions. Nonetheless, our attention will be directed towards analyzing Euler's column theory as applied to the four notable end conditions as shown in Fig. 5.4.

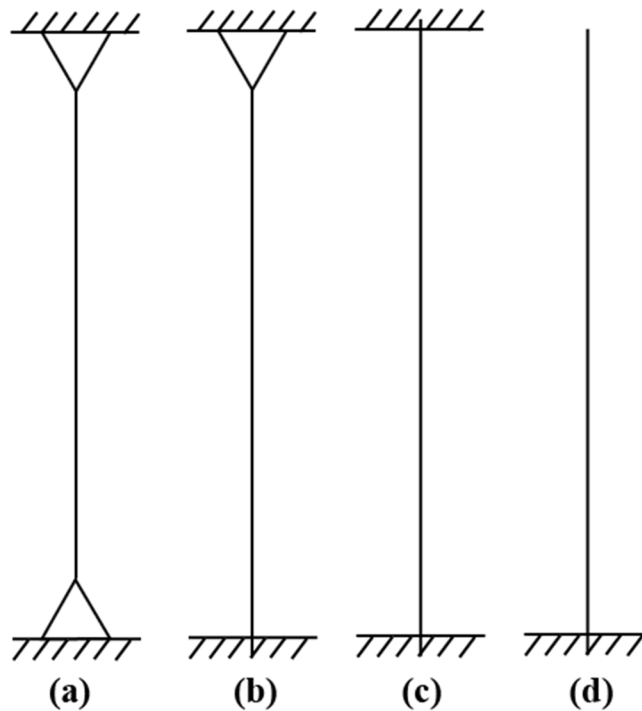


Fig. 5.4: Figure showing the different end conditions: (a) Both ends are hinged, (b) One end is fixed, while the other is hinged, (c) Both ends are fixed, (d) One end is fixed, and the other end is free.

These conditions are sufficient enough to understand the behaviour of the column for any loading condition, these include:

- Both ends are hinged,
- One end is fixed, and the other is hinged,
- Both ends are fixed, and
- One end is fixed, and the other end is free.

5.2.1.4 STABILITY

Stability is an important factor in the behaviour of long columns. Consider the case of ball resting on three different surfaces as shown in Fig. 5.5 which can be stated as stable, neutral, and unstable equilibrium.

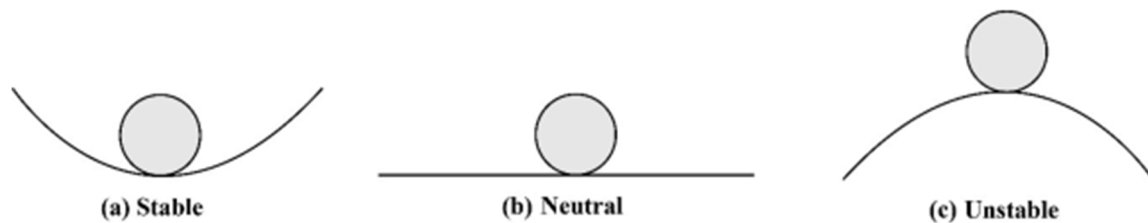


Fig. 5.5: Fig. showing the equilibrium positions: (a) Stable, (b) Neutral equilibrium and (c) Unstable

Assume a small force is applied to all the three cases. The ball in Fig. 5.5(a) will be back to equilibrium after removal of the load is known as stable equilibrium. The ball in Fig. 5.5(b) changes to a new equilibrium position which is known as neutral equilibrium. The ball in Fig. 5.5(c) will not be back to equilibrium even after some time. Hence, we can say that unstable equilibrium.

Let us consider a column as shown in Fig. 5.6, the column is in stable equilibrium when the load P is small. It can be demonstrated by applying a lateral load and deflecting the column. The deflected shape shown by dotted lines can be maintained only so long as the lateral force exists. Upon the removal of the lateral force, the column reverts back to its initial straight position, which is the stable equilibrium of the column.

When the value of P is progressively increased and reaches a particular value P , the column does not return to the initial position when deflected by a small lateral force, even after this force is removed.

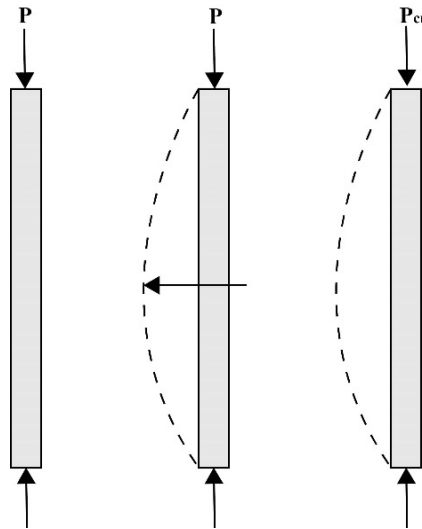


Fig. 5.6: Stable equilibrium of column with load P

The deflected position is the new equilibrium position, and hence we can say that the column is in neutral equilibrium. Further increase in the value of P makes increases the deflection and causes the failure of the column by buckling and the column is in the state of unstable equilibrium.

5.2.1.5 CRITICAL OR BUCKLING LOAD

The maximum value of the load, P_{cr} , up to which the column remains in a position of neutral equilibrium, is known as the critical load of the column. At the critical load, the column can potentially maintain equilibrium in either a straight or deflected position.

5.3 EULER'S COLUMN THEORY

This theory generally deals with the stability and strength of slender columns under compressive loads. Euler took the initial systematic approach to analyze the stability of long columns. He developed an equation to describe the buckling of long columns, primarily accounting for bending stress. The impact of direct stress is neglected during the formulation of this equation, since the direct stress induced

in a long column is negligible in comparison to the bending stress. It is to be noted that Euler's formula is not applicable for short columns due to the significant presence of direct stress, which cannot be ignored.

5.3.1 ASSUMPTION IN EULER'S COLUMN THEORY

Euler's column theory relies on the following simplifying assumptions:

- The column is initially straight, and the applied load is purely axial.
- The column maintains a uniform cross-section throughout the length.
- The material is homogeneous, and isotropic.
- Load applied is within the elastic limit and the material obeys Hooke's law.
- The column's length significantly exceeds its cross-sectional dimensions.
- Any reduction in column length caused by direct compression is disregarded.
- Column failure exclusively results from buckling.

5.3.2 SIGN CONVENTIONS

The following sign convention are followed in considering the moment:

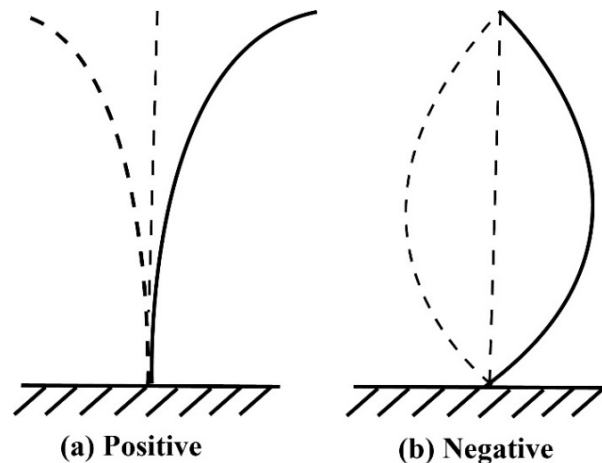


Fig. 5.7: Sign convention: (a) Positive bending moment and
(b) Negative bending moment

- A **positive** moment causes the column to bend with its curvature directed away from its original centre line, as seen in Fig. 5.7(a).
- A **negative** moment causes the column to bend with its curvature orientated towards its original centre line, as shown in Fig. 5.7(b).

5.3.3 BOTH ENDS ARE HINGED

Consider a column AB, of length "L," and hinged at both ends. This column bears a critical load at point B. Upon applying the load, the column undergoes deformation, adopting a curved shape represented as ABC, as shown in Fig. 5.8.

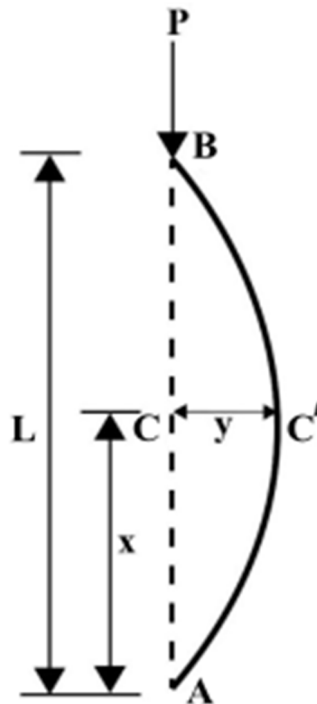


Fig. 5.8: Figure showing the both ends are hinged

Consider the section CC_1 , which is at a distance x from point A

Let

P - Critical load

y - The central deflection at CC_1

The moment induced by the (P) critical load can be expressed as:

$$M = - P \times y \quad \text{..... (5.2)}$$

We know that Moment (M) = $EI \frac{d^2y}{dx^2}$

$$\therefore EI \frac{d^2y}{dx^2} = -P \times y$$

(- sign owing to concavity towards actual centre line)

$$\therefore EI \frac{d^2y}{dx^2} + (P \times y) = 0 \quad \text{Or}$$

$$\therefore \frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = 0$$

To find the value of y, we need to solve this differential equation

$$y = A. \cos \left(x \sqrt{\frac{P}{EI}} \right) + B. \sin \left(x \sqrt{\frac{P}{EI}} \right)$$

Where A and B are integration constants. To find the values of constants, we will use the boundary conditions. Notably, when $x = 0$, $y = 0$ gives $A = 0$. Similarly, at $x = L$, $y = 0$, implies that

$$0 = B \sin \left(L \sqrt{\frac{P}{EI}} \right)$$

It becomes evident that either B must be zero or $\sin \left(L \sqrt{\frac{P}{EI}} \right)$ should be zero.

Hence, for non-trivial solutions B cannot be zero.

Thus

$$\sin \left(L \sqrt{\frac{P}{EI}} \right) = 0 \quad \dots\dots (5.3)$$

$$\text{gives } \left(\frac{P}{EI} \right)^{\frac{1}{2}} L = n\pi, \text{ where } n = 1, 2, 3, \dots\dots$$

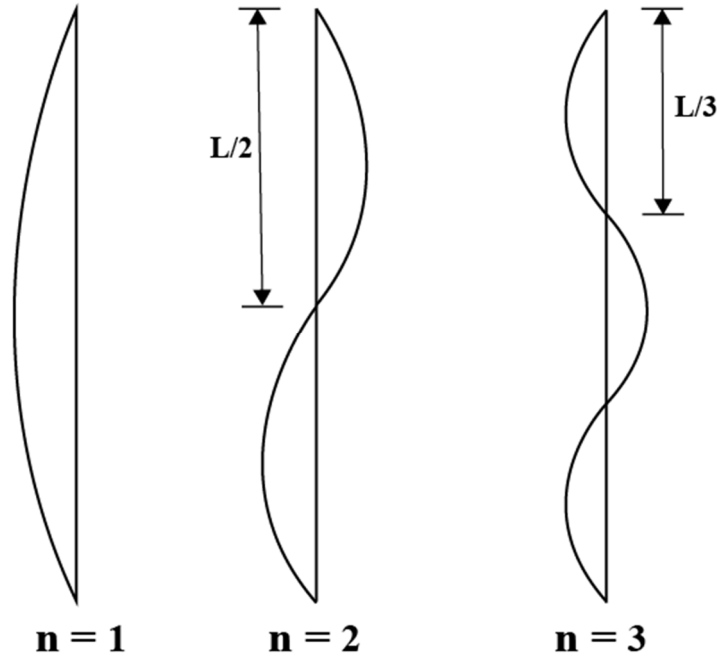


Fig. 5.9: Deflected shapes for various n values, ranging from $n = 1$ to 3.

For different values of n , the deflected shapes will have different shapes as shown in Fig. 5.9. $n = 1$ gives the deflected shape with the elastic curve which is half sin wave. $n = 2$ gives the elastic shape showing the full sin wave. The lowest buckling load corresponding to $n = 1$ is given by

$$\left(\frac{P}{EI} \right)^{\frac{1}{2}} L = \pi \quad \text{(Squaring on both the sides)}$$

$$P = \frac{\pi^2 EI}{L^2} \quad \dots\dots (5.4)$$

This critical load pertains to a column of length ' L ' with both ends hinged.

5.3.4 ONE END IS FIXED, AND THE OTHER FREE

Consider a column AB of length 'L' which is fixed at point A and is free at point B. It bears a critical load at point B. Due to this loading the column deflects. Let the deflected shape be as AC₁B₁. As a result, the free end B moves by a distance of "a," as shown in Fig 5.10. Let us consider the point C at distance x from point A.

P - Critical load

y - Column deflection at point C

The moment induced by the applied load P can be expressed as:

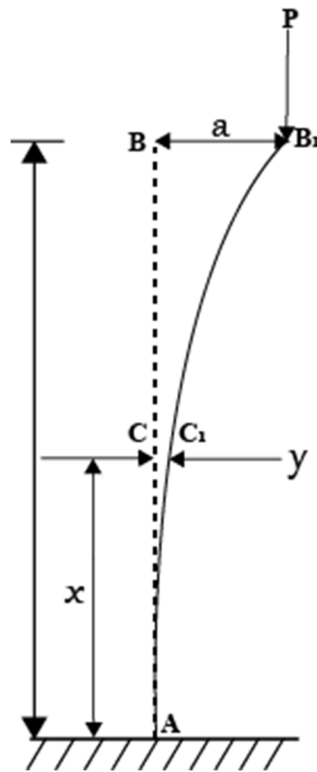


Fig. 5.10: Figure showing the one end is fixed, while the other free

$$M = + P. (a - y)$$

$$M = (P. a) - (P. y)$$

(- sign owing to concavity towards actual center line)

We know that $M = EI \frac{d^2y}{dx^2}$

$$\therefore EI \frac{d^2y}{dx^2} = (P \cdot a) - (P \cdot y)$$

Or

$$EI \frac{d^2y}{dx^2} + (P \cdot y) = (P \cdot a) \quad \dots \dots (5.5)$$

Dividing both the side by EI

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI} \cdot y \right) = \frac{(P \cdot a)}{EI}$$

To find the value of y, will have to solve this differential equation:

$$y = A \cos \left(x \sqrt{\frac{P}{EI}} \right) + B \sin \left(x \sqrt{\frac{P}{EI}} \right) + a \quad \dots \dots (5.6)$$

Where A and B are integration constants. To find the values of A and B, we apply the boundary condition. Given that when $x = 0$, $y = 0$, it follows that $A = -a$. Now, differentiating the previous equation:

$$\frac{dy}{dx} = -A \sqrt{\frac{P}{EI}} \sin \left(x \sqrt{\frac{P}{EI}} \right) + B \sqrt{\frac{P}{EI}} \cos \left(x \sqrt{\frac{P}{EI}} \right)$$

We know that when $x = 0$, the value of slope $\frac{dy}{dx}$ is 0. Thus:

$$0 = B \sqrt{\frac{P}{EI}}$$

With further consideration, it becomes clear that either B must be zero or $\sqrt{\frac{P}{EI}}$ must be zero. Given that load P is not equal to zero, it is evident that B equals zero. Substituting the values $A = -a$ and $B = 0$ in the Eqn. (5.6):

$$y = a - a \cos \left(x \sqrt{\frac{P}{EI}} \right)$$

$$y = a \left[1 - \cos \left(x \sqrt{\frac{P}{EI}} \right) \right] \quad \dots \dots (5.7)$$

Additionally, applying the boundary conditions when $x = L$, $y = a$. Thus:

$$a = a \left[1 - \cos \left(L \sqrt{\frac{P}{EI}} \right) \right]$$

$$\cos \left(L \sqrt{\frac{P}{EI}} \right) = 0$$

gives $\left(\frac{P}{EI} \right)^{\frac{1}{2}} L = \frac{n\pi}{2}$, Where $n = 1, 2, 3, \dots$

$$\left(\frac{P}{EI} \right)^{\frac{1}{2}} L = \frac{n\pi}{2} \quad \dots \dots (5.8)$$

By squaring both sides of the equation (5.8), we obtain:

$$\left(\frac{P}{EI} \right) L^2 = \left(\frac{n\pi}{2} \right)^2$$

$$\left(\frac{P}{EI} \right) L^2 = \left(\frac{n^2 \pi^2}{4} \right)$$

$$P = \left(\frac{n^2 \pi^2 EI}{4L^2} \right) \quad \dots \dots (5.9)$$

The lowest buckling load corresponding to $n = 1$, is given by

$$P_{cr} = \left(\frac{\pi^2 EI}{4L^2} \right) \quad \dots \dots (5.10)$$

5.3.5 BOTH ENDS FIXED

Consider a column AB of length L fixed at both ends, A and B. This column is subjected to a critical load at point B. The column deflects due to this loading, as shown in Fig. 5.11.

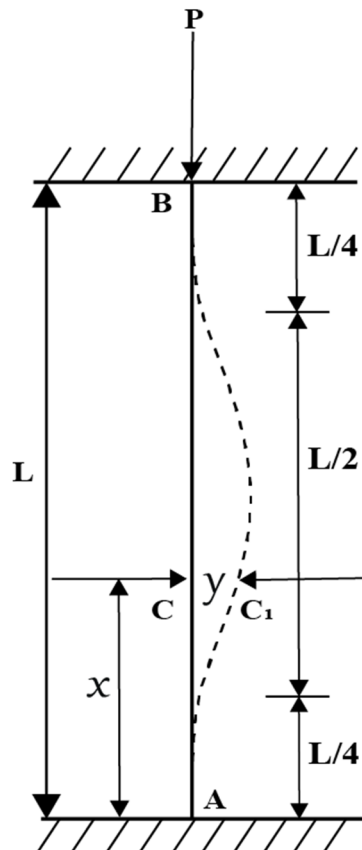


Fig. 5.11: Figure showing the both ends are fixed

Consider the section CC_1 , positioned at a distance x from point A.

Let

P_{cr} - Critical load

y - Column deflection at section C

Due to the fixed end the moment at the end is non-zero.

Let, M_o represents the fixed end moment at points A. This implies that the moment caused by the critical load P is:

$$M = -P \cdot y$$

We know that $M = EI \frac{d^2y}{dx^2}$, thus

$$EI \frac{d^2y}{dx^2} = M_o - (P \cdot y)$$

[$\therefore M_o$ is the moment at the fixed end]

Dividing both the side by EI

$$\frac{d^2y}{dx^2} = \frac{M_o}{EI} - \frac{(P \cdot y)}{EI}$$

(- sign owing to the concavity of the actual center line)

$$\therefore \frac{d^2y}{dx^2} + \left(\frac{P}{EI} \cdot y \right) = \frac{M_o}{EI} \quad \dots \dots (5.11)$$

To find the value of y, we have to solve the differential equation:

$$y = A \cos \left(x \sqrt{\frac{P}{EI}} \right) + B \sin \left(x \sqrt{\frac{P}{EI}} \right) + \frac{M_o}{P} \quad \dots \dots (5.12)$$

Here A and B are integration constants. Now, applying the boundary conditions, the column is fixed at both ends $y = 0$ when $x = 0$, it follows that $A = -\frac{M_o}{P}$. Now, differentiating the equation above:

$$\frac{dy}{dx} = -A \sqrt{\frac{P}{EI}} \sin \left(x \sqrt{\frac{P}{EI}} \right) + B \sqrt{\frac{P}{EI}} \cos \left(x \sqrt{\frac{P}{EI}} \right) \quad \dots \dots (5.13)$$

We know that $x = 0$, the value of $\frac{dy}{dx} = 0$ Thus:

$$0 = B \sqrt{\frac{P}{EI}} \quad \dots \dots (5.14)$$

It evident that either $\sqrt{\frac{P}{EI}}$ must be zero, or B should be zero. Since the load P is non-zero and B is zero. Substituting the values $A = -\frac{M_o}{P}$ and $B = 0$ in the Eqn. (5.12):

$$y = -\frac{M_o}{P} \cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_o}{P}$$

$$y = \frac{M_o}{P} \left[1 - \cos\left(x\sqrt{\frac{P}{EI}}\right)\right] \quad \dots \dots (5.15)$$

Also, at $x = L$, $y = 0$,

Thus,

$$0 = \frac{M_o}{P} \left[1 - \cos\left(L\sqrt{\frac{P}{EI}}\right)\right]$$

$$\cos\left(L\sqrt{\frac{P}{EI}}\right) = 1$$

Or

$$\left(L\sqrt{\frac{P}{EI}}\right) = 2n\pi \quad n = 1, 2, 3 \dots$$

$n = 1$, give the lowest bucking value which is given by

$$\left(L\sqrt{\frac{P}{EI}}\right) = 2\pi \quad \dots \dots (5.16)$$

$$\left(\frac{P}{EI}\right)^{\frac{1}{2}} L = 2\pi$$

Squaring on both the sides

$$\left(\frac{P}{EI}\right) L^2 = (2\pi)^2$$

$$P = \frac{4\pi^2 EI}{L^2} \quad \dots \dots (5.17)$$

5.3.6 ONE END IS FIXED AND THE OTHER HINGED

Consider a column AB of length 'L' fixed at point A and hinged at point B. This column is subjected to load P at B. Due to the applied loading, the column undergoes deflection, as shown in Fig. 5.12.

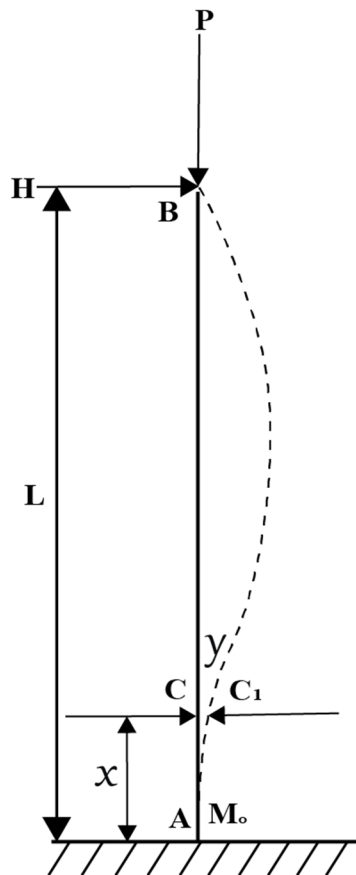


Fig. 5.12: Fixed at one end while hinged the other end

Consider section CC_1 , at a distance x distance from point A.

Let

P - Critical load

y - Column deflection at C

Due to the fixed condition of column AB at point A, a fixed end moment will be present at A. The horizontal reaction will emerge at point B to offset the fixed moment at A.

Let

M_A - Fixed end moment at point A.

H - Horizontal reaction at point B.

This implies that the moment caused by the critical load P can be expressed as:

$$M = (-P \times y) + H(L - x)$$

(- sign owing to concavity towards the actual center line)

We know that $M = EI \frac{d^2y}{dx^2}$

$$EI \frac{d^2y}{dx^2} = H(L - x) - (P \cdot y)$$

$$\therefore \frac{d^2y}{dx^2} + \left(\frac{P}{EI} \cdot y\right) = \frac{H(L - x)}{EI} \quad \dots \dots (5.18)$$

To find the value of y, we have to solve the differential equation:

$$y = A \cos\left(x \sqrt{\frac{P}{EI}}\right) + B \sin\left(x \sqrt{\frac{P}{EI}}\right) + \frac{H(L - x)}{P} \quad \dots \dots (5.19)$$

Where A and B are integration constants. Now applying the boundary condition as the column is fixed $y = 0$ when $x = 0$, it follows that $A = \frac{HL}{P}$. Now, differentiating the equation above:

$$\frac{dy}{dx} = -A \sqrt{\frac{P}{EI}} \sin\left(x \sqrt{\frac{P}{EI}}\right) + B \sqrt{\frac{P}{EI}} \cos\left(x \sqrt{\frac{P}{EI}}\right) - \frac{H}{P} \quad \dots \dots (5.20)$$

When $x = 0$, the value of $\frac{dy}{dx} = 0$ Thus:

$$0 = B \sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$\frac{H}{P} = B \sqrt{\frac{P}{EI}}$$

$$B = \frac{H}{P} \sqrt{\frac{EI}{P}} \quad \dots \dots (5.21)$$

As the column is hinged at $x = L$, only $y = 0$. Thus, substituting the values of A, B, and x , into Eqn (5.19):

$$0 = \frac{Hl}{P} \cos \left(L \sqrt{\frac{P}{EI}} \right) + \frac{H}{P} \sqrt{\frac{EI}{P}} \sin \left(L \sqrt{\frac{P}{EI}} \right)$$

$$\frac{H}{P} \sqrt{\frac{EI}{P}} \sin \left(L \sqrt{\frac{P}{EI}} \right) = \frac{Hl}{P} \cos \left(L \sqrt{\frac{P}{EI}} \right)$$

$$\frac{\sin \left(L \sqrt{\frac{P}{EI}} \right)}{\cos \left(L \sqrt{\frac{P}{EI}} \right)} = L \sqrt{\frac{P}{EI}}$$

$$\tan \left(L \sqrt{\frac{P}{EI}} \right) = \left(L \sqrt{\frac{P}{EI}} \right) \quad \dots \dots (5.22)$$

It becomes apparent that $\left(L \sqrt{\frac{P}{EI}} \right)$ value in radians must satisfy the condition that its tangent equals itself. It worth mentioning that the only angle where the tangent is equivalent to itself is around 4.5 radians.

$$\therefore \left(L \sqrt{\frac{P}{EI}} \right) = 4.5 \quad \dots \dots (5.23)$$

By squaring both sides, we obtain:

$$\left(L^2 \cdot \frac{P}{EI} \right) = 20.25$$

$$\therefore P = \frac{2\pi^2 EI}{L^2} \quad \dots \dots (5.24)$$

It becomes evident that 20.25 is not precisely equivalent to $2\pi^2$ but is closely equal to $2\pi^2$.

5.4 EFFECTIVE LENGTH OF THE COLUMN

Euler buckling load can be experimented in a column that considers all end conditions by introducing effective length.

$$P_{cr} = \frac{\pi^2 EI}{(L_e)^2} \quad \dots \dots (5.25)$$

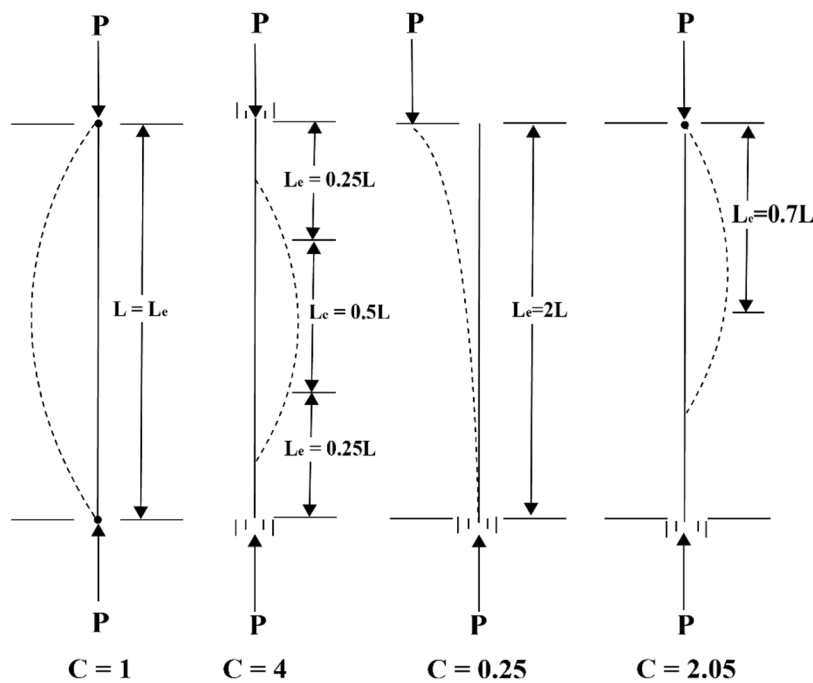


Fig. 5.13: Effective length of the column

Where L_e is referred as the equivalent length of column.

$$\begin{aligned}
 L_e &= L \text{ if both ends are hinged} \\
 &= \frac{L}{\sqrt{2}} = 0.707 L \text{ if fixed at one end while hinged at the other end} \\
 &= 2L \text{ for a column with one end fixed while the other end is free} \\
 &= \frac{1}{2} L \text{ for a column fixed at both ends}
 \end{aligned}$$

Where L represents the actual column length. The concept of equivalent length can be applied to the general formula for critical load, $P_{cr} = \frac{\pi^2 EI}{(L_e)^2}$.

The concept of equivalent length can also be understood from the defective shapes shown in Fig. 5.13, where the equivalent length is the length between the inflection points on the deflected shape.

5.5 RADIUS OF GYRATION

It is the property inherent to a specific section. It depends on the second moment of area (also known as the moment of inertia). The radius of gyration measures the stiffness of a section. Typically, this parameter is employed in analyzing compressed members, including columns. The expression for calculating the Radius of Gyration (r) is given by:

$$r = \sqrt{\frac{I}{A}} \quad \dots \dots (5.26)$$

Where,

I represent the second moment of the area

A denotes the cross-sectional area of the member

The unit of measurement for the radius of gyration is millimeters (mm).

For the calculations involving the structural stiffness of a member, the smallest value of the radius of gyration is taken into account. It signifies the plane where the member is most susceptible to failure or buckling. In the case of square or circular shapes, their radius of gyration remains the same regardless of the plane.

5.6 SLENDERNESS RATIO

It is the ratio to calculate buckling phenomena that occurs in a compression member.

The critical buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad \dots \dots (5.27)$$

It can be stated by substituting $I = Ar^2$,

$$P_{cr} = \frac{\pi^2 EAr^2}{L_e^2} \quad \dots \dots (5.28)$$

The corresponding stress

$$\begin{aligned} \frac{P_{cr}}{A} &= \frac{\pi^2 Er^2}{L_e^2} \\ \frac{P_{cr}}{A} &= \frac{\pi^2 E}{\frac{L_e^2}{r^2}} \\ \frac{P_{cr}}{A} &= \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2} \quad \dots \dots (5.29) \end{aligned}$$

Where r represents the radius of gyration, and A signifies the area of the cross-section of the member.

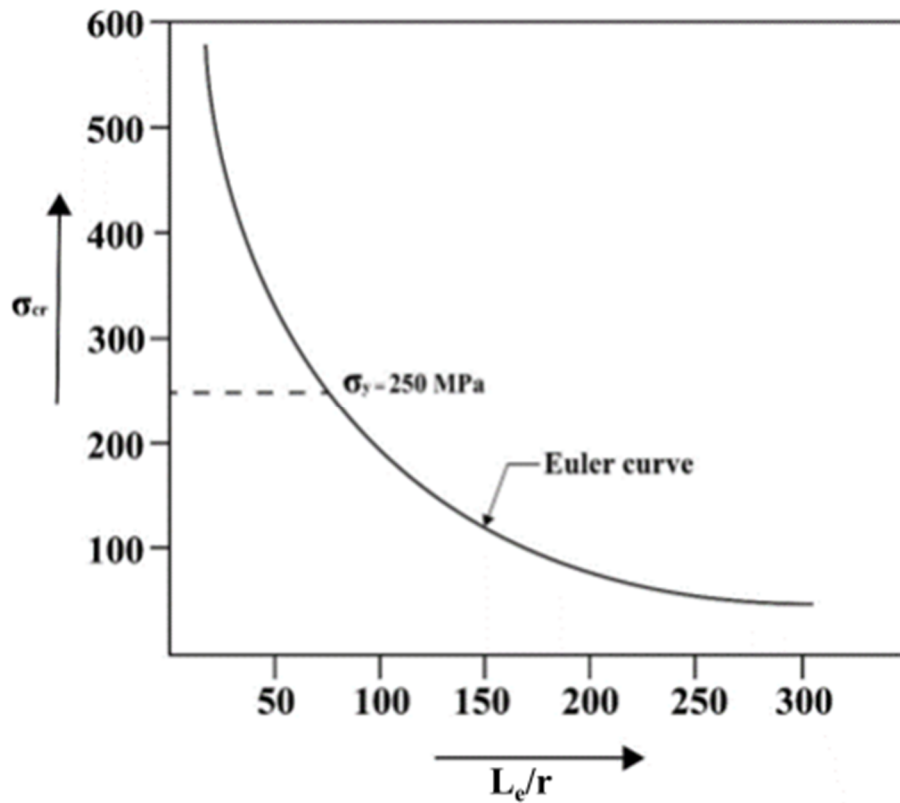


Fig. 5.14: Figure showing the plot of σ_{cr} against $\left(\frac{L_e}{r}\right)$

Thus,

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

$\left(\frac{L_e}{r}\right)$ is called as the slenderness ratio

$$\text{Slenderness ratio} = \frac{\text{Equivalent length}}{\text{Radius of gyration}}$$

The critical compressive stress is thus inversely proportional to the square of the slenderness ratio. A plot of σ_{cr} against $\left(\frac{L_e}{r}\right)$ will appear as shown in Fig. 5.14. The

curve is asymptotic to both the σ_{cr} and $\left(\frac{L_e}{r}\right)$ axes, σ_{cr} tends to infinity as $\left(\frac{L_e}{r}\right)$ tends to zero and σ_{cr} tends to become zero as $\left(\frac{L_e}{r}\right)$ tends to infinity.

5.7 LIMITATIONS AND APPLICABILITY OF EULER'S FORMULA

Euler's theory relies on the following assumptions:

- The column is initially straight.
- The load applied is purely axial in nature.
- The material constituting the column is isotropic and homogeneous.
- The material behaves elastically up to a point very close to the critical load.

These assumptions serve as a basis for understanding column stability using Euler's theory.

The Euler formula, $P_{cr} = \frac{\pi^2 EI}{L^2}$ is employed for the theory of columns. This formula primarily considers the geometric and elastic properties of the column rather than explicitly involving material strength characteristics of materials.

From Fig. 5.14, the stress σ_{cr} obtained is high when the L_e/r ratio is small or when a column is short. Such a high-stress value has no meaning since the material would have already failed by crushing or yielding of the material. Therefore, considering the average stress, the critical load must be such that the elastic limit is not exceeded. For the superposition principle, the stress should not exceed the proportional limit.

Taking the case of a steel column with both ends hinged and length L , if the proportional limit is equal to 250 MPa, then

$$\sigma_{cr} = 250 = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

$$\left(\frac{L_e}{r}\right)^2 = \frac{\pi^2 E}{250}$$

$$\left(\frac{L_e}{r}\right)^2 = \frac{\pi^2 \times 200,000}{250}$$

$$\frac{L_e}{r} = 89$$

It is the lower limit of the L/r ratio up to which Euler's formula can be applicable for steel. Since it is known that short columns do fail by crushing or yielding, in the case of steel, the σ_{cr} vs L/r curve will be as shown in Fig. 5.15. Note that for extremely low values of slenderness ratio, the critical stress is the yield stress, and in long columns, the critical stress decreases according to Euler's formula. In the transition zone the column is neither short nor long. Such a column may fail by a combination of compressive stress and bending stress.

SOLVED PROBLEM

Problem 5.1

Determine the buckling load of a solid round bar, which has a length of 5m and a diameter of 70 mm. Strut is hinged at both ends. The modulus of elasticity E is given as $E = 2.0 \times 10^5 \text{ N/mm}^2$

Solution:

Length (L) = 5 m = 5000 mm

Diameter (d) = 70 mm

Modulus of elasticity (E) = $2.0 \times 10^5 \text{ N/mm}^2$

$$I = \frac{\pi}{64} \times 7^4 = 117.85 \text{ cm}^4 = 117.85 \times 10^4 \text{ mm}^4$$

Let

P = Buckling load.

Given that the bar is hinged at both ends, the buckling load can be determined by using the formula.

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 \times 117.85 \times 10^4 \times 2.0 \times 10^5}{5000^2}$$

$$P_{cr} = 93050 \text{ N} = 93.050 \text{ kN}.$$

Problem 5.2

Determine the buckling load of a solid round bar, which has a length of 5m and a diameter of 70 mm. Strut is hinged at both ends. The modulus of elasticity E is given as $E = 2.0 \times 10^5 \text{ N/mm}^2$

- (i) When the column is fixed at one end while free at the other end.
- (ii) When the column is fixed at both ends
- (iii) When the column is fixed at one end and the hinged other ends

Solution:

The data from problem 5.1,

Length (l) = 5000mm,

Diameter (d) = 70mm,

Modulus of elasticity (E) = $2.0 \times 10^5 \text{ N/mm}^2$ and

$I = 117.85 \times 10^4 \text{ mm}^4$.

- (i) The buckling load when the strut is fixed at one end while free at the other end can be determined using Euler's critical load formula:

Using the equation,

$$P = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 \times 117.85 \times 10^4 \times 2 \times 10^5}{4 \times 5000^2} = 23262 \text{ N} = 23.26 \text{ kN}.$$

- (ii) To determine the buckling load when the strut is fixed at both ends can be determined using critical load formula:

Using the equation,

$$P = \frac{4\pi^2 EI}{L^2} = \frac{4 \times \pi^2 \times 117.85 \times 10^4 \times 2 \times 10^5}{5000^2} = 372202 \text{ N} = 372.202 \text{ kN}.$$

- (iii) The buckling load when the strut is fixed at one end and the hinged other ends can be determined using Euler's critical load formula:

Using the equation,

$$P = \frac{2 \times \pi^2 EI}{L^2} = \frac{2 \times \pi^2 \times 117.85 \times 10^4 \times 2 \times 10^5}{5000^2} = 186101 \text{ N} = 186.101 \text{ kN}.$$

Problem 5.3

A timber column with dimensions 200 mm × 250 mm and a length of 8m is fixed at both ends. Given that timber Young's modulus is 17.5 kN/mm². Find the

- (i) The buckling load, and
- (ii) The column safe load, considering a factor of safety 3.

Solution:

Section dimension = 200 mm × 250 mm

Young's modulus, $E = 17.5 \text{ kN/mm}^2$

Length of the column, $L = 8 \text{ m} = 8000 \text{ mm}$

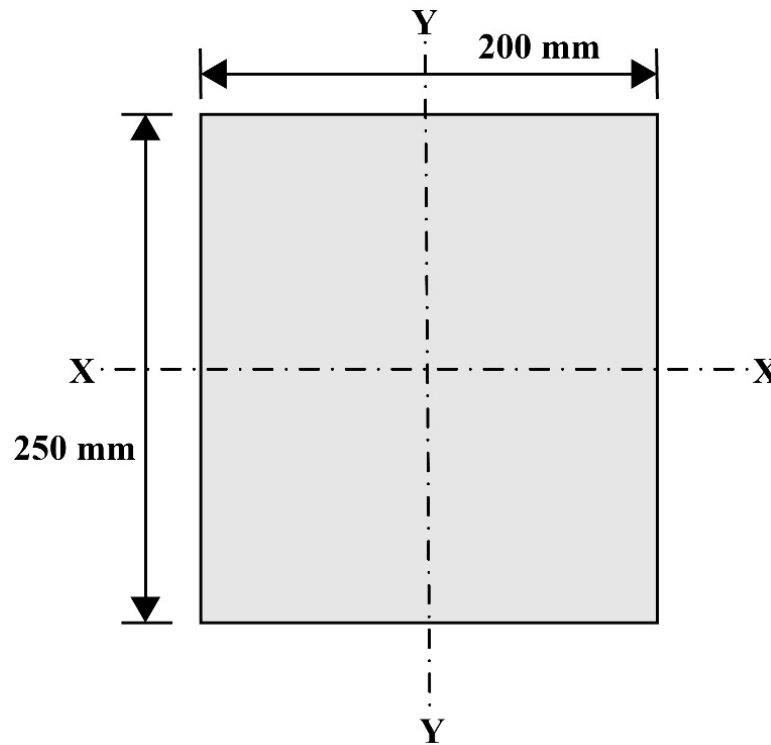


Fig. 5.15: Figure showing the cross-section dimension of a column

(i) Consider P as the buckling load

Using the equation (5.26)

$$P = \frac{\pi^2 EI}{L_e^2},$$

We get

$$L_e = \text{Effective length} = \frac{L}{2} \quad (\text{Where both the ends are fixed})$$

$$L_e = \frac{L}{2} = \frac{8000}{2} = 4000 \text{ mm}$$

Where I represent the least value of the MI and I_{xx} signifies the moment of inertia of the section about the X-X axis.

$$I_{xx} = \frac{200 \times 250^3}{12} \quad \therefore \left(I_{xx} = \frac{bh^3}{12} \right)$$

$$I_{xx} = 26041 \times 10^4 \text{ mm}^4$$

Similarly, I_{yy} represents the MI of the section about the Y-Y axis.

$$I_{yy} = \frac{250 \times 200^3}{12} \quad \therefore \left(I_{yy} = \frac{hb^3}{12} \right)$$

$$I_{yy} = 16666 \times 10^4 \text{ mm}^4$$

Considering that I_{yy} is smaller than I_{xx} , the column's buckling tendency will be in the Y-Y direction. Selecting the smaller of the two moments of inertia will provide the value of I .

$$\therefore I = 16666 \text{ cm}^4 = 16666 \times 10^4 \text{ mm}^4$$

Substituting the values of $I = 16666 \times 10^4 \text{ mm}^4$ and $L_e = 4000 \text{ mm}$ in Eqn (5.26), we get,

$$P = \frac{\pi^2 \times 17.5 \times 16666 \times 10^4}{4000^2}$$

$$P = 1799.07 \text{ kN}$$

(ii) The safe load for the column can be calculated, considering a factor of safety of 3.0 (as given).

$$\therefore \text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{1799.07}{3} = 599.69, \text{ say } 600 \text{ kN.}$$

Problem 5.4

An 8-meter-long hollow mild steel tube is being used as a strut with both ends hinged. The tube has a 6 cm inner diameter and 6 mm thickness. The modulus of elasticity is $E = 2 \times 10^5 \text{ N/mm}^2$ and a safety factor of 3 is considered. Determine:

- (i) The buckling loads
- (ii) The safe load

Solution:

Length, $L = 8 \text{ m} = 800 \text{ cm}$

Inner diameter, $d = 6 \text{ cm}$

Thickness, $t = 6 \text{ mm} = 0.6 \text{ cm}$

$\therefore$ Outer diameter, $D = d + 2t = 6 + 2(0.6) = 7.2 \text{ cm}$

$E = 2 \times 10^5 \text{ N/mm}^2$

Factor of safety = 3.0

$$\begin{aligned} \text{MI of a section, } I &= \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (7.2^4 - 6^4) \text{ cm}^2 \\ I &= 68.29 \text{ cm}^2 = 68.29 \times 10^4 \text{ mm}^4 \end{aligned}$$

Given that both ends of the strut are hinged,

$\therefore$ Effective length, $L_e = L = 800 \text{ cm} = 8000 \text{ mm}$

Let

$P =$ Buckling load

Using the equation (5.26) $P = \frac{\pi^2 EI}{L_e^2}$, we get

$$\begin{aligned} P &= \frac{\pi^2 \times 2 \times 10^5 \times 68.29 \times 10^4}{8000^2} \\ P &= 21062 \text{ N} \end{aligned}$$

$$\therefore \text{ Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{21062}{3} = 7020.66 \text{ N}$$

Problem 5.5

A simply supported beam with a length of 5 meters is subjected to a UDL of 40 kN/m across its entire span, resulting in a center deflection of 20 mm. Determine the buckling load when this beam is utilized as a column under the following conditions:

- (i) One end is fixed, and the other end is hinged.
- (ii) Both ends pin jointed.

Solution:

Length, $L = 5 \text{ m} = 5000 \text{ mm}$

UDL, $P = 40 \text{ kN/m} = 40000 \text{ N/m}$

$$P = \frac{40000}{1000} \text{ N/mm}$$

$$P = 40 \text{ N/mm}$$

Center deflection, $\delta = 20 \text{ mm}$

The formula for calculating the deflection at the center of a simply supported beam subjected to a UDL over its entire span is:

$$\delta = \frac{5}{384} \times \frac{P \times L^4}{EI}$$

$$15 = \frac{5}{384} \times \frac{40 \times 5000^4}{EI}$$

$$EI = \frac{5}{384} \times \frac{40 \times 5000^4}{15}$$

$$EI = \frac{5}{384} \times \frac{4 \times 625}{15} \times 10^{13}$$

$$EI = 2.170 \times 10^{13} \text{ N mm}^2$$

- (i) Determine the buckling load while using the beam as a column with one end fixed and the other end hinged.

The following equation provides the buckling load P for this case, given in terms of the actual length of the column:

$$P = \frac{2\pi^2 EI}{L^2}$$

(Where L = actual length = 5000 mm)

$$P = \frac{2 \times \pi^2 \times 2.170 \times 10^{13}}{5000^2} = 17133.63 \text{ kN}$$

(ii) When both column ends are pin-jointed, the buckling load is calculated

Using the equation (5.26)

$$P = \frac{\pi^2 EI}{L^2}$$

(Where L = actual length = 5000 mm)

$$P = \frac{\pi^2 \times 2.170 \times 10^{13}}{5000^2} = 8566.81 \text{ kN}$$

Problem 5.6

A solid round bar, 5m in length and with a diameter of 60 mm, experienced an extension of 5 mm under a 60 kN tensile load. The same bar is now being utilized as a strut hinged at both ends. Calculate the buckling and safe loads, considering a safety factor of 4.0.

Solution:

Actual length, $L = 5\text{m} = 5000 \text{ mm}$

Diameter, $d = 6 \text{ cm}$

$$\therefore A = \frac{\pi}{4} \times 60^2 = 900\pi \text{ mm}^2$$

Extension, $\delta L = 5.0 \text{ mm}$

$W = 60\text{kN} = 60000 \text{ N}$.

Young's modulus E values are not clearly given in this problem. However, using the supplied data, it is feasible to derive Young's modulus.

$$\therefore E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{\left(\frac{\text{Tensile load}}{\text{Area}}\right)}{\left(\frac{\text{Extension of bar}}{\text{length of bar}}\right)}$$

$$\therefore \text{Stress} = \frac{\text{Load}}{\text{Area}} \text{ and Strain} = \frac{\delta L}{L}$$

$$E = \frac{\left(\frac{W}{A}\right)}{\left(\frac{\delta L}{L}\right)} = \left(\frac{W}{A}\right) \cdot \left(\frac{L}{\delta L}\right) = \frac{60000}{900 \pi} \times \frac{5000}{5} = 2.122 \times 10^4 \text{ N/mm}^2$$

Given that both ends of the strut are hinged,

$\therefore$ Effective length, $L_e = 5000 \text{ mm}$

Using the given equation (5.26)

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$\therefore I = \frac{\pi}{64} \times 6^4 \times 10^4 \text{ mm}^4$$

$$P = \frac{\pi^2 \times 2.122 \times 10^4 \times \frac{\pi}{64} \times 6^4 \times 10^4}{5000^2}$$

$$P = 5329.42 \text{ N}$$

$$\therefore \text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{5329.42}{4} = 1332.35 \text{ N}$$

Problem 5.7

When a tensile force of 70 kN is applied to a hollow alloy tube having a length of 6m, an outer diameter of 45 mm, and an inner diameter of 35 mm, the longitudinal extension is 6.5 mm. This tube is designed to be used as a column, with both ends

pinned. Calculate the buckling load of the tube and the safe load using a safety factor of 4.0.

Solution:

$$L = 6\text{m} = 6000\text{mm}$$

$$\text{Outer diameter, } D = 45 \text{ mm}$$

$$\text{Inner diameter, } d = 35 \text{ mm}$$

$$\text{Extension, } \delta L = 6.5 \text{ mm}$$

$$\text{Safety factor} = 4$$

$$\text{Tensile load, } W = 70\text{kN} = 70,000 \text{ N}$$

$$\text{Area, } A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (45^2 - 35^2) = 628.31 \text{ mm}^2$$

$$\text{MI, } I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (45^4 - 35^4) = 127627 \text{ mm}^4$$

The value of E can be determined using the following approach:

$$\therefore E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{\left(\frac{\text{Tensile load}}{\text{Area}}\right)}{\left(\frac{\text{Extension of bar}}{\text{length of bar}}\right)}$$

$$\therefore \text{Stress} = \frac{\text{Load}}{\text{Area}} \text{ and Strain} = \frac{\delta L}{L}$$

$$E = \frac{\left(\frac{W}{A}\right)}{\left(\frac{\delta L}{L}\right)} = \left(\frac{W}{A}\right) \cdot \left(\frac{L}{\delta L}\right) = \frac{70000}{628.31} \times \frac{6000}{6.5} = 10.283 \times 10^4 \text{ N/mm}^2$$

Since both the ends of the column is pinned.

$$\therefore \text{Effective length } L_e = 6000 \text{ mm}$$

Let

P - Buckling load,

Using the equation (5.26),

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$P = \frac{\pi^2 \times 10.283 \times 10^4 \times 127627}{6000^2} = 3598 \text{ N}$$

$$\text{And Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{3598}{4} = 899.5 \text{ N.}$$

Problem 5.8

Determine the permissible compressive load for a cast iron hollow column with a rigidly fixed one end while hinged at the other end. The column has an outer diameter of 20 cm, an inner diameter of 15 cm, and a length of 12 m. Use Euler's formula with a safety factor is 5, considering a modulus of elasticity E of 95 kN/mm².

Solution:

Outer diameter, D = 20 cm

Inner diameter, d = 15 cm

L = 12 m = 12000 mm

E = 95 kN/mm² = 95000 N/mm²

Safety factor = 5

MI of the hollow column,

$$I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (20^4 - 15^4) = 5368.93 \times 10^4 \text{ mm}^4$$

Given that column one end is rigidly fixed while hinged at other end,

$$\therefore \text{Effective length, } L_e = \frac{L}{\sqrt{2}} = \frac{12000}{\sqrt{2}}$$

Let P = Buckling load.

Using Eqn (5.26)

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$P = \frac{\pi^2 \times 95000 \times 5368.93 \times 10^4}{\left(\frac{12000}{\sqrt{2}}\right)^2}$$

$$P = \frac{\pi^2 \times 95000 \times 5368.93 \times 10^4 \times 2}{(12000)^2}$$

$$P = 699163 \text{ N} = 699.3163 \text{ kN}$$

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}} = \frac{699.316}{5} = 139.86 \text{ kN}.$$

Problem 5.9

Calculate the critical load of an I-section joist with dimensions 500 mm × 300 mm × 10 mm and a length of 6m (Fig. 5.18). The joist is being used as a strut fixed at both ends. Consider the modulus of elasticity E as $2.1 \times 10^5 \text{ N/mm}^2$.

Solution:

I-section dimension = 500 mm × 300 mm × 10 mm

L = 6m = 6000 mm

E = $2.1 \times 10^5 \text{ N/mm}^2$,

MI of the section about the X-X axis,

$$I_{xx} = \text{MI of rectangle } [300 \text{ mm} \times 500 \text{ mm}] - \text{MI of rectangle } [(300 \text{ mm} - 10 \text{ mm}) \times (500 \text{ mm} - 10 \text{ mm} - 10 \text{ mm})]$$

$$= \frac{1}{12}bd^3 - \frac{1}{12}b_1d_1^3$$

$$= \left(\frac{1}{12} \times 300 \times 500^3\right) - \left(\frac{1}{12} \times 290 \times 480^3\right)$$

$$= 3125000000 - 2672640000 = 452360000 \text{ mm}^4$$

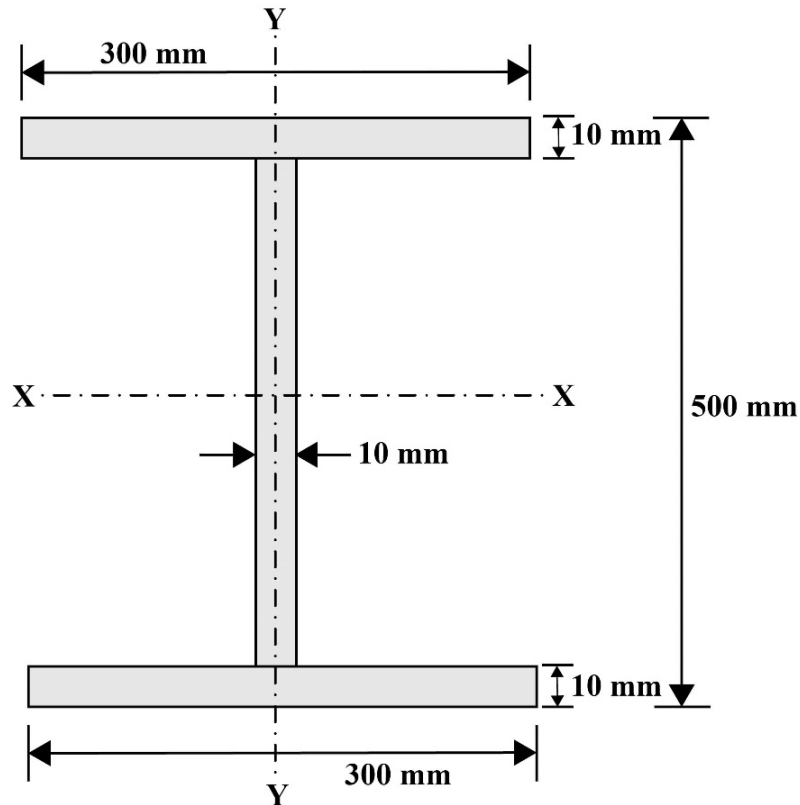


Fig. 5.16: I-Section strut fixed at both ends

Similarly, the MI of the section about the Y-Y axis

$$\begin{aligned}
 I_{YY} &= \text{Rectangle MI } (480 \times 10) + \text{Two rectangle MI } (10 \times 300) \\
 &= \left(\frac{1}{12} \times 480 \times 10^3 \right) + \left(\frac{1}{12} \times 10 \times 300^3 \right) \\
 &= 40000 + 22500000 = 22540000 \text{ mm}^4
 \end{aligned}$$

$\therefore$ Least value of the MI is about the YY axis

$$I_{YY} = 22540000 \text{ mm}^4$$

When rigidly fixed at both ends.

$$\therefore \text{Effective length, } L_e = \frac{l}{2} = \frac{6000}{2} = 3000 \text{ mm}$$

Let P = Buckling load.

Using the equation (5.26)

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$P = \frac{\pi^2 \times 2.1 \times 10^5 \times 2254 \times 10^4}{3000^2}$$

$$P = 5190753 \text{ N} = 5.1907 \text{ MN.}$$

Problem 5.10

Calculate the buckling load of the T-section with $150 \text{ mm} \times 150 \text{ mm} \times 20 \text{ mm}$ (Fig. 5.17) dimensions and a 6 m length when used as a strut hinged at both ends.

The elasticity modulus is $E = 2 \times 10^5 \text{ N/mm}^2$

Solution

T-section dimension = $150 \text{ mm} \times 150 \text{ mm} \times 20 \text{ mm}$

$L = 6 \text{ m} = 6000 \text{ mm}$

$E = 2.0 \times 10^5 \text{ N/mm}^2$

Determine the C.G of the given section. The section is symmetric and the Y-axis and hence the C.G lies on the Y axis.

y_1 = C.G of area a_1 distance from bottom end

We have the flange.

$$a_1 = 150 \times 20 = 3000 \text{ mm}^2$$

$$y_1 = 130 + 10 = 140 \text{ mm}$$

For the web.

$$a_2 = 130 \times 20 = 2600 \text{ mm}^2$$

y_2 - C.G of area a_2 distance from bottom end

$$y_2 = \frac{130}{2} = 65 \text{ mm}$$

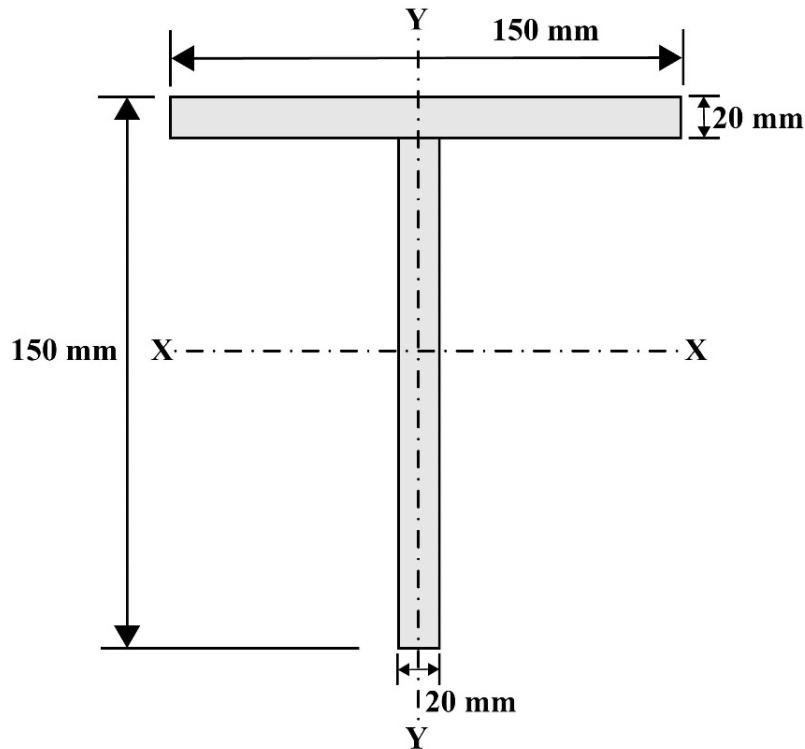


Fig. 5.17: T-Section strut hinged at both ends

Using the expression

$$\begin{aligned}\bar{y} &= \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} \\ &= \frac{(3000 \times 140) + (2600 \times 65)}{3000 + 2600} = \frac{420000 + 169000}{5600} = 105.17 \text{ mm}\end{aligned}$$

MI of the X-X axis section

$$I_{xx} = [(I_{G1}) + (A_1 \times h_1^2)] + [(I_{G2}) + (A_2 \times h_2^2)]$$

$$\text{where } h_1 = (y_1 - \bar{y}) \text{ and } h_2 = (y_2 - \bar{y})$$

$$I_{xx} = \left[\frac{150 \times 20^3}{12} + (3000 \times (140 - 105.17)^2) \right] + \left[\frac{20 \times 130^3}{12} + (2600 \times (65 - 105.17)^2) \right]$$

$$I_{xx} = (100000 + 3639386.7) + (3661666.6 + 4195435.14) = 11596488.83 \text{ mm}^4$$

MI of the Y-Y axis section

$$I_{YY} = \left(\frac{20 \times 150^3}{12} \right) + \left(\frac{130 \times 20^3}{12} \right)$$

$$I_{YY} = (5625000) + (86666.66) = 5711666.66 \text{ mm}^4$$

The least value of the MI is about the Y-Y axis

$$I = 571.66 \times 10^4 \text{ mm}^4$$

Given that the strut is hinged at both of its ends

∴ Effective length,

$$L_e = L = 6000 \text{ mm}$$

Let P - Buckling load

Using the given equation

$$P = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 2 \times 10^5 \times 571.16 \times 10^4}{6000^2}$$

$$P = 313173.51 \text{ N} = 313.174 \text{ kN}$$

Problem 5.11

Calculate the critical stress for the struts with slenderness ratios of 50, 100, 150, 200, and 250 using Euler's formula, taking into account the following cases:

- (i) Hinged at Both ends.
- (ii) Fixed at Both ends.

Consider Young's modulus E as $2.05 \times 10^5 \text{ N/mm}^2$.

Solution:

(i) Determine the critical stress for the case when both ends are hinged.

Slenderness ratios,

$$\frac{L}{r} = 50, 100, 150, 200 \text{ and } 250$$

The equation (5.26) provides the critical stress or Buckling stress.

$$\therefore \text{Critical stress} = \frac{\pi^2 \times E}{\left(\frac{L_e}{r}\right)^2}$$

Where L_e = Effective length

But hinged at both ends, $L_e = L$

Where L = actual length

$$\therefore \text{Critical stress} = \frac{\pi^2 \times E}{\left(\frac{L}{r}\right)^2} \quad (\because L_e = L)$$

When $\frac{L}{r} = 50$, the critical stress become as

$$= \frac{\pi^2 \times E}{50^2} = \frac{\pi^2 \times 2.05 \times 10^5}{2500} = 809.307 \text{ N/mm}^2$$

When $\frac{L}{r} = 100$, the critical stress become as

$$= \frac{\pi^2 \times E}{100^2} = \frac{\pi^2 \times 2.05 \times 10^5}{10000} = 202.326 \text{ N/mm}^2$$

When $\frac{L}{r} = 150$, the critical stress become as

$$= \frac{\pi^2 \times E}{150^2} = \frac{\pi^2 \times 2.05 \times 10^5}{22500} = 89.923 \text{ N/mm}^2$$

When $\frac{L}{r} = 200$, the critical stress become as

$$= \frac{\pi^2 \times E}{200^2} = \frac{\pi^2 \times 2.05 \times 10^5}{40000} = 50.581 \text{ N/mm}^2$$

When $\frac{L}{r} = 250$, the critical stress become as

$$= \frac{\pi^2 \times E}{250^2} = \frac{\pi^2 \times 2.05 \times 10^5}{62500} = 32.372 \text{ N/mm}^2$$

(ii) Calculate the critical stress when fixed at both ends

Using the equation (5.26)

$$\text{critical stress} = \frac{\pi^2 \times E}{\left(\frac{L_e}{r}\right)^2}$$

When both ends are fixed, $L_e = \frac{L}{2}$

$$\therefore \text{Critical stress} = \frac{\pi^2 \times E}{\left(\frac{L}{2r}\right)^2} = \frac{4 \times \pi^2 \times E}{\left(\frac{L}{r}\right)^2}$$

When $\frac{L}{r} = 50$, the critical stress become as

$$\begin{aligned} &= \frac{4 \times \pi^2 \times E}{\left(\frac{L}{r}\right)^2} = \frac{4 \times \pi^2 \times 2.05 \times 10^5}{(50)^2} \\ &= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{2500} = 3237.230 \text{ N/mm}^2 \end{aligned}$$

When $\frac{L}{r} = 100$, the critical stress become as

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{(100)^2} = 809.307 \text{ N/mm}^2$$

When $\frac{L}{r} = 150$, the critical stress become as

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{(150)^2} = 359.692 \text{ N/mm}^2$$

When $\frac{L}{r} = 200$, the critical stress become as

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{(200)^2} = 202.326 \text{ N/mm}^2$$

When $\frac{L}{r} = 250$, the critical stress become as

$$= \frac{4 \times \pi^2 \times 2.05 \times 10^5}{(250)^2} = 129.489 \text{ N/mm}^2$$

5.8 EMPIRICAL FORMULA

Euler's bucking formula, for columns with different end conditions, is useful to understand the behaviour of column. Euler's formula is basically useful for long columns and suffers for short column due to the limitations listed earlier. Many empirical formulae have been developed from the theoretical considerations supported by experimentations. One of the most commonly used empirical formula was established by Rankine. Many other formulae developed from practical results on testing columns are also discussed below.

5.8.1 RANKINE-GORDON FORMULA/ RANKINE'S FORMULA

Euler's formula is applicable to columns above a certain minimum value of the L/r ratio or for long columns. Short columns fail by crushing. There are cases of intermediate columns which may not fail by crushing and for which Euler's formula is not applicable. Such columns may fail by the combination of buckling and crushing.

Rankine established an empirical relationship for such intermediate columns, considering the direct compressive and bending stress. The relationship is given by

$$\frac{1}{P_r} = \frac{1}{P_s} + \frac{1}{P_e}$$

Where P_r is the Rankine buckling load, P_s is the failure load due to direct compressive stress as a short column, and P_e is the Euler's buckling load. Thus,

$$P_r = \frac{P_s P_e}{P_s + P_e} = \frac{P_s}{\left(1 + \frac{P_s}{P_e}\right)}$$

$$P_s = \sigma_c A$$

$$P_e = \frac{\pi^2 EI}{L_e^2}$$

Where σ_c = Crushing stress and L_e = Effective length. Therefore,

$$P_r = \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A}{\left(\frac{\pi^2 EI}{L_e^2}\right)}}$$

Since $I = A(r)^2$, Where r = least radius of gyration

$$P_r = \frac{\sigma_c \cdot A}{1 + \left(\frac{\sigma_c \cdot A}{\pi^2 \cdot E}\right) \left(\frac{L_e^2}{A \cdot r^2}\right)}$$

$$P_r = \frac{\sigma_c \cdot A}{1 + \left(\frac{\sigma_c}{\pi^2 \cdot E}\right) \left(\frac{L_e}{r}\right)^2}$$

$$P_r = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{L_e}{r}\right)^2}$$

Where $a = \left(\frac{\sigma_c}{\pi^2 E}\right)$ is called the Rankine constant. The Rankine constants for different materials are given in Table 5.1.

Rankine's formula can be applied to columns with different end conditions by substituting the equivalent length of the columns. The following examples illustrate the use of this formula.

Table 5.1 The Rankine constant for some materials

Material	σ_Y	Rankine constant
Mild Steel	330	$\frac{1}{7500}$
Medium carbon steel	500	$\frac{1}{500}$
Cast iron	550	$\frac{1}{1600}$
Strong timber	50	$\frac{1}{750}$
Wrought iron	250	$\frac{1}{9000}$

Problem 5.12

For the hollow cast iron column with an outer diameter of 70 mm, inner diameter of 50 mm, and length of 4m, find the buckling load using Rankine's formula. Consider that the ends of the column are fixed. The parameters for Rankine's formula are $a = \frac{1}{1600}$ and $\sigma_c = 550 \text{ N/mm}^2$.

Solution:

Outer diameter (D) = 70 mm

Inner diameter (d) = 50 mm

Cross sectional area, $(A) = \frac{\pi}{4} (70^2 - 50^2) = 6\pi \times 10^2 \text{ mm}^2 = 600\pi \text{ mm}^2$

Moment of inertia, $(I) = \frac{\pi}{64} (70^4 - 50^4) = 27.75\pi \times 10^4 \text{ mm}^4$

$$I = 277500\pi \text{ mm}^4$$

$\therefore$ Least radius of gyration

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{277500\pi}{600\pi}} = 21.505 \text{ mm}$$

length (L) = 4 m = 4000 mm

As fixed at both ends.

∴ Effective length,

$$L_e = \frac{L}{2} = \frac{4000}{2} = 2000 \text{ mm}$$

$$\sigma_c = 500 \text{ N/mm}^2$$

$$\text{Rankine constant, (a)} = \frac{1}{1600}$$

Let P = Buckling load

Using the Rankine's equation

$$P = \frac{\sigma_c \times A}{1 + a \left(\frac{L_e}{r} \right)^2}$$

$$P = \frac{550 \times 600 \pi}{1 + \left(\frac{1}{1600} \right) \left(\frac{2000}{21.505} \right)^2}$$

$$P = \frac{550 \times 600 \pi}{1 + (0.000625)(93.0016)^2} = 161841.42 \text{ N} = 161.841 \text{ kN}$$

Problem 5.13

Calculate the required minimum diameter of a hollow cylindrical cast iron column of length 5m which has both ends fixed. The column has to support a safe load of 300 kN, with a factor of safety of 5. The inner diameter is set at 0.8 times the outer diameter. Use $a = \frac{1}{1600}$ and $\sigma_c = 550 \text{ N/mm}^2$ for Rankine's formula.

Solution

Length (L) = 5m = 5000mm

End condition fixed at both the ends

$$\therefore \text{Effective length } (L_e) = \frac{L}{2} = \frac{5000}{2} = 2500 \text{ mm}$$

$$\text{Safe load} = 300 \text{ kN}$$

$$\text{Safety Factor} = 5$$

Let,

$$\text{Outer diameter} = D$$

$$\text{Inner diameter} = 0.8 \times D$$

$$\text{Crushing stress, } \sigma_c = 550 \text{ N/mm}^2$$

$$\text{Rankine constant (a)} = \frac{1}{1600}$$

$$\text{Now Safety factor} = \frac{\text{Buckling load}}{\text{Safe load}} \text{ or } 5 = \frac{\text{Buckling load}}{300}$$

$$\therefore \text{Buckling load (P)} = 1500 \text{ kN} = 1500000 \text{ N}$$

Area of the column

$$\begin{aligned} \therefore A &= \frac{\pi}{4} [D^2 - (0.8D)^2] \\ &= \frac{\pi}{4} [D^2 - 0.64D^2] = \frac{\pi}{4} [0.36D^2] = \pi \times 0.09D^2 \end{aligned}$$

Moment of inertia,

$$\begin{aligned} I &= \frac{\pi}{64} (D^4 - (0.8D)^4) \\ &= \frac{\pi}{64} (D^4 - 0.4096D^4) \\ &= \frac{\pi}{64} (0.5904D^4) \\ I &= 0.009225\pi D^4 \end{aligned}$$

However, $I = A \times r^2$, where radius of gyration is r

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.009225\pi D^4}{\pi \times 0.09D^2}} = 0.3201 D$$

Let P = Buckling load

Using the Rankine's equation

$$P = \frac{\sigma_c \times A}{1 + a \left(\frac{L_e}{r} \right)^2}$$

$$1500000 = \frac{550 \times \pi \times 0.09D^2}{1 + \left(\frac{1}{1600} \right) \left(\frac{2500}{0.3201 D} \right)^2}$$

$$\frac{1500000}{550 \times \pi \times 0.09} = \frac{D^2}{1 + \left(\frac{38123}{D^2} \right)}$$

$$\frac{1500000}{550 \times \pi \times 0.09} = \frac{D^2}{\left(\frac{D^2 + 38123}{D^2} \right)}$$

$$9645 = \frac{D^2 \times D^2}{(D^2 + 38123)}$$

$$9645(D^2 + 38123) = D^4$$

$$9645D^2 + 367696335 = D^4$$

$$D^4 - 9645D^2 - 367696335 = 0$$

The given equation is quadratic in D^2 . The approach is

$$D^2 = \frac{9645 \pm \sqrt{9645^2 + 4(367696335)}}{2}$$

$$\therefore \left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$D^2 = \frac{9645 \pm \sqrt{93026025 + 1470785340}}{2}$$

$$D^2 = \frac{9645 \pm \sqrt{1563811365}}{2}$$

$$D^2 = \frac{9645 \pm 39545}{2}$$

$$D^2 = 24595 \text{ mm}^2$$

[As diameter cannot be negative, only positive value is taken]

$$D = 156.827 \text{ mm}$$

∴ Outer diameter (D) = 156.827 mm

Inner diameter (d) = $0.8 \times D = 0.8 \times 156.827 = 125.46 \text{ mm}$

Problem 5.14

Determine the safe load of a 2m long column with a 6cm diameter circular cross-section. One end of the column is fixed, while the other end is free. To calculate the safe load, use the following methods:

(i) Rankine's formula, considering crushing stress of $\sigma_c = 550 \text{ N/mm}^2$ and constant value of $a = \frac{1}{1600}$

(ii) Euler's formula, with Young's modulus of $E = 1.2 \times 10^5 \text{ N/mm}^2$ for fixed-free ends.

Solution

Length (L) = 2.0m = 2000mm

Diameter (d) = 6cm

Cross sectional area, $A = \frac{\pi}{4} \times 6^2 = 28.274 \text{ cm}^2 = 28.274 \times 10^2 \text{ mm}^2$

Moment of inertia, $I = \frac{\pi}{64} \times 6^4 = 63.617 \text{ cm}^4 = 63.617 \times 10^4 \text{ mm}^4$

Least radius of gyration,

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{63.617 \times 10^4}{28.274 \times 10^2}} = 15 \text{ mm}$$

End conditions both ends pinned

Effective length, $L_e = L = 2000 \text{ mm}$

Safety Factor = 3.0

(i) Safe load by Rankine's formula

Yield stress $\sigma_c = 550 \text{ N/mm}^2$

Rankine constant (a) = $\frac{1}{1600}$

Let

P - Buckling load

Using the Rankine's equation

$$P = \frac{\sigma_c \times A}{1 + a \left(\frac{L_e}{r} \right)^2}$$

$$P = \frac{550 \times 2827.4}{1 + \left(\frac{1}{1600} \right) \left(\frac{2000}{15} \right)^2}$$

$$P = \frac{1555070}{12.11} = 128412 \text{ N}$$

$\therefore$ Safe load = $\frac{\text{Buckling load}}{\text{Factor of safety}}$

$$= \frac{128412}{3} = 42804 \text{ N}$$

(ii) Euler's formula for safe load [fixed-free ends].

One end is fixed while free at the other end

∴ Effective length, $L_e = 2 \times L = 2 \times 2000 = 4000 \text{ mm}$

$$E = 1.2 \times 10^5 \text{ N/mm}^2$$

Let

P - Buckling load

Using the Euler Eqn,

$$P = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 1.2 \times 10^5 \times 63.617 \times 10^4}{4000^2}$$

$$P = 47090 \text{ N}$$

$$\therefore \text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}}$$

$$= \frac{47090}{3} = 15696.66 \text{ N}$$

5.9 SAFE WORKING LOAD

The term Safe Working Load (SWL), is known as Normal Working Load (NWL), signifies the maximum force or load that lifting equipment, such as cranes, winches, hoists, or accessories, can safely manage without the risk of failure or breakage.

Safe Working Load (SWL) is determined by the equipment manufacturer and calculated based on the Minimum Breaking Strength (MBS) or Minimum Breaking Load (MBL), which is the minimum load that could cause the equipment to fail under force. The MBS is then divided by a safety factor to determine the SWL.

The safety factor is a numerical value generally falling within the range of 4 to 6, chosen based on factors such as the type of equipment, its intended purpose, and the potential consequences of equipment failure. In cases where equipment failure could threaten human safety, the safety factor may even be elevated to 10.

Thus

$$\text{Safe working load} = \frac{\text{Minimum Breaking Strength}}{\text{Safety Factor}}$$

5.10 WORKING LOAD LIMIT

The Working Load Limit (WLL) denotes the maximum load a lifting device, such as a crane, hoist, chain, sling, or similar equipment, can safely bear during normal operational conditions. Manufacturers establish this limit through rigorous testing to ensure the safe operation of the equipment, and it should never be exceeded.

The concept of Safe Working Load (SWL), the WLL is derived by dividing the Minimum Breaking Strength (MBS) or Minimum Breaking Load (MBL) of the equipment by the safety factor. This safety factor varies depending on the type of equipment and its intended usage. Generally, the safety factor falls between 4 to 6, but when equipment failure could potentially endanger human life, it can reach up to 10.

In the formula form:

$$\text{Working load limit} = \frac{\text{Minimum Breaking Strength}}{\text{Safety Factor}}$$

5.11 DIFFERENCE BETWEEN SAFE WORKING LOAD (SWL) AND WORKING LOAD LIMIT (WLL)

Working Load Limit (WLL) and Safe Working Load (SWL) are used in material handling and engineering sectors to indicate the maximum capacity that equipment can safely manage. Although they have been used interchangeably in certain contexts, they hold separate definitions in various regions and standards. Below is a concise overview outlining the distinctions between them:

- Safe Working Load, denotes the maximum weight or force that equipment or machinery can safely manage without risking deformation or failure. This maximum load can be applied to a component or equipment without causing any structural compromise. The calculation of SWL involves dividing the Minimum Breaking Load (MBL) or Minimum Breaking Strength (MBS) by the safety factor, typically falling within the range of 4 to 6.
- Working Load Limit, is a term that has largely replaced SWL in the United States, European, and ISO standards due to the more specific and legally clear nature of its definition. The WLL is the maximum mass or force a product can support in general service when the pull is applied inline.
- In practical terms, both SWL and WLL refer to the maximum safe load that a piece of lifting equipment can handle. However, the transition from SWL to WLL has been driven by efforts to enhance specificity and clarity in safety regulations, as well as to mitigate potential legal implications linked to the term "safe."
- It is important to note that these terms provide a guideline for safe operation, and loads should not exceed these limits. Proper training, inspection, and maintenance of lifting equipment are essential in ensuring safe operations.

5.12 FACTORS OF SAFETY

The safety factor represents the ratio of the critical load (CL) to the safe load (SL), reflecting the degree of structural strength beyond the applied load. Below are the customary safety factors utilized for different materials:

Column material	Safety Factor
Cast iron	5
Mild steel	3
Timber	6
Wrought iron	3

UNIT SUMMARY

- The unit on columns in strength of materials is a comprehensive exploration of these critical structural elements. Beginning with an introduction to their significance in engineering and architecture, the unit progresses to discuss various types of columns based on material, shape, and construction.
- It delves into the analysis of column buckling, covering Euler's formula and design principles crucial for ensuring structural stability and integrity.
- Throughout the unit, emphasis is placed on understanding effective length, end conditions, and practical applications of column design in actual engineering projects, including topics such as column stability, bracing methods, and the influence of load types.
- This unit equips readers with essential knowledge for effectively designing and analyzing columns in diverse engineering contexts.

EXERCISE

A. Multiple Choice Questions

5.1 A loaded column fails due to

- | | |
|-------------------------------|-----------------------|
| (a) Stress due to direct load | (c) Both (a) and (b) |
| (b) Stress due to bending | (d) None of the above |

5.2 Rankine's formula is an empirical formula which is used for

- | | |
|-------------------|---------------------------------|
| (a) Long columns | (c) Both long and short columns |
| (b) Short columns | (d) None of the above |

5.3 The slenderness ratio of a long column is

- (a) 10-20 (c) 50-60
(b) 20-30 (d) Above 80

5.4 The Rankine's constant (a) for a given material of a column depends upon the

- (a) Length of column (c) length and diameter
(b) Diameter of the column (d) None of the above

5.5 The value of Rankine's constant for mild steel is

- (a) 1/9000 (c) 1/1600
(b) 1/7500 (d) 1/750

5.6 The effective length is equal to actual length of a column when

- (a) One end fixed other end free (c) One end is fixed and other end hinged
(b) Both ends fixed (d) Both ends hinged

5.7 A column is fixed at both ends. Its effective length will be equal to

- (a) $2l$ (c) $0.5l$
(b) l (d) $0.707l$

5.8 The nature of buckling load is

- (a) Tensile load (c) Live load
(b) Compressive load (d) Dead load

5.9 The Rankine's constant 'a' in the Rankine's formula is equal to

- (a) $\pi^2 E / \sigma_c$ (c) $E \sigma_c / \pi^2$
(b) $\pi^2 / E \sigma_c$ (d) $\sigma_c / \pi^2 E$

Answer to Multiple-Choice Questions

5.1 (c), 5.2 (c), 5.3 (d), 5.4 (d), 5.5 (b), 5.6 (d), 5.7 (b), 5.8 (b), 5.9 (d)

B. Subjective Questions

5.1 What are the different end condition of a column? Explain.

5.2 Explain the effect of eccentric loading on a column.

5.3 Explain the failure of short and long columns.

5.4 Define the terms: column, strut, and buckling load.

- 5.5 Derive the expression for the Euler's buckling load of a long column with following end conditions. (a) Both ends are hinged (b) Both ends are fixed.
- 5.6 Obtain the expression for the Rankine's buckling load.
- 5.7 Define slenderness ratio. State the limitations of Euler's formula.
- 5.8 What is equivalent length of a column? How is the concept used in the column theory?
- 5.9 Explain the assumptions in the Euler's column theory.

B. Numerical Problems

1. For a column with a timber section measuring $150\text{ mm} \times 150\text{ mm}$ and a length of 6 m, both ends are fixed, and Young's modulus of 18 kN/mm^2 , determine the following: (i) buckling load (ii) column safe load when the safety factor is 3.
(Ans: 832.74 kN, 278 kN)
2. Calculate the buckling load and safe load for a hollow mild steel tube with a length of 6 m, inner diameter of 5 cm, and thickness of 6 mm. The tube is employed as a strut with both ends hinged. Use a factor of safety of 3 and consider the modulus of elasticity E as $2 \times 10^5\text{ N/mm}^2$.
(Ans: 22948.77 N, 7649.59 N)
3. Calculate the I-section Euler's buckling load of a beam with $400\text{ mm} \times 200\text{ mm} \times 20\text{ mm}$ dimensions and a 6 m length. The joist is utilized as a strut fixed with both ends. Use Young's modulus of $2 \times 10^5\text{ N/mm}^2$ for the joist.
(Ans: 3125813.37 N)
4. Calculate the buckling load of T-section with $100\text{ mm} \times 100\text{ mm} \times 10\text{ mm}$ dimensions and of length 5000 mm. The column is hinged at both ends. Consider the modulus of elasticity E as $2 \times 10^5\text{ N/mm}^2$ (Ans: 135805.7 N)
5. The hollow cast iron column has an inner diameter of 3 cm and an external diameter of 5 cm. Its length is 4 m, and both ends are fixed. Using Rankine's

formula, determine the buckling load considering $f_c = 550 \text{ N/mm}^2$ and $a = \frac{1}{1600}$.

The calculation involves evaluating the moment of inertia and effective length and applying Rankine's formula to find the buckling load.

(Ans: 540.09 kN)

6. Calculate the minimum diameter required for a cylindrical cast iron hollow column that is 6m meters long and has both ends fixed. The column must safely bear a load of 300 kN while maintaining a factor of safety of 4. The inner diameter of the column is specified as 0.7 times the external diameter. The calculation has to be based on Rankine's formula, using $f_c = 550 \text{ N/mm}^2$ and $a = \frac{1}{1600}$

(Ans: D = 95.3 mm, d = 66.7 mm)

7. Calculate the buckling load for a solid circular bar with a length of 5m and diameter of 40 mm. This bar, which experienced a 4.5 mm extension under a tensile load of 48 kN, is utilized as a strut with both ends hinged. Also, determine the safe load, considering a factor of safety 3.0.

(Ans: 2105.5 N and 701.8 N)

8. Calculate the buckling load of a solid round bar with 3 meters long and has a diameter of 5 cm. This bar is employed as a strut and hinged at both ends. Consider Young's modulus E to be $2 \times 10^5 \text{ N/mm}^2$.

(Ans: 67.287 kN)

9. Calculate the Euler's buckling load of a hollow cylindrical column of cast iron with an outer 150 mm diameter and a 25 mm thickness. This 7-meter-long column is hinged on both ends. Consider Young's modulus E to be $8 \times 10^4 \text{ N/mm}^2$. Compare this calculated crushing load with the crushing load obtained using Rankine's formula, with $f_c = 550 \text{ N/mm}^2$ and $a = \frac{1}{1600}$.

(Ans: 665 kN, 260.77 kN)

KNOW MORE

- *Structural Engineering:* In the design and construction of buildings, bridges, and other structures, columns play a critical role in supporting vertical loads and resisting compressive forces. Engineers use column concepts to determine the appropriate size, shape, and material for columns to ensure structural stability and safety.
- *Civil Engineering:* Columns are essential components in the construction of infrastructure projects such as highways, tunnels, and dams. Engineers apply column concepts to design support structures that can withstand the loads and pressures associated with these projects, ensuring their long-term durability and reliability.
- *Manufacturing and Industrial Engineering:* In manufacturing and industrial settings, columns are used in support structures for machinery, equipment, and storage systems. Engineers apply column concepts to design robust support systems that can withstand the weight and forces associated with industrial operations, ensuring workplace safety and efficiency.

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Dynamic QR Code for Further Reading

Concepts of columns



Assumption made in Euler's theory



Design of columns



Types of columns



Key concepts of effective length of columns



Failure modes of columns



6

LABORATORY MANUAL

EXPERIMENT – 1

PREPARE A SPECIMEN AND EXAMINE THE MICROSTRUCTURE OF THE FERROUS AND NON-FERROUS METALS USING THE METALLURGICAL MICROSCOPE

AIM: To describe metallography and to study the microstructure of ferrous and non-ferrous metals.

APPARATUS: Metallurgical microscope, polishing machine, emery papers of various grades, etchants.

THEORY:

Metallography is the study of the microstructure of materials, particularly metals and alloys. It is basically the study of the structures and composition of these materials using metallurgical microscopes through magnifications, so that the physical and mechanical properties of an alloy can be related to its observed microstructure. Such microscopic studies can provide an abundance of constitutional information about the specimen under investigation, including the size and shape of the grains (crystallites), the presence of micro defects (such as segregation, hair cracks, and non-metallic inclusions), and the nature and distribution of secondary phases.

The metallographic examination can be used in quality control, to predict and/or explain the mechanical properties and to know whether the material has been processed correctly and is therefore a critical step for determining the product reliability and investigating the reasons behind material failure.

PREPATION OF SPECIMEN FOR METALLOGRAPHIC EXAMINATION:

The process of sample preparation is schematically summarized in Fig. 6.1. The preparation of a metallographic sample typically involves the following steps:

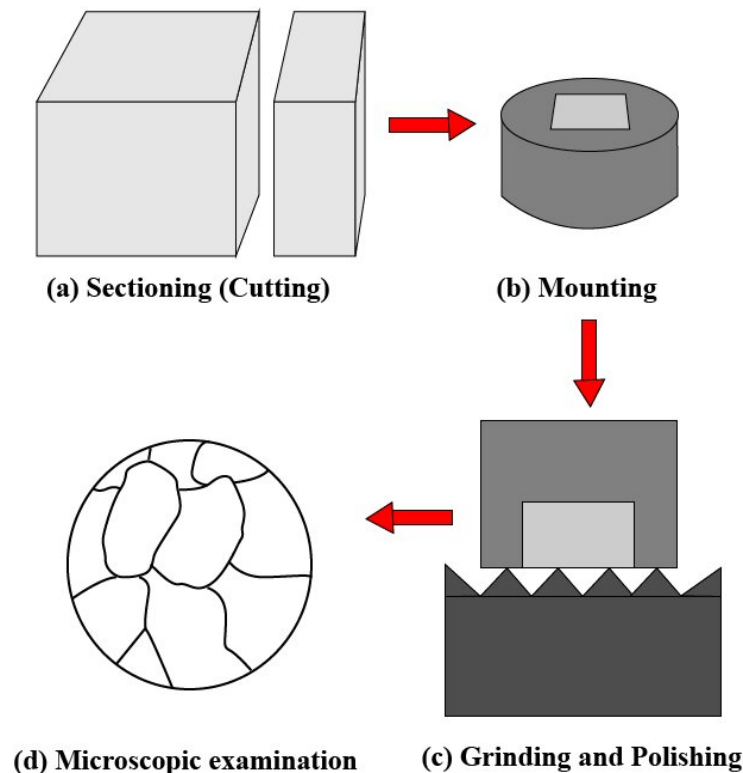


Fig. 6.1(a)-(d): Process of sample preparation

- 1. SECTIONING (CUTTING):** When cutting a specimen from a larger piece of material, care must be taken to ensure that it accurately represents the features found in the larger sample, or contains all the necessary information for investigation.

Proper cutting requires the correct selection of abrasive type, bonding, and size; as well as proper cutting speed, load and coolant. Depending upon the material, the sectioning operation can be obtained by abrasive cutting for metals and metal matrix composites, diamond wafer cutting for ceramics, electronics, biomaterials, minerals, or thin sectioning with a microtome for plastics. The specimen (a square 10mm or diameter 15 mm) is cut to a height of about 15 mm.

2. **MOUNTING:** Mounting of specimens is usually necessary to allow them to be handled easily. It also minimizes the amount of damage likely to be caused to the specimen itself as shown in Fig. 6.2.

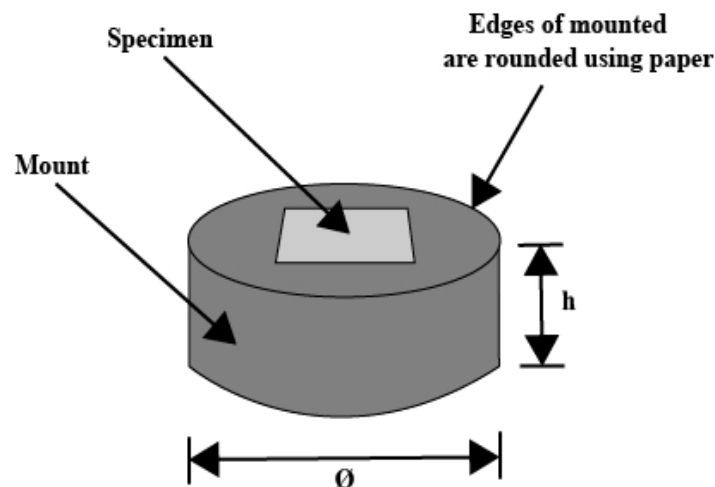


Fig. 6.2: Specimen with mounting

3. **GRINDING:** Surface layers damaged by cutting must be removed by grinding. The grinding procedure involves several stages, with each stage using a finer paper (higher number) than the previous one. Each grinding stage removes the scratches from the previous coarser paper. This can be easily achieved by orienting the specimen perpendicular to the previous scratches. Between each grade the specimen is washed thoroughly with soapy water to prevent contamination from coarser grit present on the specimen surface.

4. **POLISHING:** It is the process of creating a smooth and shiny surface by rubbing it or using a chemical action, leaving a surface with a significant reflection. Polishing involves the use of abrasives (usually alumina powder), suspended in a water solution, on a cloth-covered electrically powered wheel. The purpose of the polishing is to remove the damage produced during cutting and grinding.
5. **ETCHING:** Etching is used to show the microstructure of the metal through selective chemical treatment. Microscopic examination of a properly polished, unetched specimen will reveal only a few structural features such as inclusions and cracks or other physical imperfections. Even in a carefully prepared sample, a surface layer of disturbed metal, resulting from the final polishing stage, is always present and must be removed. Etchants are usually dilute acid or dilute alkalis in a water, alcohol or some other solvent. Etching occurs when the acid or base is placed on the specimen surface because of the difference in rate of attack of the various phases present and their orientation. The etching process is usually accomplished by merely applying the appropriate solution to the specimen surface for several seconds to several minutes.

MICROSCOPIC EXAMINATION

Detailed viewing is done with a Metallurgical Microscope (Fig. 6.3). A metallurgical microscope has a system of lenses (objectives and eyepiece) so that different magnifications (25X to 1000X) can be achieved.

The important characteristics of the microscope are:

1. Magnification,
2. Resolution and
3. Flatness of field.

The resultant magnification is the product of the magnifying power of the objective lens and that of the eyepiece lens.

Scanning Electron Microscopes (SEMs) are capable of magnifying up to 20,000X and Transmission Electron Microscopes (TEMs) are utilized to view at magnifications up to 100,000X for highly detailed microstructural study.

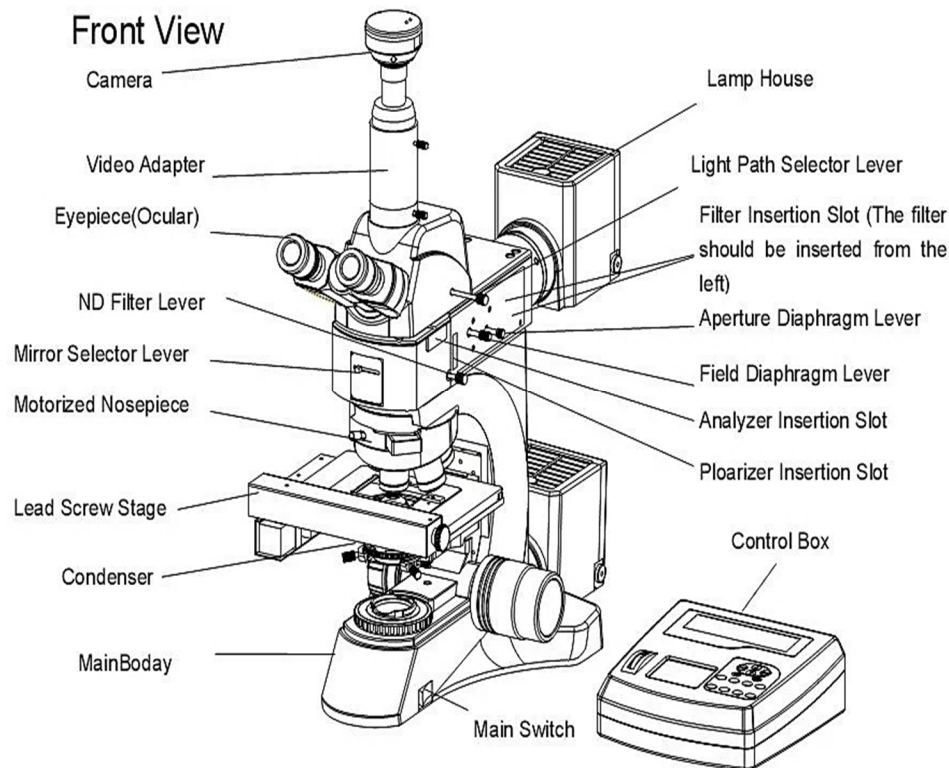


Fig. 6.3: Metallurgical Microscope

The specimen is placed on the stage of the microscope so that its surface is perpendicular to the optical axis. Put on the source of light. Adjust the longitudinal and transverse movements of the table to focus the light on the specimen. Make coarse and fine adjustments to focus. Observe and record the microstructure. The microstructures of some ferrous and non-ferrous alloys are shown in Fig. 6.4.

GRAIN SIZE CALCULATION:

The micro structure of many metallic or ceramic materials consists of numerous grains. A grain is the portion of the materials within which the arrangement of the atoms is nearly identical but the orientation or crystal structure of atoms may differ.

The properties of any alloy are affected not only by the character of phases present, but also by the size of grains that are present in the structure. Larger the size of the grains, greater will be the hardenability of the steel, whereas smaller grain size led to greater strength.

The grain size is measured by comparison, under microscope with a magnification of X100, with standard charts shown in the Fig. 6.4. Steels with grain size number 1 to 5 are coarse grained and number 6, 7 and 8 are fine grained steels.

According to ASTM, the grain size N is given by $m = 8 \times 2^N$, where m is the number of grains per mm^2 .

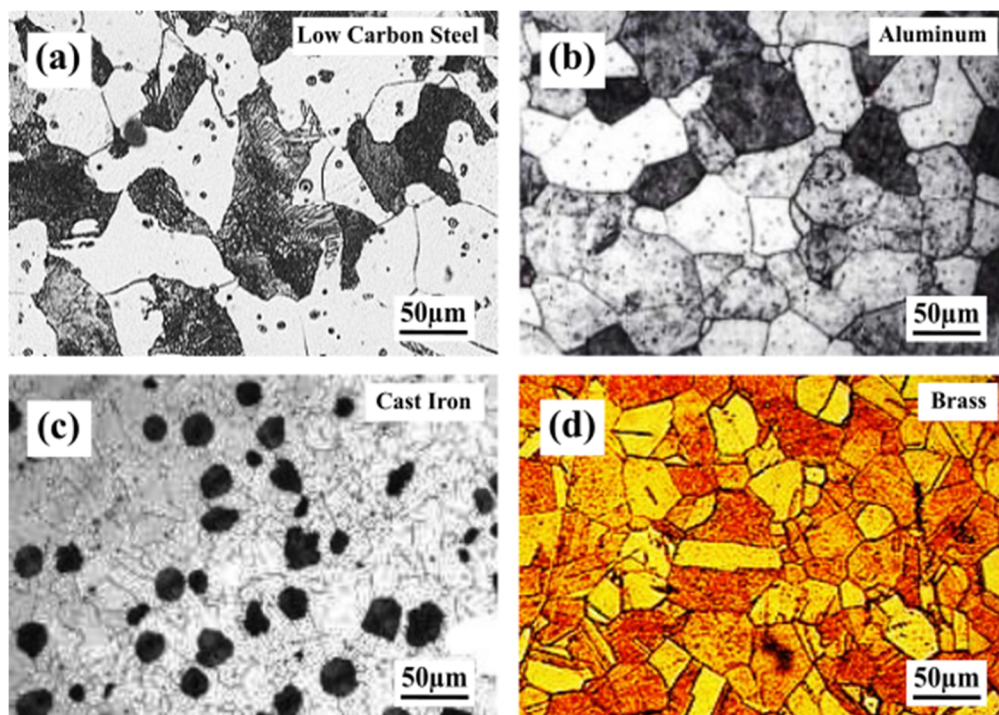


Fig. 6.4: Microstructures of some ferrous and non-ferrous alloys

METALLOGRAPHY OBSERVATION RECORD:

Draw the microstructures observed in the specimens.

1. Magnification:
2. Specimen:
3. Etchant:
4. Observation:
5. Grain size number:

RESULT:

Proper sample preparation techniques are essential for accurately revealing the microstructure of a metallic specimen. Otherwise, it will be difficult to distinguish between different phase in the microstructure, and the fine scratches on metal surface and grain boundaries.

EXPERIMENT – 2

DETECT THE CRACKS IN THE SPECIMEN USING (i) VISUAL INSPECTION AND RING TEST (ii) DYE PENETRATION TEST (iii) MAGNETIC PARTICLE TEST

(i) VISUAL INSPECTION AND RING TEST

AIM: To detect the cracks in the specimen using visual inspection and ring test

APPARATUS REQUIRED:

1. Grinding wheel,
2. Mounting shaft or pin
3. A hard plastic or hard wood object such as the handle of a screwdriver or other tool.

THEORY:

VISUAL INSPECTION:

Visual inspection is one of the most common and most powerful means of non-destructive testing. Visual testing requires adequate illumination of the test surface and proper eye-sight of the tester. For optimal effectiveness, visual inspection demands special attention due to the necessity for training and the utilization of specific equipment and instrumentation.

RING TEST:

Grinding wheels must be inspected and ring tested before they are mounted to ensure that they are free from cracks or other defects. Wheels should be tapped gently with a light, non-metallic instrument. A stable and undamaged wheel will

give a clear metallic tone. The unique ringing sound is attributed to the hardness of the material in the wheel and its ability to transmit vibrations as sound. If the wheel is cracked, the vibrations stop at the crack and there is no ring.

PROCEDURE:**VISUAL INSPECTION:**

1. Check the surface condition of the test specimen.
2. Examine the shape of the component.
3. Checking for internal side defects.

RING TEST:

1. Make sure the wheel is dry and free of sawdust that could deaden the sound of the ring.
2. Suspend the wheel on a pin that fits through the hole so that it will be easy to turn, but do not mount the wheel on the grinder.
3. Make sure the wheel is at dry condition.
4. Imagine a vertical plumb line up the centre of the wheel.
5. Tap the wheel about 4° on each side of the vertical line, about one or two inches from the wheel edge.
6. Turn the wheel about 180° so that the bottom of the wheel is now on top.
7. The wheel passes the test if it gives a clear metallic tone when tapped at all four points. If the wheel sounds dead at any of the four points, it is cracked.

RESULT:

Thus, the cracks in the specimen using visual inspection and ring test are detected.

(ii) DYE PENETRATION TEST

AIM: To detect the cracks in the specimen using dye penetration test

APPARATUS REQUIRED:

Dye penetrant kit consists of cleaner, penetrant and developer.

THEORY:

A suitable liquid penetrant is applied to the surface of the component under examination and is permitted to remain there for sufficient time to allow the liquid to penetrate into any defects open at the surface. After the penetrant time, the excess penetrant, which remain on the surface, is removed. Then a light coloured, powder absorbent called a developer is applied to the surface. This developer acts as a blotter and draws out a portion of the penetrant which had previously seeped into the surface openings. As the penetrant is drawn out, it diffuses into the coating of the developer, forming indications of the surface discontinuities or flaws.

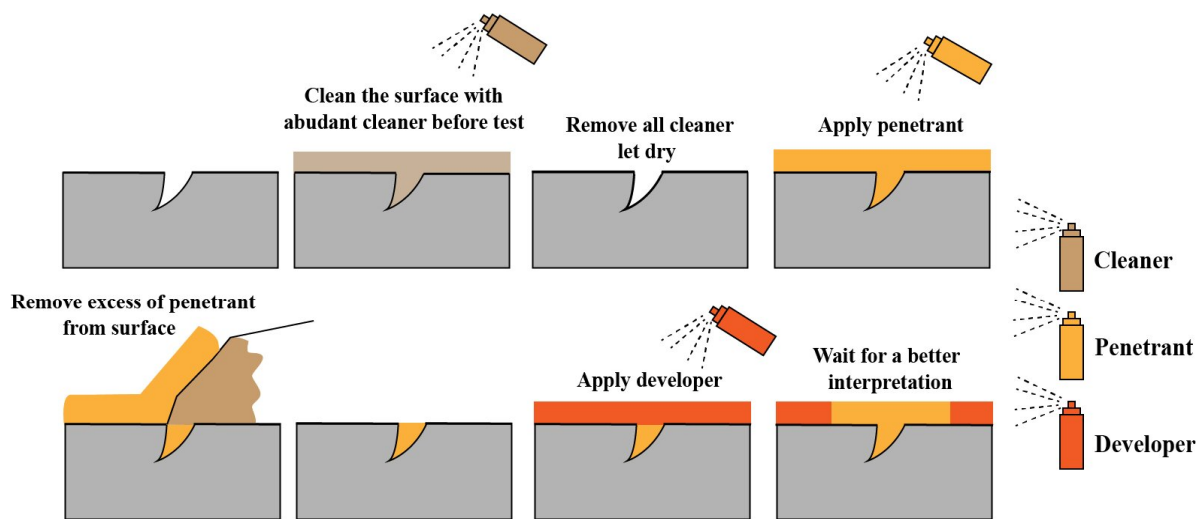
PROCEDURE:

The general steps can be summarized as follow:

1. **Surface preparation:** Wipe the test surface clean from oil, grease, water or other contaminants that may prevent penetrant from entering flaws.
2. **Penetrant application:** Once the surface has been thoroughly cleaned and dried, the penetrant material is applied by spraying onto surface.
3. **Penetrant dwell:** The penetrant is left on the surface for a sufficient time to allow as much penetrant as possible to be drawn from or to seep into defects.
4. **Excess penetrant removal:** Remove excess penetrant from the surface of the sample while removing small amount of penetrant from defects.

5. **Developer application:** Apply a thin layer of developer to draw penetrant trapped in flaws back to the surface where it will be visible.
6. **Indication development:** The developer is allowed to stand on the part surface for a period of time sufficient to permit the extraction of the trapped penetrant out of any surface flaws. This development time is usually minimum of 10 minutes. Significantly longer time may be necessary for tight cracks.
7. **Inspection:** Perform visual inspection under appropriate lighting to detect indications from any flaws which may be present.
8. **Clean surface:** The final step in the process is to clean the part surface properly to remove the developer from the parts that found to be acceptable.

OBSERVATION:



Dye penetration testing procedure

Fig. 6.5: Dye penetration testing

RESULT:

Thus, the cracks in the specimen using dye penetration test are detected.

(iii) MAGNETIC PARTICLE TEST

AIM: To detect the cracks in the specimen using magnetic particle test

APPARATUS REQUIRED:

1. Magnetic Crack Detector
2. Iron powder

DIAGRAM:

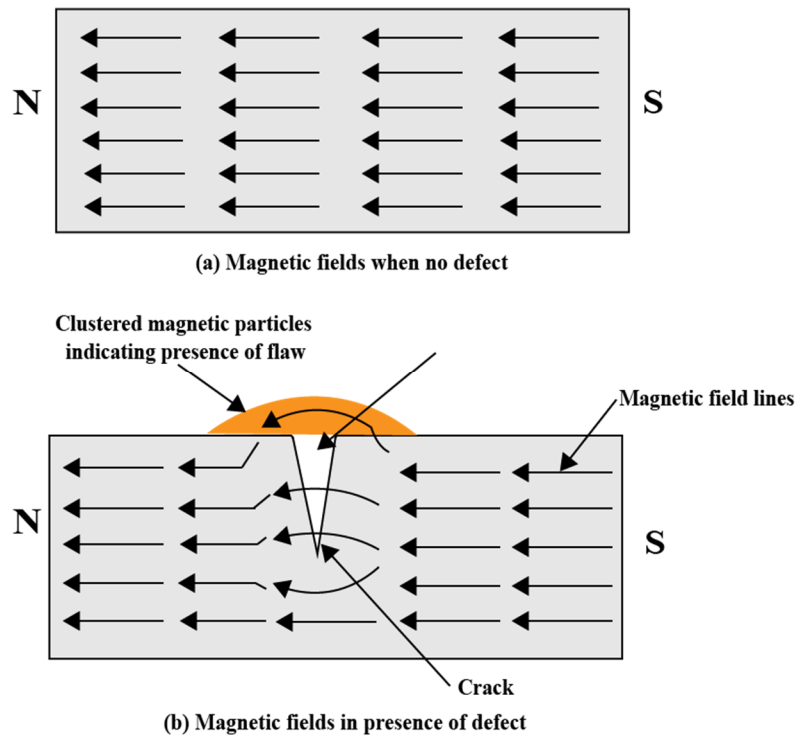


Fig. 6.6(a)-(b): Principle of magnetic particle inspection

THEORY:

Magnetic particle inspection uses magnetic fields and magnetic particles for detecting defects in ferromagnetic components. The basic principle of this inspection method is that the component specimen is magnetized to generate

magnetic flux in the material which travels from the north pole to the south pole (magnetic flux exits at the north pole and enters at the south pole). Now if there is any discontinuity or flaws in the specimen, secondary magnetic poles are produced in the cracked faces. In this location, the magnetic field spreads out due to the air gap in the defect causing a magnetic flux leakage field. Such regions can be detected easily by using magnetic particles (iron powder), or a liquid suspension on the surface. Due to the magnetic effect, such particles are attracted to the flux leakage and make a cluster around the flaw making it visible. Fig. 6.6 shows the basic principle of magnetic particle inspection.

PROCEDURE:

The general steps can be summarized as follow:

1. Surface Preparation:

All surfaces and adjacent areas (within 1 inch) that will be examined must be free of rust, scale, sand, grease, paint, slag, oily films, or other interfering conditions. Unusually rough or non-uniform surfaces may interfere with magnetic particle cluster formation making interpretation of the magnetic particle inspection method's (indications) difficult.

2. Inducing a Magnetic Field:

This is the most important step in the magnetic particle inspection procedure. In this step, place the equipment on the area to be tested and induce the magnetic field. Various types of magnetic particle inspection equipment are available. Mostly used industrial equipment are Permanent magnets, Electromagnetic Yokes, Current flow probes, Magnetic Flow, Flexible coils, Threading bars, Adjacent cables, etc. The magnetization technique can be Longitudinal, Circular, or Multidirectional Magnetization. Equipment spacing in the inspection area is normally kept in between 3 inches to 8 inches.

3. Applying magnetic particles on the test surface:

Magnetic particles which are dry or wet can be either fluorescent or non-fluorescent (visible, colour contrast) and are available in a variety of colours to contrast with the tested material. After the specimen is magnetized choose magnetic particles accordingly for magnetic particle inspection and apply them on the surface of specimen.

4. Examine the component surface for defects:

Remove the excess particles using light airflow and inspect the specimen for defects as per the standards.

5. Repeat the test by changing the magnetic field:

Two separate examinations are to be used on each area. The second examination is performed with the flux lines perpendicular to those used for the first examination in that area.

6. Demagnetization and Cleaning:

Once the magnetic particle inspection is completed the specimen should be demagnetized otherwise the residual magnetism on the specimen may interfere with subsequent usage. The presence of residual magnetism can be verified using a calibrated Gaussmeter, Magnetic Field Meter, or a hall Probe Gauss meter. Residual magnetism must not exceed (+/-) 2 gauss. After that, the parts shall be cleaned to remove all residual magnetic particle materials. If wet fluorescent MPI was performed, the part shall be scanned with the backlight to assure that the cleaning is adequate.

RESULT:

The cracks in the specimen using magnetic particle test are detected.

EXPERIMENT – 3

DETERMINATION OF ROCKWELL'S HARDNESS NUMBER FOR VARIOUS MATERIALS LIKE MILD STEEL, HIGH CARBON STEEL, BRASS, COPPER AND ALUMINIUM.

AIM: To determine the hardness of the given specimen using Rockwell hardness test.

APPARATUS: Rockwell hardness testing machine, Black diamond cone indenter, Hard steel specimen.

DIAGRAM:

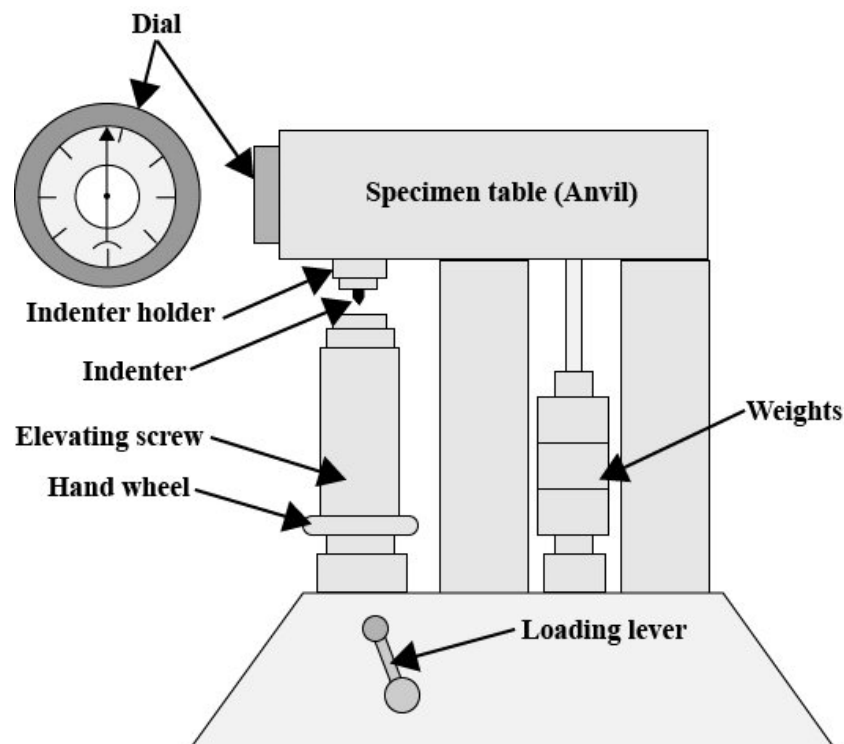


Fig. 6.7: Schematic of a Rockwell hardness tester

SPECIFICATION

1. Maximum test height = 295 mm
2. Depth of throat = 150 mm
3. Maximum depth of screw below base = 280 mm
4. Dimensions of machine = 210 mm × 470 mm
5. Height = 850 mm
6. Net weight = 1225.831 N

THEORY

Hardness is defined as the resistance of a metal to plastic deformation against indentation, scratching, abrasion or cutting. The hardness of a material is measured by Rockwell hardness test method through the depth of penetration of the indenter. The depth of penetration is inversely proportional to the hardness. Both ball or diamond cone types of indenters are used in this test. There are three scales on the machine for taking hardness readings. Scale “A” with load 60 kgf or 588.4 N and diamond indenter is used for performing tests on thin steel and shallow case-hardened steel. Scale “B” with load 100 kgf or 980.7 N and 1.588 mm diameter ball indenter is used for performing tests on soft steel, malleable iron, copper and aluminum alloys. At first, minor load is applied to overcome the film thickness on the metal surface. Minor load also eliminates errors in the measurements of depth due to spring of the machine frame or setting down of the specimen and table attachments.

The Rockwell hardness is derived from the measurement of the depth of the impression

E_P = Depth of penetration due to a Minor load of 98.07 N.

E_a = Increase in the depth of penetration due to Major load.

E = Permanent increase in depth of indentation under minor load of 98.07 N even after the removal of Major load.

This method of test is suitable for finished or machined parts of simple shapes.

ADVANTAGES:

1. It is quicker.
2. The dial indicator gives the hardness directly (no need of microscope to measure indentation).
3. The use of diamond cone penetrator enables the test to be used for high ranges of hardness which are not possible with Brinell test.

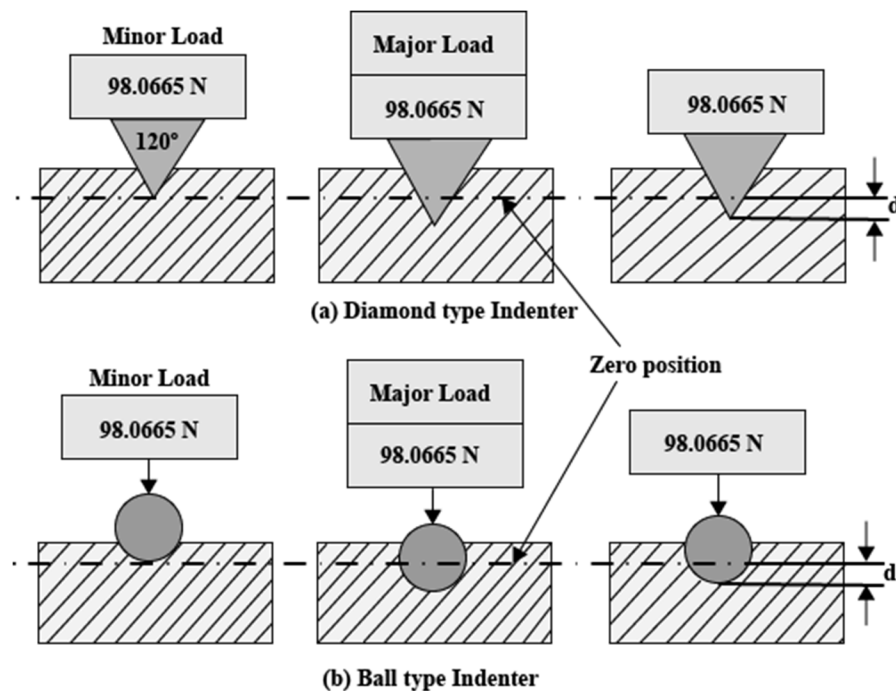


Fig. 6.8: Measurement of the depth of penetration of the indenter

PRINCIPLE OF THE TEST:

This test is based on the measurement of the depth of penetration of the indenter. In order to eliminate zero error and possible surface effects due to roughness,

initial load (called the minor load) of 98.0665 N is first applied to the indenter, and the depth of penetration then reached is taken as zero for further measurements as shown in Fig. 6.8. The details of load and indenters are given in Table.

In direct reading hardness tester, B scale is marked in red and C scale in black. Zero of C scale is opposite to 30 on B scale, so that there is a difference of 30 hardness number.

Table : Details of Indenter and loading:

Particulars	Rockwell A	Rockwell B	Rockwell C
Indenter	Diamond cone with 120 deg. apex angle	Inch (1.59 mm) dia ball	Diamond cone with 120 deg. apex angle
Minor load kgf	10	10	10
Major load kgf	50	90	140
Total load kgf	60	100	150
Dial scale mark	Black (outer)	Red (inner)	Black (outer)
Pointer position on dial	0 (outer scale)	30 (inner scale)	0 (outer scale)
Type of material	Much harder material such as carburized steel, cement	Soft steels, copper, aluminum, brass, grey cast iron	Hard steels, Ti, W, Va, etc.

PROCEDURE:

1. To examine hardness testing machine.
2. Place the specimen on the platform of the machine. Using the elevating screw raise the platform and bring the specimen just in contact with the ball. Apply an initial load until the small pointer shows red mark.
3. Release the operating valve to apply additional load. Immediately after the additional load is applied, bring back operating valve to its position.
4. Read the position of the pointer on the C scale, which gives the hardness number.
5. Repeat the procedure several times on the specimen selecting different points for indentation.

OBSERVATION:

Indenter = Ball type

Diameter of steel ball indenter, d = ----- mm

Material of test piece = -----

Thickness of test piece = ----- mm

Hardness scale used = -----

Minor load = ----- N

Major load = ----- N

PRECAUTIONS:

- The specimen should be cleaned properly.
- Place the specimen properly.
- Take reading carefully and correctly.

TABULAR COLUMN

Sl. No	Specimen material	Type of indenter	Diameter of indenter (mm)	Load applied (N)	Rockwell reading HRA/HRB/HRC
1.					
2.					

RESULTS: Rockwell hardness values of the different materials are

- Mild steel = -----
- High carbon steel = -----
- Brass = -----
- Copper = -----
- Aluminum = -----

EXPERIMENT – 4

DETERMINE THE RESISTANCE OF MATERIALS TO IMPACT LOADS BY IZOD TEST AND CHARPY TEST.

AIM: To determine the impact toughness (strain energy) through Izod test and Charpy test

APPARATUS: Pendulum type impact testing machine, setting gauge, vernier caliper, test specimen and steel rule.

THEORY:

An impact test signifies toughness of material which is the ability of material to absorb energy during plastic deformation. Static tension tests of unnotched specimens do not always reveal the susceptibility of a metal to brittle fracture. This important factor is determined by impact test. Toughness takes into account both strength and ductility of the material. Several engineering materials have to withstand the impact or suddenly applied loads while in service. Impact strengths are generally lower as compared to strengths achieved under gradually applied loads. For all types of impact tests, the notch bar tests are most extensively used. Therefore, the impact test measures the energy necessary to fracture a standard notch bar by applying an impulse load. The test measures the notch toughness of material under shock loading. Values obtained from these tests are not of much utility to design problems directly and are highly arbitrary. Still, it is important to note that it provides a good way of comparing toughness of various materials or toughness of the same material under different condition. This test can also be used to assess the ductile brittle transition temperature of the material occurring due to the lowering of temperature.

PROCEDURE:

1. With the striking hammer (pendulum) in safe test position, firmly hold the steel specimen in impact testing machine's vice in such a way that the notch faces the hammer and is half inside and half above the top surface of the vice.
2. Bring the striking hammer to its top most striking position unless it is already there, and lock it at that position.

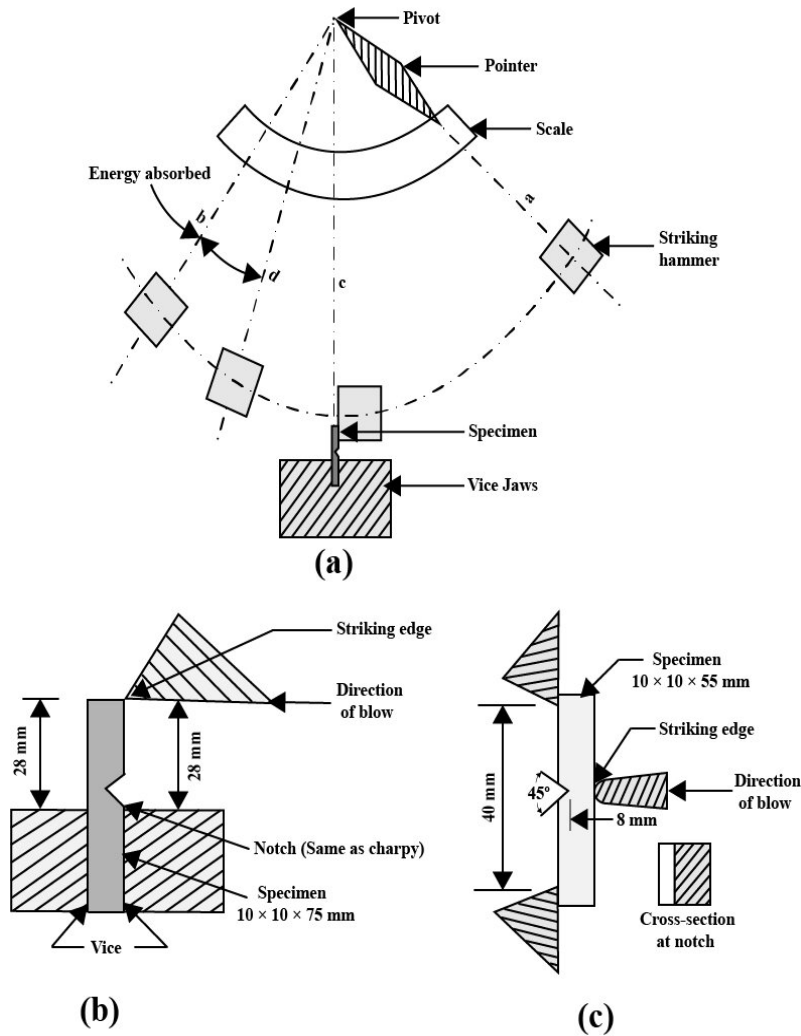


Fig. 6.9: Impact testing details: (a) Pendulum type impact test machine, (b) Izod impact specimen, and (c) Charpy impact specimen.

3. Bring the indicator of the machine to zero, or follow the instructions of the operating manual supplied with the machine.

4. Release the hammer. It will fall due to gravity and break the specimen through its momentum, the total energy is not absorbed by the specimen. Then it continues to swing. At its topmost height after breaking the specimen, the indicator stops moving, while the pendulum falls back. Again, bring back the hammer to its idle position.
5. For Charpy test the specimen is placed on supports or anvil so that the blow of hammer is opposite to the notch.

OBSERVATION:

IZOD IMPACT

1. Original size of the specimen = $10 \times 10 \times 75$ mm
2. Depth of V-notch, $d = 2$ mm
3. Cross-sectional area at notch, $A_{\text{notch}} = 8 \times 10 = \text{-----}$ mm²
4. Weight of hammer $W = \text{-----}$ kg
5. Length/ radius of pendulum, $R = \text{-----}$ m
6. Angle of notch, $\theta = 45^\circ$
7. Angle of fall, $\alpha = \text{-----}$
8. Impact strength, $\frac{\text{Fracture Energy}}{\text{Area of Notch}} = \text{-----}$ N-m/m²

Table : Test results

Sl.No	Material	Fracture energy from scale N-m (J)	Impact strength of the material N-m/m ² (J/m ²)
1.			
2.			
3.			

CHARPY IMPACT

1. Original size of the specimen = $10 \times 10 \times 55$ mm
2. Depth of V-notch, $d = 5$ mm
3. Width of notch, $b = 2$ mm
4. Cross-sectional area at notch, $A_{\text{notch}} = 5 \times 10 = \text{-----}$ mm²
5. Weight of hammer $W = \text{-----}$ kg
6. Length/ radius of pendulum, $R = \text{-----}$ m
7. Angle of fall, $\alpha = \text{-----}$
8. Impact strength, $\frac{\text{Fracture Energy}}{\text{Area of Notch}} = \text{-----}$ N-m/m²

Table : Test results for Charpy Impact

Sl.No	Material	Fracture energy from scale N-m (J)	Impact strength of the material N-m/m ² (J/m ²)
1.			
2.			
3.			

PRECAUTION:

1. Measure the dimensions of the specimen carefully.
3. Locate the specimen in such a way that the hammer, strikes it at the middle
4. Note down the readings carefully.

RESULT:

The Impact strength of material is found out to be ----- J/m².

EXPERIMENT – 5

TORSION TEST ON MILD STEEL – RELATIONSHIP BETWEEN TORQUE AND ANGLE OF TWIST, DETERMINATION OF SHEAR MODULUS AND SHEAR STRESS

AIM: To conduct a torsion test on mild steel.

APPARATUS:

- (i) A torsion test machine along with angle of twist measuring attachment.
- (ii) Standard specimen of mild steel.
- (iii) Steel ruler.
- (iv) Vernier caliper or a micrometer.

THEORY:

For transmitting power through a rotating shaft, it is necessary to apply a turning force. The force is applied tangentially and in the plane of transverse cross section. The torque or twisting moment can be calculated by multiplying two opposite turning moments. It is said to be understood that under pure torsion and it will exhibit the tendency of shearing off at every cross section which is perpendicular to the longitudinal axis.

TORSION EQUATION:

$$\frac{T}{I_p} = \frac{G\theta}{L} = \frac{\tau}{R}$$

T = Maximum twisting torque (N·mm)

I_p = Polar moment of inertia (mm⁴)

τ = Shear stress (N/mm²)

G = Modulus of rigidity (N/mm²)

θ = Angle of twist in radians

L = Length of shaft under torsion (mm)

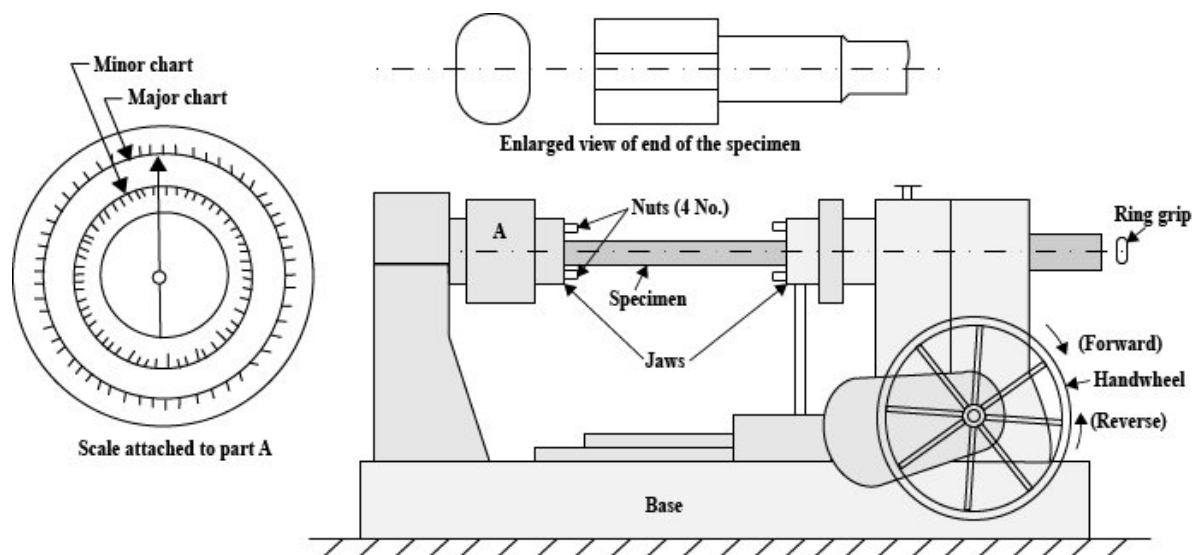


Fig. 6.10: Schematic of a torsion test machine

TORSION TEST MACHINE

Torsion test machine has ring grip jaws to hold the specimen securely in place. Part A, typically a grip or fixture, includes a manually readable scale for monitoring torsional deformation as shown in Fig. 6.10. This setup allows operators to apply torque manually while observing deformation directly on the scale, providing basic yet effective measurements of mechanical properties.

PROCEDURE:

1. Select the suitable grips to suit the size of the specimen and clamp it in the machine by adjusting the length of the specimen by means of a sliding spindle.
2. Measure the diameter at about three places and take the average value.

3. Choose the appropriate range with the capacity change lever.
4. Set the maximum load pointer to zero.
5. Set the protractor to zero for convenience and clamp it by means of knurled screw.
6. Carry out straining by rotating the hand wheel in either direction.
7. Load the machine in suitable increments, observe and record the strain reading.
8. Continue till failure of the specimen.
9. Calculate the modulus of rigidity G by using the torsion equation.
10. Plot the torque–twist graph (T vs θ).

OBSERVATIONS:

1. Polar torsion equation,

$$\frac{T}{I_p} = \frac{G\theta}{L} = \frac{\tau}{R}$$

2. Polar area moment of inertia, $I_p = \frac{\pi d^4}{32} = \text{----- } m^4$
3. Polar section modulus, $Z_p = \frac{\pi d^3}{16} = \text{----- } m^3$
4. Yield torque, $T_y = \text{----- } N\cdot m$
5. Ultimate torque, $T_u = \text{----- } N\cdot m$
6. Modulus of elasticity in shear, $G = \text{----- } N/m^2$

Where G = Slope of the initial linear part of the shear stress v/s shear strain diagram

7. Modulus of rupture in torsion, $\frac{T_u}{Z_p} = \text{----- } N/m^2$
8. Yield stress in torsion, $\frac{T_y}{Z_p} = \text{----- } N/m^2$
9. Torsional stiffness, $K_t = \frac{T}{\theta} \text{ Nm/rad}$
10. (In the elastic region of torque v/s Angular twist diagram)

Table : Torsional test readings

Sl.No	Torque (T) (N-m)	Angle of twist (θ)		Shear stress $\tau = \frac{T}{I_p}$ (N/mm ²)	Shear strain $\phi = \frac{r\theta}{L}$ (rad)
		Degree	Radian		
1.					
2.					

PRECAUTIONS:

1. Measure the dimensions of the specimen carefully
2. Measure the Angle of twist accurately for the corresponding value of Torque.
3. The specimen should be properly placed between the jaws.

GRAPH:

Draw graph of (a) Torque v/s Angular Twist (b) Shear stress v/s Strain

RESULTS:

Sl.No.	Mechanical Properties	Material
		Mild Steel
1.	Torque	
2.	Angle of twist	
3.	Shear stress	
4.	Shear strain	

EXPERIMENT - 6

DETERMINE THE YOUNG'S MODULUS, YIELD POINTS, PERCENTAGE OF ELONGATION AND PERCENTAGE OF REDUCTION IN AREA, STRESS STRAIN DIAGRAM FOR MILD STEEL.

AIM: Determine tensile strength of a given mild steel specimen using UTM.

APPARATUS:

1. Universal Testing Machine (UTM)
2. Mild steel specimens
3. Graph paper
4. Scale
5. Vernier Caliper

UNIVERSALTENSILE MACHINE

The tensile test is conducted on a UTM. It is hydraulically operated by a pump, oil in oil sump, load dial indicator and central buttons. The left part has upper, middle and lower cross heads i.e., specimen grips (or jaws). Idle cross head can be moved up and down for adjustment. The pipes connecting the left and right parts are oil pipes through which the pumped oil under pressure flows on left parts to move the cross-heads as shown in Fig. 6.11.

SPECIFICATIONS:

1. Load capacity = 40 Tonnes.
2. Least count = 8 kgf.
3. Power supply = 440 V

THEORY:

The tensile test is one of the most commonly applied mechanical test. In this test, the ends of test piece are fixed into grips connected to a straining device and a load measuring device. When the applied load is small enough, the deformation of any solid body is entirely elastic.

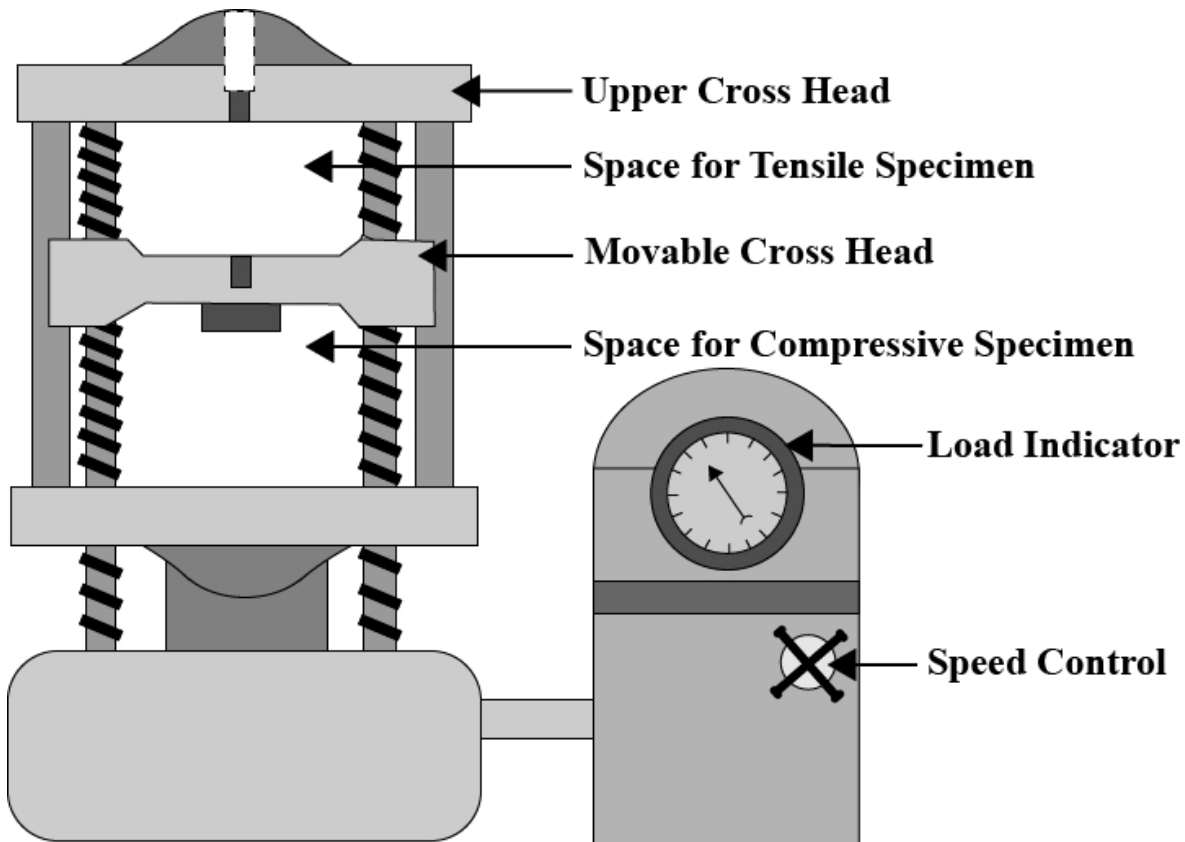


Fig. 6.11: Schematic diagram of Universal Testing Machine

An elastically deformed solid will return to its original form as soon as load is removed. However, if the load is too large, the material can be deformed permanently. The initial part of the tension curve, which is recoverable immediately after unloading is termed as elastic and the rest of the curve, which represents the manner in which solid undergoes permanent deformation is termed as plastic.

The stress below which the deformations are essentially entirely elastic is known as the yield strength of material. In some materials the onset of plastic deformation is denoted by a sudden drop in load indicating both an upper and a lower yield point. However, some materials do not exhibit a sharp yield point. During plastic deformation, at larger extensions strain hardening cannot compensate for the decrease in section and thus the load passes through a maximum and then begins to decrease. At this stage the “ultimate strength” which is defined as the ratio of the load on the specimen to original cross-sectional area, reaches a maximum value. Further loading will eventually cause ‘neck’ formation and rupture as shown in Fig. 6.12.

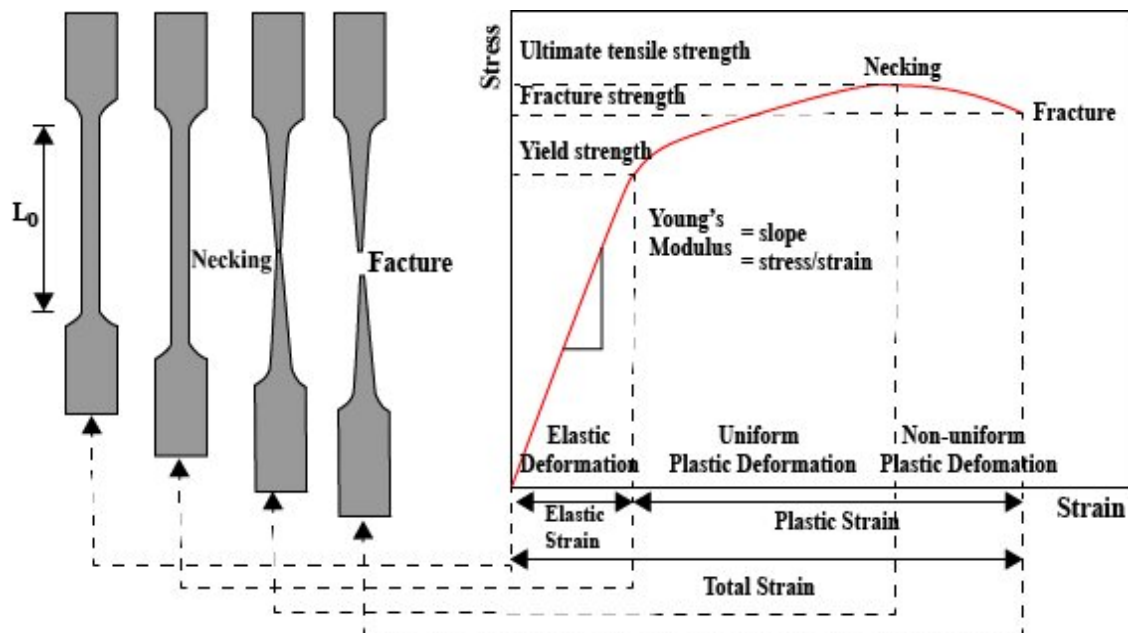


Fig. 6.12: Stress-strain curve for mild steel

PROCEDURE:

1. Measure the original length and diameter of the specimen. The length is the gauge length of the section which is marked on the specimen with a present punch or the total length of the specimen

2. Insert the specimen into grips of the test machine and attach strain-measuring device on it
3. Begin the load application and record load versus elongation data.
4. Take readings more frequently as the yield point is approached.
5. Measure elongation values with the help of dividers and a ruler.
6. Continue the test till fracture occurs.
7. By joining the two broken halves of the specimen together, measure the final length and diameter of the specimen.

OBSERVATIONS:

1. Initial diameter of specimen, $d_0 = \text{----- mm}$
2. Initial gauge length of specimen, $L_0 = \text{----- mm}$
3. Initial cross-section area of specimen, $A_0 = \frac{\pi}{4} d_0^2 = \text{----- mm}^2$
4. Final length after specimen breaking, $L = \text{----- mm}$
5. Diameter of specimen at breaking, $d = \text{----- mm}$
6. Cross section area at breaking, $A = \frac{\pi}{4} d^2 = \text{----- mm}^2$
7. Yield stress, $\sigma_y = \frac{P_y}{A_0} = \text{----- N/mm}^2$
8. Ultimate Stress, $\sigma = \frac{\text{Maximum Load}}{\text{Area}} = \frac{P}{A} = \text{----- N/mm}^2$
9. Strain, $\epsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\sigma_y}{L_0} = \text{-----}$
10. Strain energy density (Modulus of resilience), $U = \frac{\sigma_y^2}{2E} = \text{----- N/mm}$
11. Young's modulus, $E = \frac{\text{Stress}}{\text{Strain}} = \text{----- N/mm}^2$
12. Yield load, $P_{uy} = N$. $P_{ly} = \text{----- N}$
13. Maximum load, $P_{\max} = \text{----- N}$
14. Fracture load, $P_{\text{fail}} = \text{----- N}$
(From the slope of the linear portion of the stress-strain diagram)
15. Percentage elongation in length, $\frac{L-L_0}{L} \times 100 = \text{----- \%}$

16. Percentage reduction in area, $\frac{d_0^2 - d^2}{d_0^2} \times 100 = \text{-----} \%$

17. Ultimate stress, $\sigma_u = \frac{P_{\max}}{A_0} = \text{-----} \text{ N/mm}^2$

18. Fracture stress, $\sigma_f = \frac{P_{\text{fail}}}{A_0} = \text{-----} \text{ N/mm}^2$

19. Limit of Proportionality = $\frac{\text{Load at limit of proportionality}}{\text{Original cross sectional area}}$

Table: Tensile test results:

SI. No	Load P (kN)	Deflection (Δ) (mm)	Stress (N/mm ²) $\sigma = \frac{P}{A_0}$	Strain $\sigma = \frac{\epsilon}{L_0}$	True Stress (N/mm ²) $\sigma_{\text{true}} = \sigma(1 + \epsilon)$	True Strain $\delta = \ln(1 + \epsilon)$
1.						
2.						
3.						

PRECAUTIONS:

1. The specimen should be prepared with proper dimensions.
2. The specimen should be properly held between the jaws.
3. Take reading carefully.

GRAPH:

Draw graphs of (a) Load v/s Deflection (b) Stress v/s Strain (c) True stress v/s True strain

RESULTS:

Sl.No.	Mechanical Properties	Material
		Mild Steel
1.	Modulus of Elasticity	
2.	Yield Stress	
3.	Modulus of Elastic Resilience	
4.	Ultimate stress	
5.	Percentage elongation in length	
6.	Percentage reduction in area	

EXPERIMENT-7

DETERMINE THE MODULUS OF RIGIDITY, STRAIN ENERGY, SHEAR STRESS AND STIFFNESS BY LOAD DEFLECTION METHOD (OPEN & CLOSED COIL SPRING)

AIM: To find the stiffness of spring and the modulus of rigidity of the materials of the specimen.

APPARATUS: Spring testing machine, spring for tension or compression test, vernier caliper and balance.

DIAGRAM:

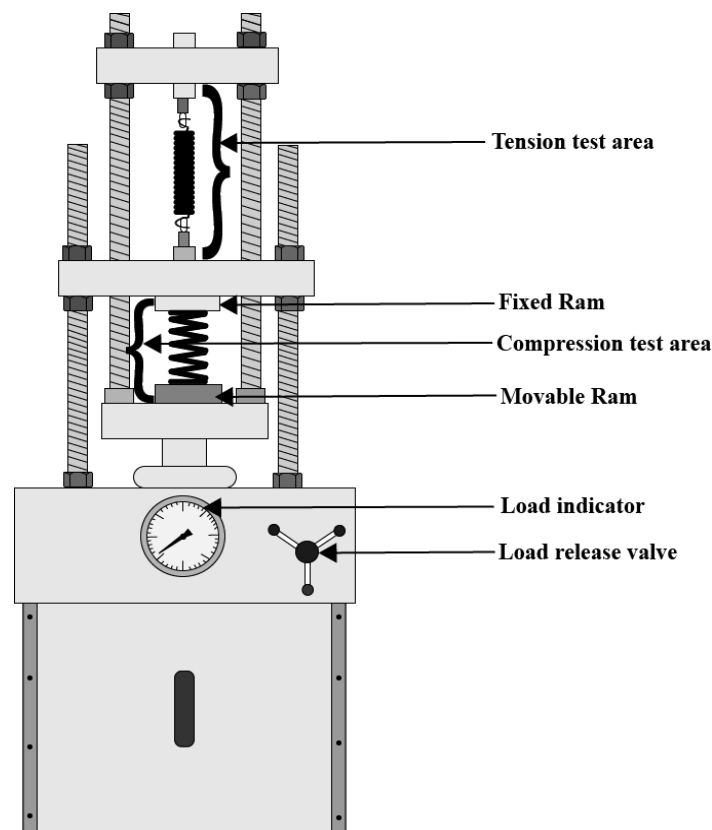


Fig. 6.13: Schematic of a spring testing machine

THEORY:

A helical spring is formed when a wire of solid circular cross section is wound on a circular core in a spiral form. Under the axial load the spring is subjected to both bending and twist. At any point in the coil, the tangent to helical centre line is not perpendicular to axial force. If 'w' is the force component parallel to the tangent at any point ($w \sin \alpha$) produces the bending moment M and the component perpendicular to the tangent ($w \cos \alpha$) produces a torque 't', where α is the angle of helix.

$$M = w R \sin \alpha \quad \text{.....(7.1)}$$

$$T = w R \cos \alpha \quad \text{.....(7.2)}$$

Where, R = Mean radius of coil

$$\text{Angle of helix, } \alpha = \tan^{-1} \left(\frac{\text{Pitch}}{2\pi R} \right) \quad \text{.....(7.3)}$$

Equating work done by the axial load to the total strain energy in bending and torsion we obtain,

$$\delta = \left(\frac{64 w R^3 n \sec \alpha}{d^4} \right) \times \left(\frac{\cos^2 \alpha}{N} + \frac{2 \sin^2 \alpha}{E} \right)$$

Where,

$$E = 2N \left(1 + \frac{1}{m} \right), \text{ Consider } \frac{1}{m} = 0.3$$

$$N = \left(\frac{64 w R^3 n \sec \alpha}{\delta d^4} \right) \times \left(\frac{\cos^2 \alpha + 2 \sin^2 \alpha}{2.6} \right) \quad \text{.....(7.4)}$$

When coils are wound very closely, angle of helix becomes very small. Hence effect of ' $w \sin \alpha$ ' may be neglected.

Open coil springs are those for which the effect of angle of helix cannot be neglected under the action of axial load, both the torsional and bending moments are to be considered. If the effect of helix is negligible, they are known as closed coil helical spring. Hence only the effect of torsion is considered.

$$\text{Shear stress max, } Z_{\max} = \frac{16 w R \cos \alpha}{\pi d^3} \quad \dots\dots (7.5)$$

Where, R = Mean radius of coil, d = Wire diameter, w = Axial load

HELICAL SPRING SETUP:

The spring testing machine consists of graduated scales with a sliding vernier fitted to a metallic angle frame. One end of the spring is passed through the hook of the sliding vernier and the other end is suspended from an adjustable hook fitted to the metallic frame. Another hook is provided at the bottom of the sliding arm on which a pan is attached and weighed.

OBSERVATION:

Open helical coil

1. Diameter of the wire, d = ----- mm
2. Mean radius of the coil, $R = \frac{D_1 + D_2}{4} = \text{----- mm}$
3. Outer diameter of coil $D_1 = \text{----- mm}$
4. Inner diameter of coil $D_2 = \text{----- mm}$
5. Number of turns, n
6. Length of wire, l = ----- mm
7. Pitch, $p = \left(\frac{l}{n}\right) = \text{-----}$
8. $\tan \alpha = \frac{p}{2\pi R} = \text{-----}$

9. $\alpha = \text{-----}$

Closed helical coil

1. Diameter of the wire, $d = \text{----- mm}$

2. Mean radius of the coil, $R = \frac{D_1 + D_2}{4} = \text{----- mm}$

3. Outer diameter of coil $D_1 = \text{----- mm}$

4. Inner diameter of coil $D_2 = \text{----- mm}$

5. Number of turns, n

6. Length of wire, $l = \text{----- mm}$

7. Pitch, $p = \left(\frac{l}{n}\right) = \text{-----}$

8. $\tan \alpha = \frac{p}{2\pi R} = \text{-----}$

9. $\alpha = \text{-----}$

Table: Load Vs deflection data of a spring due to applied load:

SI. No	Load in (N)	Compression scale reading			Load in (N)	Extension scale reading		
		Loading (mm)	Unloading (mm)	Mean deflectio n (mm)		Load increasin g (mm)	Load decreasing (mm)	Mean deflectio n (mm)
1.								
2.								
3.								
4.								

PROCEDURE:

1. Suspend the spring between sliding vernier and adjustable hook.

2. Adjust the top hook so that the zero of the vernier coincides exactly with the zero of the graduated scale.
3. Suspend the pan of known weight from bottom hook of sliding vernier and note the deflection.
4. Gradually add weights to the pan and note the corresponding deflection. Note the deflection on removing the weights one by one. Find the mean deflection.
5. Draw a curve for load vs deflection. A straight-line curve proves that deflection is proportional to the load.
6. The procedure is repeated for tension test and compression test.
7. Measure the diameter 'd' of the wire, internal and external diameter 'D₁' and 'D₂' of the coil, number of turns of the spring, length of the spring etc.
8. 'w' and 'δ' are obtained from the graph.

TABULAR COLUMN:

Sl. No	Open coil (Compression)				Closed coil (Tension)			
	Load (N)	Shear stress $I = \frac{16 \times w \times R}{(\pi d^3)}$ (kg/cm ²)	Stiffness $y = \frac{w}{\delta}$ (kg/cm)	Strain energy $u = \frac{w\delta}{2}$ (kgcm ² /s ²)	Load (N)	Shear stress $I = \frac{16 \times w \times R}{(\pi d^3)}$ (kg/cm ²)	Stiffness $y = \frac{w}{\delta}$ (kg/cm)	Strain energy $u = \frac{w\delta}{2}$ (kgcm ² /s)
1.								
2.								

1. Slope of load vs deflection graph (Open coil) = ----- kg/mm
2. Stiffness of spring (Compression) = ----- N/mm

3. Slope of load vs deflection graph (Closed coil) = ----- kg/mm
4. Stiffness of spring (Tension) = ----- N/mm

RESULT:

1. Modulus of rigidity of the material of spring under tension =
2. Stiffness of spring under tension =
3. Modulus of rigidity of the material of spring under compression =
4. Stiffness of spring under compression =

EXPERIMENT-8

SINGLE OR DOUBLE SHEAR TEST ON MILD STEEL BAR IN FINDING THE RESISTANCE OF MATERIAL TO SHEAR LOAD

AIM: To determine experimentally, the ultimate shear strength in single shear and double shear of mild steel rod.

APPARATUS: UTM, shear test attachment, vernier caliper / screw gauge.

THEORY:

Shear stress refers to the type of stress within a body or part that is more likely to cause sliding or shearing rather than stretching or bending.

A shearing stress acts parallel to a plane whereas tensile and compressive stresses act normal to a plane. There are two main types of shear stresses used in laboratory tests. One is called direct or transverse shear stress and corresponds to the type of stress encountered in rivets, bolts, and beams. The other type of shear stress is called pure or torsion shear and represents the kind of shear stress encountered in a shaft subjected to pure torsion.

In actual practice when a beam is loaded the shear force at a section always comes to play along with the bending moment. It has been observed that the effect of shear stress as compared to bending stress is quite negligible. But sometimes, the shearing stress at a section is considered to be much more important in design calculations.

This test is useful for determining ultimate shear strength of most metals as well as other rigid plastic materials where the shear characteristics finds importance in characterizing structural integrity of bond in the form of weld, adhesive bond or friction joint delivered between two surfaces.

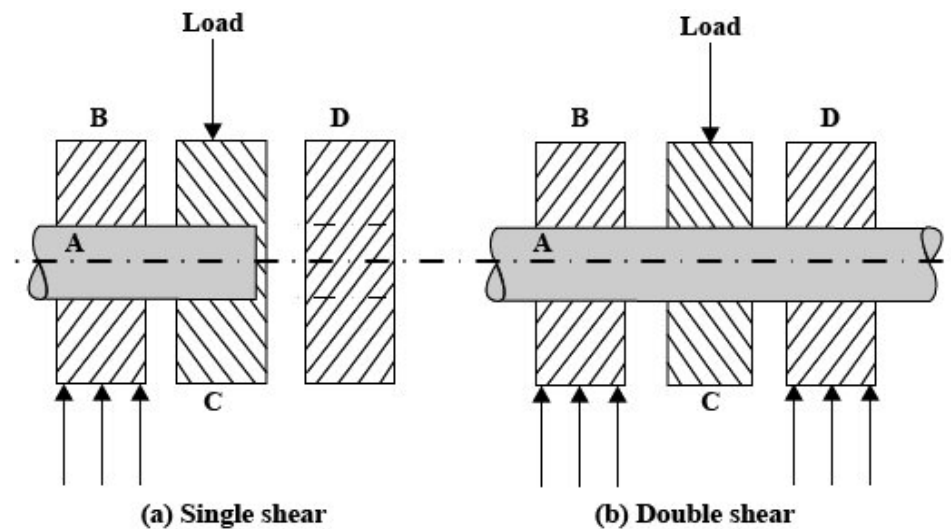


Fig. 6.14 Specimen position in shear attachment for single and double shear

PROCEDURE:

1. Measure the diameter of the test specimen by means of a screw gauge at least at three sections and obtain the mean value. Also measure the length of the specimen.
2. Fix the specimen in the shear attachment accordingly for single and double shear as shown in Fig. 6.14.
3. The shear assembly is placed in UTM, and the load is applied, at right angle to the axis of the specimen, through the central block, and the load at shear is recorded.
4. The shape and texture of fractured surface is observed.

OBSERVATION:

1. Material of the specimen -----
2. Average diameter, d = ----- mm

3. Length, $L = \text{----- mm}$

4. Single shear, $\tau = \frac{P}{A} = \text{----- N/mm}^2$

5. Double shear, $\tau = \frac{P}{2A} = \text{----- N/mm}^2$

Table: Single and double shear test results:

Sl. No	Diameter (mm)	Area of cross section $\left(A = \frac{\pi d^2}{4}\right) \text{ mm}^2$	Failure load P (N)	Single shear strength (N/mm ²)	Double shear strength (N/mm ²)
1.					
2.					
3.					

RESULT:

Sl.No	Type of shear	Material	Failure load P (N)	Shear strength (N/mm ²)
1.	Single shear	Mild steel		
2.	Double shear	Mild steel		

The Shear strength of mild steel specimen is found to be ----- N/mm²

KNOW MORE

- **Visual Inspection, Ring Test, Dye Penetration Test, Magnetic Particle Test for Crack Detection:**

Structural Integrity Assurance: These tests are crucial in industries like manufacturing, aerospace, and construction to ensure the safety and reliability of components subjected to stress and fatigue.

- **Rockwell Hardness Testing for Various Materials:**

Quality Control in Manufacturing: Determining hardness helps assess material suitability for specific applications, aiding in the selection of appropriate materials during manufacturing processes.

- **Impact Testing (Izod and Charpy Tests):**

Material Selection for Safety Critical Applications: These tests provide insights into a material's resistance to sudden impact or shock, guiding material selection for applications where safety is paramount, such as automotive crash testing or structural engineering.

- **Torsion Test on Mild Steel:**

Design Optimization in Engineering: Understanding the relationship between torque and angle of twist aids in designing components for optimal performance, especially in applications where torsional forces are prevalent, like shafts in machinery.

- **Young's Modulus, Yield Points, Stress-Strain Diagram for Mild Steel:**

Material Characterization: These tests provide essential data for engineers to understand the mechanical properties of materials, enabling the design of structures and components with predictable performance under varying loads.

- **Determination of Modulus of Rigidity by Load Deflection Method:**

Spring Design and Optimization: Knowing the modulus of rigidity helps in designing and selecting appropriate springs for various applications, ensuring that they can withstand required loads while maintaining their desired characteristics.

- **Shear Test on MS Bar:**

Structural Component Design: Understanding the materials resistance to shear loads is vital in designing structural components like bolts, rivets, and connectors, ensuring they can withstand shear forces without failure.

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7. Rattan S.S., Strength of Materials, McGraw Hill Education; New Delhi.
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Dynamic QR Code for Further Reading

Microstructure Analysis of Ferrous
and Non Ferrous Metals



Magnetic Particle Testing



Rockwell Hardness Test



Charpy Impact Test



Izod Impact Test



Torsion Test



Universal Testing Machine



Single and Double Shear Test



Open and Closed Coil Test



APPENDICES

APPENDIX-A: Suggestive Template for Practicals

- **Aim**
Explain verify about the aim of the experiment.
- **Relevance**
Explain about the relevancy of the experiment in your own words.
- **Requirements**
List out all the required apparatus along with their proper specifications.
- **Procedure, Observations and Inference**
Explain the procedure of the experiment step-wise and note down the observations properly.
On the basis of observations certain inference is to be made. You can use a table similar to that given below:

Step No.	Procedure	Observation	Inference
1			
2			

- **Video / animation**
If possible, you can go through some video/animation to visualize the steps physically.
- **Calculations**
Properly calculate all the required physical quantities essential for your experiment.
- **Result and Discussion (Error measurement)**
Obtain the final result and discuss about it with proper considerations of errors which can be introduced during your experiment.
- **Conclusions**
Finally give your conclusion based on the obtained results.
- **Validation of the topics in Experiment**
Try to validate the result of the experiment in real life scenario.
- **Use of ICT**
You can also study using the available online resources. These are useful as there is no time constraint at all. Some of which are listed (not limited to) below:

<http://swayam.gov.in/>

<https://nptel.ac.in/>

APPENDIX-B: Indicative Evaluation Guidelines for Practicals/Projects/Activities in Group

Process Related Skills

Criteria and Level	Developing	Competent	Proficient
Handling the Set-up			
Recording of Data			
Time Management			
Team Work			
Individual Work			
Safety Precautions			

Process Related Skills

Criteria and Level	Developing	Competent	Proficient
Content			
Research/Survey			
Use of latest Technology			
Stays on Topic			
Preparedness			
Confidence of Presentation			
ICT Usage including PPT Making Skill			
Time Management			
Group Efforts			
Individual Efforts			

APPENDIX-C: Assessments Aligned to Bloom's Level

- Bloom's Taxonomy – It has been coupled into following two categories for Development of Questions for this Book as given below:

Category I Questions	Category II Questions <i>Higher Order Thinking Skills</i>
Bloom's Level 1: Remember	Bloom's Level 4: Analyse
Bloom's Level 2: Understand	Bloom's Level 5: Evaluate
Bloom's Level 3: Apply	Bloom's Level 6: Create

APPENDIX-D: Records for Practical

S. No	Exp. No.	Name of the Experiment	Date			Marks	Signature
			Actual	Repeat	Remarks		
1.							
2.							
3.							
4.							
5.							
6.							
7.							

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1. Bedi D.S., Strength of Materials, Khanna Publishing House, Delhi, Ed. 2018
2. Timoshenko, S., Strength of Materials, Vol. I, CBS, New Delhi.
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11. A Text Book strength of Material– R.K. Bansal, Laxmi Publication New Delhi

CO AND PO ATTAINMENT TABLE

Course outcomes (COs) for this course can be mapped with the programme outcomes (POs) after the completion of the course and a correlation can be made for the attainment of POs to analyze the gap. After proper analysis of the gap in the attainment of POs necessary measures can be taken to overcome the gaps.

Table for CO and PO attainment

Course Outcomes	Expected Mapping with Programme Outcomes (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)						
	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO-1							
CO-2							
CO-3							
CO-4							
CO-5							

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Mechanics of Materials and their Testing

R. Velmurugan, P. Pragathi

The textbook titled “Mechanics of Materials and their Testing” holds significant importance for Mechanical Engineering students across various technical boards and universities. The book is highly beneficial for diploma students engaged in the designing of structures and machinery component parts. The authors have crafted the content in a simple language, facilitating easy comprehension of complex concepts. Whether you are a student embarking on your academic journey or an instructor guiding aspiring minds this book aims to be your reliable companion.

Salient Features:

- Content of the book aligned with the mapping of course outcomes, and program outcomes.
- In beginning of each unit learning objective are listed to make the student understand what is expected out of him/her after completing that unit.
- Figures, tables and screen shots are inserted to improve clarity of the topics.
- Numerical problems with answers are given for practice of students after every chapter.
- Solved and unsolved problems including numerical examples are solved with systematic steps.

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